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NORMAL

ARITHMETIC:

A

TEXT-BOOK,

THEORETICAL AND PRACTICAL,

IN SIX PARTS, VIZ:

PART I. DECIMAL NUMBERS.
PART II. DENOMINATE NUMBERS.
PART III. PER CENTAGE.
PART IV. PROPORTION.
PART V. POWERS, ROOTS AND SERIES.
PART VI. MISCELLANEOUS SUBJECTS.

BY

SILAS L. LOOMIS, A.M., M.D.

PHILADELPHIA:
J. B. LIPPINCOTT & CO.
1859.

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PREFACE.

THE following work has been mainly suggested by the experience of twenty years as a mathematical instructor. Its aim is, to present the subject of Arithmetic in a plain and concise manner.

The end of Arithmetic is not merely to teach rules and work examples, but to give the mind capability, to fit it for research; hence, to multiply rules till every species of problem has its own specific rule, is to defeat the possibility of a good arithmetician. It leaves the student nothing more than to commit the rules and work the examples. He becomes a book student, and a book student only, and, after having passed brilliantly through his Arithmetic, may find himself utterly unable to calculate the worth of the first load of wood brought to his door. What is needed is less book and more thought, less rule and more judgment. The student must be taught to think for himself. When a mathematical principle is stated, it is a thousand-fold better for him to make the application by his own common sense than to be guided by either teacher or rule. To master a problem unaided is victory; it gives new courage and conscious ability. There is moral power in "I did it without aid."

It has been the purpose of the author to draw as much as possible from the pupil, and, induce that accuracy which will give him confidence in his own abilities and computations.

The method of classification and arrangement has been most successful with the author in his own school room, and, it is believed, will commend itself to every teacher of mathematics.

The more prominent features of the work are embraced in the following points, viz:

1st. The Analysis of Arithmetic on page 11, and that of each Part, on pages 12, 114, 144, 196, 227, and 268. These will be found to aid both teacher and pupil, and should be carefully examined.

2d. The omission of all that may be readily understood, thereby materially condensing the whole work.

3d. The generalization of rules. Thus, the Rule of Addition is first applied to abstract numbers, and to this extent it must be understood. It is not necessary that the pupil understand all its various applications before advancing; it is sufficient that he understand its application to the particular subject in hand. As he advances the pupil will perceive its application to Decimal Fractions, afterward to Denominate Numbers, and thus finally comprehend it in its fullest extent. The rule then becomes to him a *General Rule for Addition.*

4th. The general use of Decimal Fractions, especially in Denominate Numbers and Interest. This will be appreciated as the pupil advances.

5th. The method of securing accuracy, by both scholar and teacher, without the disadvantages of a key, or of the answers in the body of the work or at the end of the book.

6th. The review at the end of each Part.

7th. The general principles and classification of Fractions.

8th. The exposition of the entire subject of Denominate Numbers, giving a complete view of the origin of the Units of Measure, and their dependence upon one another; also the U. S. Standards of Weights and Measures.

9th. The classification of subjects under Per Centage, surmounting the less before the greater difficulty.

And 10th. The addition of new rules and subjects.

Besides these, there are many minor points which will aid in elevating the standard of qualification among teachers, improve their methods of imparting instruction, and assist the pupil in gaining a thorough, practical knowledge of the principles of Arithmetic.

SILAS L. LOOMIS.

WASHINGTON, D. C., Nov. 1st, 1858.

INDEX.

A. PAGE

Above Par ... 146
Abstract Unit ... 20, 118
Abstract Numbers ... 118
Accents ... 91
Acceptances ... 270
Accurate Interest ... 182
Addition ... 22
Addition of Decimals ... 80
Addition of Duodecimals ... 90
Addition of Fractions ... 102
Addition of Denominate Numbers. 134
Ad Valorem Duty ... 154, 155
Aliquot Parts ... 43
Alligation Alternate ... 213
Alligation Medial ... 213
Amount ... 22, 163
Analysis ... 288
Angular Measures ... 122
Annuities ... 257
Annuity Certain ... 257
Annuity Contingent ... 257
Annuity in Possession ... 257
Annuity in Reversion ... 257
Annuity in Arrears ... 257
Answer ... 20
Antecedent ... 197
Antiquarian Paper ... 126
Apothecaries' Measures ... 123
Arabic Method ... 14
Arabic Notation ... 16
Arc ... 124
Area of N. E. States ... 40
Area of Triangle ... 237
Arithmetic ... 13
Arithmetical Progression ... 252
Army and Navy of Great Britain.. 26
Ascent of Bodies ... 278
Assessment of Taxes ... 156
Astronomical Questions ... 281

PAGE

Atlas Paper ... 126
Average of Accounts ... 218
Average Date ... 217
Average Due ... 218
Average Storage ... 219
Avoirdupois Measures ... 123
Axiom ... 20

B.

Bank Interest ... 172
Bankable Notes ... 273
Barleycorns ... 125
Base ... 237
Bill of Exchange ... 274
Below Par ... 146
Broker ... 146
Brokerage ... 146
Building Associations ... 259
Bullion in India, 1850 ... 112
Bushels in U. S. ... 128

C.

Cæsar ... 69
Cancelling ... 66
Cap Paper ... 126
Cask ... 249
Chain Rule ... 205
Chaldron ... 126
Change Improper Fractions ... 97
Change a Whole or Mixed Number ... 97
Change Common Fractions to Decimals ... 98
Change Common Fractions to Duodecimals ... 98
Change Fractions to Common Denominators ... 99
Change to L. C. Denominator ... 100

PAGE
Change to Common Numerator 101
Change to a given Numerator or Denominator 101
Change a Fraction to lower terms 96
Change in form of Common Fractions 95
Change Duodecimals to Common Fractions 94
Change Duodecimals to Decimals 94
Change Decimals to Common Fractions 87
Change Decimals to Duodecimals 86
Chronometer U. S. A. 82
Churches N. England, 1852 27
Ciphers 16
Ciphering 16
Circles 118, 238
Circular Measures 122
Classification of Denominate Numbers 118
Classification of Fractions 75
Classification of Common Fractions 95
Cloth Measures 122
Coal, Soft (Charcoal) 126
Coal, Hard (Anthracite) 126
Coffee Imported, 1852 26
Columbia Paper 126
Commercial Par 274
Commission 146
Commission Merchants 146
Common Divisors 58
Common Fractions 95
Common Multiple 61
Common Multiple of Fractions 106
Common Ratio 255
Complete Division 70
Complex Fractions 95
Composite Numbers 53
Compound Fellowship 208
Compound Fractions 95, 103
Compound Interest 183
Compound Proportion 200
Compound Repetend 283
Concrete 20
Cones 239, 248
Conjoined Proportion 205
Consequents 197
Conterminous Repetend 283
Contributory Interests 210
Corporation 146
Cotton imported, 1852 26
Couplets 199
Course of Exchange 274
Cube 248

PAGE
Cube Root 242
Cubic Foot of Water 120
Cubits 125
Custom House Business 154
Cylinder 239, 248

D.

Day of the month 125
Day of the week 281
Days of Grace 172
Decimal Fractions 75, 76
Decimal Numbers 13, 17
Decimal Point 77, 78
Decrease of Numerator 72
Demy Paper 125
Denominate Numbers 115
Denomination 16
Denominator 71
Denominator of Decimals 76, 89
Difference 36
Digits 16
Diplomatic Corps, 1857 111
Discount 185
Dissimilar Repetend 283
Divide by any power of ten 49
Divide by aliquot parts of ten or any power of ten 49
Divide by Fractions 50
Divide by Cancelling 51
Divide by Arrangement 51
Divide by Mental Process 52
Dividend 37
Dividends 146
Divisibility of Numbers 53, 54, 55
Division 37, 49
Division, Complete 70
Division, Exact 70
Division, Incomplete 70
Division of Decimals 84
Division of Duodecimals 93
Division of Fractions 105
Division of Denominate Numbers 138
Divisor 37
Divisors 58
Divisors and Dividends 106
Domestic Exchange 269
Dominical Letter 281
Double Elephant Paper 126
Double Fellowship 208
Draft 154
Drawee 274
Dry Measures 123
Duodecimal 75, 90
Duties 154

E. PAGE

Eagle 121
Earth's Diameter 79
Elephant 126
Ellipse 239
Ells, English, French, and Scotch. 125
Empereal Paper 126
Endorsement 272
Engineer's Chain 125
English Standard Yard 116
English Money 277
English Method 18
English Numeration Table 19
Equality 21
Equation of Accounts 217
Equation of Payments 217
Equi-different Series 252
Equi-distant Numbers 43
Eratosthenes' Sieve 55
Even Number 54
Evolution 230
Exact Division 70
Example 20
Exchange 269
Exponent 228
Extremes 199, 252, 255

F.

Factor 146
Factors 28
Falling Bodies 278
Fathom 125
Fellowship 207
Finite Decimals 76
Fire Insurance 151
Foreign Coins 277
Foreign Exchange 274
Formulas for Interest 163
Fractions 70
Fractional Unit 70
Franc 124
French Ells 125
French Method 18
French Metre 119, 124
French Numeration Table 18
French System of Weights and Measures 124
French System of Denominate Numbers 124

G.

Gauging 249
General Average 210
General Principles of Fractions 70
Geometrical Progression 255
Gold Dollar 121
Grade 124
Gramme 124
Greatest Common Divisor 58, 59, 60
Greatest Common Measure 58, 59, 60
Gregorian Year 79
Gross Weight 156
Gunter's Chain 83, 125

H.

Hand 125
Hide 126
Height of Animals 125
Hungarian Mile 81
Hypothenuse 237

I.

Imperial Paper 126
Improper Fraction 95
Inclined Plane 279
Incomplete Division 70
Increase of the Numerator 71
Index 229
Infinite Decimals 76, 87, 283
Inscribed Square 238
Insurance 151
Integral Number 53
Integral Divisors 58
Interest 163
Interest at any per cent 174
Interest, Compound 182
Intrinsic Par 274
Involution 228
Irish Mile 81

J.

Jettison 210
Joint Note 273
Joint and Several Note 273

L.

Land Measures 122
Laws of English Notation 19
Laws of French Notation 18
Laws of Notation 17
Leagues 125
Leakage 154
Least Common Multiple 61
Legal Interest 163
Length of a Degree of a Circle 79

PAGE

Length of a Degree of Latitude.... 79
Length of a Degree of Longitude at the Equator, and at 40° Latitude........ 79
Lever........ 278
Life Insurance........ 151
Linear Units........ 118, 120
Linear Measures........ 122, 125
Lines........ 118, 125
Litre........ 124
Liquid Measures........ 123
Loan and Fund Associations........ 259

M.

Marine Insurance........ 151
Marine Mile........ 125
Means........ 199, 252, 255
Mean Number........ 43
Measures........ 58
Measures of Time........ 119
Measures of Length........ 122, 125
Measures of Surface........ 122, 126
Measures of Volume........ 123, 126
Measures of Money........ 123
Medial Proportion........ 213
Medium Paper........ 126
Mensuration of Surfaces........ 237
Mensuration of Solids........ 248
Merchants' Discount........ 185
Metre........ 124
Mile, Statute........ 125
Mile, Nautical........ 125
Mile, Nautical U. S........ 125
Minus........ 21
Minuend........ 36
Miscellaneous Tables........ 125
Mixed Fractions........ 95
Mixed Repetend........ 283
Money Measures........ 123
Money Units........ 118, 121
Multecimal Fractions........ 76
Multiple........ 61
Multiplicand........ 28
Multiplication........ 28
Multiplier........ 28
Multiplication of Decimals...... 81, 88
Multiplication of Denominate Numbers........ 136
Multiplication of Duodecimals...... 91
Multiplication of Fractions........ 103
Multiply by ten or any power of ten........ 43
Multiply the aliquot parts of ten or any power of ten........ 44

PAGE

Multiply by Factors........ 45, 46
Multiply by Mean and Equi-distant Numbers........ 47
Multiply without partial Products. 48

N.

Names of the periods........ 18
Nautical Mile........ 145
Negotiable Notes........ 271
Net Weight........ 154
New Cent........ 121
New Par........ 275
New York Gallon........ 117
Notation........ 14
Notation of Decimals........ 76
Notation of Duodecimals........ 90
Notation of Denominate Numbers........ 115, 122
Notes........ 270
Notes on Time........ 271
Number........ 13
Number is written........ 19
Number is read........ 19
Numbers below Unity........ 76
Numeration........ 18
Numeration Decimals........ 78
Numeration Duodecimals........ 90
Numerator........ 70

O.

Odd Numbers........ 54
Old Cent........ 121
Old Par........ 274
Operation........ 20
Orders........ 269
Origin of Denominate Numbers... 115
Overlay........ 156

P.

Palm........ 125
Paper........ 126
Parallelogram........ 237
Parallelopipedon........ 248
Partial Payments........ 176
Partitive Proportion........ 207
Partnership........ 207
Par Value of Stocks........ 146
Payee........ 274
Pendulum........ 119, 120, 125
Pennsylvania Schools, 1848........ 112
Per Centage........ 145, 163
Perpendicular........ 237

PAGE

Perch, Stone 126
Perpetuity 257
Pi, π 79, 238
Philosophical Questions 278
Plus 21
Points 125
Poll 156
Poll Tax 156
Policy 151
Polyhedron 239
Polygon 238
Population of Great Britain, 1851. 26
Population of Middle States, 1850. 25
Population of N. E. States, 1850... 27
Population of United States, 1850. 68
Power 228
Powers of a Number 43
Premium 151
Presidents of U. S. 68
Pressure of Liquids 279
Primary Rules 13
Primes, Seconds, &c. 90
Prime Numbers 34, 53
Prime Factor 53, 54, 55, 56
Principle 163
Prism 248
Problem 20
Problems in Interest 175
Product 28
Profit and Loss 160
Promissory Note 270
Proof 22
Property Tax 156
Proper Fractions 95
Properties of Fractions 71, 72, 73
Properties of Numbers 53, 54
Proportion 199
Proportional 199
Pulley 278
Pure Repetend 283
Pyramid 239, 248

Q.

Quadrangle 237
Quantity 20
Quarters 125
Question 20, 200
Quotient 37

R.

Ratio 197
Rate per cent 163
Rate of Insurance 152

PAGE

Rate, legal 163
Rectangle 237
Real Estate 156
Remainder 37
Remitter 274
Repetend 283
Rhomboid 237
Rhombus 237
Roman Notation 14
Root 228
Royal Paper 126
Rule 20

S.

School Rates 159
Screw 279
Sea Measures 122
Series 252
Short Division 37
Short Method of Interest 166
Silver Dollar 121
Signs 21
Significant Figures 16
Similar Repetend 283
Simple Fractions 95
Single Ratio 199
Single Repetend 283
Solid Measures 123
Solar Year 82
Span 125
Spheres 239, 248
Spheroid 249
Specific Duty 154, 155
Square 126, 237
Square Measures 122
Square Root 28, 230
Square of π 79
Square Root of π 79
Square Number 28
Standard Gold 121
Standard Silver 121
Standard Copper 121
Statement 200
Statute Mile 125
Stere 124
Stocks 146
Stockholders 146
Strength of Ropes 278
Subtrahend 36
Subtraction 36
Subtraction of Decimals 83
Subtraction of Denominate Numbers 137
Subtraction of Duodecimals 92

PAGE
Subtraction of Fractions.............. 104
Sum 22
Superficial Measures............. 122, 126
Superficial Units................. 118, 120
Super Royal Paper...................... 126
Surfaces................................... 118
Surveyors' Measures 122

T.

Table of Powers.......................... 229
Tare 154
Tax.. 156
Teas imported, 1852.................... 26
Terms.. 22
Term of Demand......................... 200
Theorem Paper........................... 126
Time.. 163
Time Measures.................... 122, 125
Tonnage............................ 156, 249
Ton, Coals.................................. 126
Ton, Hay, Sand, Timber.............. 126
Trapezoid................................... 238
Triangle..................................... 237
Troy Measures............................ 123
Troy Pound................................. 67

U.

Uncle Sam Paper 126
Underwriters............................... 151
Unit.. 13
Unit, Abstract.............................. 20
Unit, Concrete............................. 20
Units of Circular Measure........... 118

PAGE
Units of Denominate Numbers..... 118
Units of the Fraction.................. 70
Units of Surface.......................... 120
Units of Volume.................. 118, 120
Units of Time 118, 119
U. S. Coin................................... 121
U. S. Gallon................................ 117
U. S. System of Denominate Numbers.................................. 122

V.

Value of a Fraction..................... 71
Velocity of Sound....................... 278
Vertical Streams......................... 278
Volume Measures................ 123, 126
Volume Units...................... 118, 120

W.

Wedge 248, 278
Weight Units...................... 118, 120
Wheel... 279
Wood Measures.......................... 123
Writing a Number....................... 17

Y.

Year, Civil.................................. 125
Year, Gregorian 79
Year, Leap.................................. 125
Year at Mercury.......................... 82
Year, Sidereal 125
Year, Solar.......................... 82, 125

	PARTS.	DIVISIONS.	CHAPTERS.	PAGE
Normal Arithmetic. Analysis.	I. DECIMAL NUMBERS.	I. WHOLE NUMBERS	1. Primary Rules	18
			2. Special Rules	43
			3. Properties of Numbers	58
		II. FRACTIONS	1. Decimal Fractions	76
			2. Duodecimal Fractions	90
			3. Common Fractions	95
	II. DENOMINATE NUMBERS		1. Notation	115
			2. Primary Rules	130
	III. PER CENTAGE	I. EQUAL TIME	1. Commission and Brokerage	146
			2. Insurance	151
			3. Custom House Business	154
			4. Assessment of Taxes	156
			5. Profit and Loss	160
		II. UNEQUAL TIME	1. Interest	168
			2. Discount	185
	IV. PROPORTION		1. Ratio	197
			2. Proportion	199
			3. Partitive Proportion	207
			4. Medial Proportion	213
			5. Equated Proportiou	217
	V. POWERS, ROOTS, AND SERIES	I. POWERS AND ROOTS	1. Involution	228
			2. Evolution	230
		II. SERIES	1. Arithmetical Progression	252
			2. Geometrical Progression	255
			3. Annuities	257
			4. Loan and Building Associations	259
	VI. MISCELLANEOUS SUBJECTS		1. Exchange	269
			2. Philosophical and Astronomical Questions	278
			3. Infinite Decimals	283
			4. Analysis	288
			5. Practice	290

Analysis of Part I. Decimal Numbers.

DIVISIONS.	CHAPTERS.	SECTIONS.	PAGE
I. WHOLE NUMBERS	I. PRIMARY RULES	1. Notation	14
		2. Numeration	18
		3. Addition	22
		4. Multiplication	28
		5. Subtraction	35
		6. Division	37
	II. SPECIAL RULES	1. Multiplication	43
		2. Division	49
	III. PROPERTIES OF NUMBERS	1. Divisibility of Numbers	53
		2. Measures or Divisors	58
		3. Multiples or Dividends	61
		4. Problems	64
		5. Cancellation	66
II. FRACTIONS	I. DECIMAL	1. Notation and Numeration	76
		2. Addition	80
		3. Multiplication	81
		4. Subtraction	83
		5. Division	84
		6. Change of Form	86
		7. Infinite Decimals	87
		8. Special Rules	88
	II. DUODECIMAL	1. Notation and Numeration	90
		2. Addition	90
		3. Multiplication	91
		4. Subtraction	92
		5. Division	93
		6. Change of Form	94
	III. COMMON	1. Classification	95
		2. Change in Form	95
		3. Addition	102
		4. Multiplication	103
		5. Subtraction	104
		6. Division	105
		7. Divisors and Multiples	106
REVIEW OF PART I.			107

ARITHMETIC.

PART I.

DECIMAL SYSTEM OF NUMBERS.

DIVISION I.

WHOLE NUMBERS.

CHAPTER I.

PRIMARY RULES.

ARTICLE 1. A *unit* is a single thing.

2. A *number* is a *unit* or a collection of units.

3. *Arithmetic* is that part of the science of mathematics which treats of the nature and properties of numbers.

Its application is called an art.

All arithmetical calculations are dependent on the following simple fundamental operations; viz., Notation, Numeration, Addition, Multiplication, Subtraction and Division

---

Of what does Part 1st treat? Division 1st? Chapter 1st? What are Primary Rules? 1. What is a unit? 2. A number? 3. Arithmetic? When is it called an art? Upon what do arithmetical calculations depend?

SECTION I.

NOTATION.

4. Numbers are generally expressed in three ways, viz.:

1. By language, called the Written Method.
2. By letters, called the Roman Method.
3. By figures, called the Arabic Method.

5. *Notation* is the art of recording numbers by any of the above methods, or by other symbols.

6. The written method is seldom used for numbers above ten.

§ 1. *Roman Notation.*

7. The Roman method of notation expresses numbers by the seven following letters, viz., I, V, X, L, C, D, M, and their combinations.

TABLE.

I	denotes	one.	XL	denotes	forty.
II	"	two.	L	"	fifty.
III	"	three.	LX	"	sixty.
IV	"	four.	LXX	"	seventy.
V	"	five.	LXXX	"	eighty.
VI	"	six.	XC	"	ninety.
VII	"	seven.	C	"	one hundred.
VIII	"	eight.	CI	"	one hundred and one.
IX	"	nine.	CX	"	one hundred and ten.
X	"	ten.	CL	"	one hundred and fifty.
XI	"	eleven.	CC	"	two hundred.
XII	"	twelve.	CCC	"	three hundred.
XIII	"	thirteen.	CD	"	four hundred.
XIV	"	fourteen.	D	"	five hundred.
XV	"	fifteen.	DC	"	six hundred.
XVI	"	sixteen.	CM	"	nine hundred.
XVII	"	seventeen.	M	"	one thousand.
XVIII	"	eighteen.	MD	"	fifteen hundred.
XIX	"	nineteen.	MDLXVI	"	fifteen hundred and sixty-six.
XX	"	twenty.	$\overline{\mathrm{MD}}$	"	fifteen hundred thousand.
XXI	"	twenty-one.	$\overline{\mathrm{M}}\overline{\mathrm{D}}\mathrm{LXVI}$	"	1500066.
XXX	"	thirty.	$\overline{\mathrm{MDLXVII}}$	"	1567000.

Of what does Section 1st treat? 4. In how many ways are numbers expressed? 5. What is Notation? 6. Why is the written method seldom used? 7. What is the Roman method of expressing numbers? What is the value of each letter by itself?

8. Repeating a letter repeats its value; thus, II. denotes two.

9. A letter of less value standing before one of greater value, takes its value from the greater; but standing after it, adds its value to the greater; thus, IV. four; VI. six.

10. A line over a letter or several letters, increases the value one thousand times; thus, V. five, $\overline{\mathrm{V}}$. five thousand. XL. forty. $\overline{\mathrm{XL}}$. forty thousand.

11. This method, formerly used by the Romans for general computation, is now used to denote the divisions of parts, chapters, sections, &c., of discourses and books, numbers on dial plates, &c.

EXAMPLES.

1. Express fifteen by the Roman method.
2. Twenty-one. Forty-one. Seventy-one.
3. Thirty-seven. Ninety-seven. One hundred and seven.
4. Eighty-six. Forty-three.
5. Two hundred eighty. Three hundred seventy-six.
6. Five hundred thirty-six. Eighty-seven.
7. What does XIX denote? LXI? LXL? XC?
8. DC? CDC? DCD? $\overline{\mathrm{DM}}$?
9. XLVIII? $\overline{\mathrm{X}}$LV? $\underline{\mathrm{XXIX}}$? $\overline{\mathrm{LI}}$XV? LXIV?
10. $\overline{\mathrm{M}}$DCCDLVIII? $\overline{\mathrm{MDC}}$CCLVIII?
11. Express forty in two ways.
12. Express fifty-seven in two ways.
13. How many ways can you express five hundred?
14. Express seventy-six in the usual form.
15. Express fourteen hundred ninety-two.

---

8. How is the value of a letter affected by repetition? 9. How is the value determined when two or more letters are combined? 10. What effect will a line, when placed over the letter, have on its value? 11. Where was this method in general use? For what is it now used? Are large numbers or small ones now expressed by it? Can you assign a reason why its use was discontinued?

§ 2. *Arabic Notation.*

12. The Arabic method of notation expresses numbers by the ten following characters or figures, viz., 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, and their combinations. These characters were called digits by the Romans, and ciphers by the Moors. Hence, the word ciphering.

TABLE.

1	denotes	one.	50	denotes	fifty.
2	"	two.	100	"	one hundred.
3	"	three.	101	"	one hundred and one.
4	"	four.	102	"	one hundred and two.
5	"	five.	110	"	one hundred and ten.
6	"	six.	111	"	one hundred and eleven.
7	"	seven.	112	"	one hundred and twelve.
8	"	eight.	120	"	one hundred and twenty.
9	"	nine.	121	"	one hundred and twenty-one.
10	"	ten.	130	"	one hundred and thirty.
11	"	eleven.	140	"	one hundred and forty.
12	"	twelve.	200	"	two hundred.
13	"	thirteen.	300	"	three hundred.
14	"	fourteen.	1000	"	one thousand.
15	"	fifteen.	1001	"	one thousand and one.
16	"	sixteen.	1010	"	one thousand and ten.
17	"	seventeen.	1100	"	one thousand and one hundred.
18	"	eighteen.	1020	"	one thousand and twenty.
19	"	nineteen.	1200	"	one thousand and two hundred.
20	"	twenty.	2456	"	two thousand four hundred and fifty-six.
21	"	twenty-one.			
22	"	twenty-two.	1859	"	one thousand eight hundred and fifty-nine.
30	"	thirty.			
40	"	forty.	2500	"	two thousand and five hundred.

13. All numbers except the first nine are expressed by combinations of figures; hence, these nine are called significant figures or digits.

---

12. What is the Arabic method of expressing numbers? What are digits? What is ciphering? 13. How are all numbers above 9 expressed? What are significant figures? Why?

14. The combinations take place according to certain laws, as follows: —

1. A figure in the first or unit place expresses its simple value, and is called a number of the first order, or order of units; thus, 6, six; 7, seven.

2. A figure in the second or tens place expresses ten times its simple value, and is called a number of the second order, or order of tens, thus; 20, twenty; 30, thirty.

3. A figure in the third place or hundreds expresses a hundred times its simple value, and is called a number of the third order, or order of hundreds; thus, 400, four hundred.

4. Hence, ten of the first order make one of the second; ten of the second make one of the third; ten of the third, one of the fourth; and generally, ten of any denomination make one of the next higher.

This is called the decimal system of numbers.

15. A number is written by writing the figure of each order as it was expressed by the language, supplying the places of all orders not named with ciphers.

EXAMPLES.

16. Write seventy-five.
17. Write four hundred and twelve.
18. Write seven hundred and forty-five.
19. Write four hundred and thirty.
20. Write four hundred and three.
21. Write fourteen hundred and eighty.
22. Write twenty-eight hundred and one.
23. Write eighteen hundred and fifty-nine.
24. Write forty hundred and seven.

---

14. 1. What does a figure in the 1st place express? 2. In the 2d place? 3. In the 3d place? 4. What is the general law of combination? What is this system called? 15. How is a number written? What is meant by supplying the places of all orders not named, with ciphers?

SECTION II.

NUMERATION.

16. Numeration is the art of reading numbers expressed by any method of notation.

17. Arabic Numeration is the art of reading numbers expressed by figures. This may be done by the French Method or by the English Method.

§ 1. *French Method.*

18. By this method the first three orders constitute a period, called a period of units.

2. Every three succeeding orders constitutes a new period.

3. The orders in every period are the same, viz., units, tens, and hundreds.

4. The names of the periods commence with units, and proceed to the left, as follows :—

TABLE.

Units.
Thousands.
M -illions.
B -illions.
Tr -illions.
Quadr-illions.
Quint -illions.
Sext -illions.
Sept -illions.
Oct -illions.
Non -illions.

Decillions.
Un -decillions.
Duo -decillions.
Tri -decillions.
Quatuor-decillions.
Quin -decillions.
Sex -decillions.
Septem -decillions.
Octo -decillions.
Novem -decillions.

Vigintillions.
Un -vigintillion.
Duo -vigintillion, &c.
Tri -gintillion, &c.
Quatuor-gintillion, &c.
Quin -gintillion, &c.
Sex -gintillion, &c.
Septem -gintillion, &c.
Octo -gintillion, &c.
Novem -gintillion, &c.

FRENCH NUMERATION TABLE.

Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
7 6 8.	4 6 5.	4 7 3.	5 2 9.	8 2 0.	5 4 2.

---

16. What is Numeration? 17. What is Arabic Numeration? 18. What is the French Method? How many orders in every period? What are they? What are the names of the periods? Do you numerate by repeating the names of the *orders* or the names of the periods?

19. A number is written by writing the figures of each period as they are expressed by the language, supplying the places of all periods and orders not named, with ciphers.

20. A number is read by expressing the units of each period by itself, together with the name of the period commencing at the left hand, and reading each period as if alone.

EXAMPLES.

25. Read 45. 67. 921. 6072.
26. Read 4021. 87601. 427. 3001
27. Read 3006. 20041. 821. 56876.
28. Read 783622. 845687845.
29. Read 87600426. 40007804004.
30. Read 87430000426. 500070821.
31. Read 78007842003261.
32. Read 84263687000007842636842684560.
33. Read 426008000000678200084260000008564321.

§ 2. *English Method*

21. By this method the first six orders constitute a period, called units.

2. Every six succeeding orders constitute a new period.

3. The orders in every period are the same, viz., units, tens, hundreds, thousands, tens of thousands, and hundreds of thousands.

4. The names of the periods are the same as in the French Method, except thousands.

5. Numbers are written and read by the same rule as in the French Method.

ENGLISH NUMERATION TABLE.

Trillions.	Billions.	Millions.	
7 6 8 4 6 5.	4 7 3 5 2 9.	8 2 0 5 4 2.	6 7 4 3 6 8.

This method is but little used.

---

19. How is a number written? 20. How read? 21. How does the English Method differ from the French? What is the difference between writing and reading numbers? Why is the English Method little used?

DEFINITIONS.

22. 1. An Abstract Unit refers to no particular thing or quantity.

2. A Concrete Unit refers to some particular thing or quantity.

23. 1. Numbers are called abstract when they do not represent quantity, or when their unit is abstract.

2. Numbers are called concrete when they do represent quantity, or when their unit is concrete.

3. A Denomination is a name for a concrete number.

4. A Quantity is anything that may be measured.

24. 1. A "Question," "Problem," or an "Example," in Arithmetic, is a statement requiring a solution by numbers.

2. The application of numbers according to some law in a solution, is called an "Operation," and

3. The statement of that law is the "Rule."

4. The result obtained is the "Answer."

25. An Axiom is a self-evident truth, as,

1. The whole is greater than any of its parts.

2. A part is less than the whole.

3. The sum of all the parts is equal to the whole.

4. If equals be added to equals, the sums will be equal.

5. If equals be taken from equals, the differences will be equal.

6. Quantities which are equal to the same quantity, are equal to each other.

7. If a quantity be multiplied, and then the product divided by the same quantity, its value will not be changed.

8. If a quantity be divided, and then the quotient be multiplied by the same quantity, its value will not be changed.

---

22. What is an Abstract Unit? Meaning of abstract? 2. What is a Concrete Unit? Meaning of concrete? 23. 1. What are Abstract Numbers? 2. What are Concrete Numbers? 3. What is a denomination? 4. What is a quantity? 24. 1. What is a Question or Problem? 2. What is an Operation? 3. What is a Rule? 4. What is an Answer? 25. What is an Axiom? Repeat axioms 1, 2, 3, 4, 5, 6, 7, 8. Repeat any other axioms you may think of

SIGNS.

26. Signs are used for indicating relations or operations; thus,

1. : : or = denotes equality between two quantities.
2. + *Plus*, denotes more, or addition.
3. × or . denotes multiplication.
4. — Minus denotes subtraction.
5. — or ÷ or : denotes division
6. —— or ( ) denotes that all the quantities under or within, are subject to the same operation.

---

26. What are signs used for in Arithmetic? What are the signs for equality? Which one is generally used? What is the sign plus used for? Why do we have it? Which sign of multiplication is preferable? Why? Can you give instances where these different signs for division are used? What is a sign for substraction called? What is meant by subject to the same operation? What is a unit? What is a number? How many ways of expressing numbers in general use? How many methods of Arabic Numeration? What is a concrete number? What is a denomination? What is Arithmetic? What is meant by Primary Rules? What are they? What is the difference between Notation and Numeration? How many kinds of notation in general use? What is the difference between them? What are the names of the orders in the French Method of numeration? What are the names of the orders in the English Method? Which method is preferable? Why? What are the names of the 1st ten periods? Of the 2d ten? Of the 3d ten? Of the 4th ten? Of the 5th ten? Is there any end to the periods? Are they the same in the English and French Methods? What is a question? What is an axiom? Repeat the French Numeration Table. Repeat the English Numeration Table. What is the difference? What is the difference in the orders? Why is the French Method preferable?

SECTION III.

ADDITION.

27. *Addition* is the process of finding a number that expresses as many units as the numbers to be added contain.

28. *Terms* are the numbers to be added.

29. The *amount* or *sum* is the result of addition.

PROB. I. — TO ADD SEVERAL NUMBERS.

RULE.—1. *Write the like denominations of the several terms under each other.*

2. *Add the numbers in the lowest denomination.*

3. *Find how many of the next higher denomination this sum will make, and (writing down the remainder) add it to the sum of the numbers in the next denomination.*

NOTE 1.—Addition may be facilitated by reading only the amounts as we add. Also by adding two columns at once.

NOTE 2.—The pupil should never accustom himself to apply proof. *The first process should be correct.*

This note is applicable to the entire course of mathematics, as well as the business transactions of life.

ILLUSTRATION.—Add 27, 185, 768 and 235.

Adding the column of units, it amounts to 25; that is, 2 of the next denomination, tens, and 5 remainder. Writing down the remainder, 5, under the column of units, we add the 2 tens to the next column, which amounts to 21; that is, 2 of the next denomination, hundreds, and 1 remainder. Writing down this remainder, 1, under the column of tens, we add the 2 to the next denomination, hundreds, which amounts to 12. This being the last denomination, we write the entire result.

	Hund.	Tens.	Units.
		2	7
	1	8	5
	7	6	8
	2	3	5
1	2	1	5

---

27. What is addition? 28. What are terms? 29. What is an amount? What is problem 1? What is first to be done in addition? Why? What second? Third? Repeat the Rule. What two methods of facilitating addition? Why is not proof required?

EXAMPLES.

34. What is the sum of forty-nine, twenty-seven, ten, fifty-six, thirty-four, eighty-six and seventeen?

35. What is the sum of one hundred and six, five hundred and twelve, one thousand forty-two, and twelve?

36. What is the sum of eight hundred and six, two thousand and four, seven thousand and thirty-one, five thousand and eighty-seven, six hundred thirty-one?

37. What is the sum of seven, nineteen, forty-seven, three hundred and forty-six, nine hundred and eight, sixteen hundred and thirty-seven, nine hundred seventy-six, sixteen thousand eight hundred and fifty-nine?

38. What is the sum of nine hundred twenty-seven thousand eight hundred and fifty-seven, six hundred seventy-three thousand and eighty-three, two hundred seventy-eight thousand nine hundred and six, and twenty-eight thousand five hundred and fifty-one?

39. What is the sum of all the *odd* numbers to fifty-seven, inclusive?

40. What is the sum of all the *even* numbers to sixty, inclusive?

41. What is the sum of all the numbers to forty-one, inclusive?

42. What is the sum of all the odd numbers up to one hundred?

43. What is the sum of all numbers to one hundred?

44. How many times does a common clock strike in a day?

45. How many days are there in the winter months?

46. How many days in the spring months?

47. How many days in the summer months?

48. How many days in the fall months?

49. How many days from the 20th of September to the 20th of March?

50. What is the sum of all the answers on this page?

Ans. 1957490 or 2.

51. How many days from the 20th of March to the 20th of September?

52. Add 71, 26, 43, 57, 93, 46 and 93.

53. Add 93, 56, 37, 93, 87, 34, 56, 82, 328, 463, 987, 576, 327 and 93.

54. Add 123, 426, 83, 8, 467, 1042, 6821, 758, 56, 787, 3274 and 33.

55. Add 764, 9423, 436, 8004, 96826, 4263, 463, 987684, 563468, 34568, 87 and 36.

56. Add 57, 2693, 4684, 3268, 4261, 76821, 426, 493, 68, 368, 426, 938, 426 and 57682.

57. Add 3156823, 4568263, 42684, 3826, 4768, 43680040, 368463, 784 and 5789.

58. Add 568426, 367846, 468263, 42682, 436, 98732, 5687, 42687 and 789.

59. Add 789, 987, 842, 678, 987, 498, 789, 678, 937, 867, 489, 569, 876, 937 and 888.

60. Add 647, 987, 567, 489, 778, 4682, 989, 797, 878, 979, 898, 979, 898 and 9.

61. Add 9346, 8784, 5679, 6787, 9899, 7687, 9799, 8687, 9899 and 9362.

62. Add 98, 79, 88, 78, 56, 98, 87, 96, 98, 87, 68, 86, 97, 79, 87, 78, 96, 69, 78 and 87.

63. Add 4684567, 987684, 5678456, 789426, 4369876, 5682681, 567856, 94156, 78421 and 74.

64. Add 5678426821, 56878426987, 78567942685728956 and 784568.

65. James Brace bought 6 cows for $176.50, a yoke of cattle for $168.75, a cart for $19.31, a carriage for $157.87, a saddle for $19.87, and a harness for $47.68. What did it all cost?

66. Henry Wallace and James Stanley went hunting and brought home the following game, viz: 78 robins, 237 rice birds, 38 woodcocks, 12 pheasants, 98 partridges, 76 squirrels, 27 rabbits and 1 deer. How many animals did they get?

67. What is the sum of all the answers on this page?

Ans. 78568005321777625.

68. There are 22 bones in the head, 56 in the trunk, 69 in the superior extremity, and 64 in the inferior extremity. How many belong to the skeleton?

69. There are 2 incisor teeth, 1 cuspid, 2 bicuspid, 3 molar on each side of each jaw in a grown person, and 20 milk teeth have been extracted. How many teeth in all?

70. The population of New York, in 1850, was 3,097,394; of New Jersey, 489,555; of Pennsylvania, 2,311,786; of Delaware, 91,532. What was the population of the Middle States in 1850?

71. There are 427 histories, 186 law books, 73 medical works, 686 scientific, 417 theological, 379 memoirs, and 1396 vols. on general subjects, in a library. How many vols. in the whole?

72. In a school there are 25 boys in the higher department, and 20 in the primary. In a quarterly report 7 boys had 867 merit-marks; 9 boys, 896; 12 boys, 917; 11 boys, 987; and 6 boys, 642. How many credit-marks in all?

73. Stephen bought 6 hens for $1.50, and gave them 1 bushel of corn at 80 cents, and 2 bushels of meal at 97 cents per bushel; also paid a carpenter $3.87 for a coop. How much money did he expend for his hens?

74. S. A. Plant bought 27986 pressed brick, 19867 hard brick, and 31876 soft brick, to build a house. How many in all?

75. There were also used 96784 feet of clear lumber, 4786 feet middling, 78465 spruce, and 47986 feet hard pine. How many feet in the house.

76. Edwin Green has in one farm 78 acres of grass land, 163 acres planted with corn, 258 acres with wheat, 437 acres of wood-land, and 38 pasturage. On another farm, 367 acres of rice, 157 of cotton, and 527 of wood-land; and has two land warrants of 160 acres each, located in Kansas. How many acres does he own?

77. What is the sum of the answers on this page?

Ans. 6309309.

78. A man paid $17.68 for 2 bbls. of flour; $11.76 for 90 of sugar; $23.98 for a bbl. of pork; $17.83 for a bbl. of beef; $38.57 for a cask of molasses; $9.87 for a bbl. of rice; $8.76 for a cask of salt. What was the amount of his bill?

79. There was imported into the United States during the year ending June 10th, 1852, unmanufactured cotton from the following places, viz.:

England	100 lbs.	Dutch West Indies..	22474 lbs.
Brit. Am. Colonies.	140280 “	British West Indies.	6756 “
New Granada	69838 “	France.................	5100 “

What was the entire amount?

80. The following was the import of teas for 1852.

Dutch East Indies...	7779 lbs.	England..........	83 lbs.
British East Indies..	49260 “	China..............	28521230 “

What was the whole amount received?

81. The following was the import of coffee for 1852.

Holland..........	2508988 lbs.	Asia generally...	580 lbs.
Dutch W. Indies.	1976286 “	Dutch E. Indies.	10203910 “
Malta..............	729 “	Dutch Guiana...	25357 “
Cape of Good Hope	238 “	British E. Indies	2074729 “
Manilla and Phillipian Islands.	49740 “	British West Indies	2291740 “
Spanish West Indies..............	125034 “	Cuba..............	2846091 “
Hayti..............	18446584 “	Cape Verde Is..	1376 “
Central America.	240321 “	Mexico	135348 “
Venzuela..........	13996027 “	New Granada...	54783 “
		Brazil.............	138156506 “
		Africa generally	524289 “

What was the whole amount of coffee imported?

82. The population of Great Britain by the census of 1851, is,

England and Wales	8762588	Males.	9160180	Females.
Scotland	1363622	“	1507162	“
Ireland	3176727	“	3339067	“
Islands in the British Seas	66511	“	76405	“

Army and Navy out of the kingdom, 167604 males.

What was the population of Great Britain in 1851?

83. Church statistics for New England, are (census 1852),

	Churches.				Cost.
Maine	851	accommodating	304477	persons,	$1712152
New Hamp.	602	"	233892	"	1401586
Vermont	564	"	226444	"	1213126
Mass	1430	"	682908	"	10205284
R. I..........	221	"	98736	"	1252900
Conn.........	719	"	305249	"	3554894

How many churches in the New England States?

84. How many persons will they accommodate?

85. What was the value of church property in New England?

86. The population of New England is as follows: (census 1850).

Maine.....	517117	Native.	31456	Foreign.	584	Unknown.
N. H	261501	"	13571	"	178	"
Vermont ..	232086	"	32831	"	323	"
Mass	695236	"	160909	"	3539	"
R. I.........	102641	"	23111	"	135	"
Conn	292653	"	37473	"	794	"

What was the native population of New England?

87. What was the foreign population of New England?

88. What was the entire population of New England?

(89.)	(90.)	(91.)	(92.)
758	8426	78957	9878642
869	3795	48674	9697879
494	9899	78788	8789988
676	8978	99899	7897897
899	7356	88789	5698987
978	5687	75437	7987896
589	6979	69798	8898987
797	8989	78983	9547899
989	3798	89879	8769989
859	7897	97798	9878778
784	9489	88989	7638899

93. What is the sum of the answers on the last two pages?

Ans. 371788852.

SECTION IV.

MULTIPLICATION.

30. *Multiplication* is a short process of obtaining the result of adding a number to itself a certain number of times.

31. The *Multiplicand* is the number to be added to itself, and *may be concrete* (23).

32. The *Multiplier* is the number of times the multiplicand is taken as a *term*, and *must always* be considered as an abstract number (23).

33. The *Product* is the result of multiplication.

34. The *Factors* of a *Number* are such numbers, as being multiplied together, will produce the given number. Any factor may be a multiplier.

35. A *Square Number* is the product of a number multiplied by itself.

36. *The Square Root* of a *Number* is the number which is multiplied by itself to produce the given number.

PROB. II.—TO MULTIPLY SEVERAL NUMBERS.

RULE.—1. *Multiply the number of the lowest denomination by the multiplier, and set down the result, if less than a unit of the next denomination.*

2. *If greater, then find how many of the next denomination it will make, and (writing down the remainder) add it to the product of the next denomination and the multiplier.*

3. *Place the numbers of like denominations under each other in the partial products.*

4. *The amount of the partial products will be the required answer.*

---

30. What is Multiplication? 31. What is the Multiplicand? What is a concrete number? What is meant by the multiplicand's being concrete? 32. What is a multiplier? What is an abstract number? Why must the multiplier be abstract? 33. What is a product? 34. What are factors? Why may any factor be a multiplier? 35. What is a square number? 36. What is a square root? What is the first thing to be done in multiplication? Second? Third? Fourth? Give the rule for multiplication. What is a denomination? What is a unit of a denomination? What are like denominations?

Illustration 1.—Multiply 478231 by 5.

Multiplicand,	478231
Multiplier,	5
Ans.	2391155

The first product, 5, being less than a *unit* of the next denomination, tens, we set it down; then say 3 times 5 are 15, which makes 1 of the next denomination and 5 remainder; set down 5 and add 1 to the next product, 10, making 11; this will make 1 of the next denomination and 1 remainder; add 1 to the next product, 40, making 41; set down 1 and add 4 to the next product, 35, making 39; set down 9 and add 3 to the next product, 20, making 23, and write the whole amount.

Illustration 2.—Multiply 468263 by 38264.

Multiplicand,	468263	Factors.		
Multiplier,	38264			
Partial Products.	1873052	=	4	times the Multiplicand.
	2809578	=	60	" "
	936526	=	200	" "
	3746104	=	8000	" "
	1404789	=	30000	" "
Ans.	17917615432	or Product.		

In the above we proceed as in Ill. 1, and multiply by each figure separately. Then add the partial products.

EXAMPLES.

94. Multiply 43243 by 2.
95. Multiply 623845 by 3.
96. Multiply 786982 by 4.
97. Multiply 426826 by 12.
98. Multiply 784278 by 7.
99. What is the sum of all the answers on this page?

Ans. 15717807.

Do you understand the illustrations? Which is the most difficult part to understand? How would you change it? Will you give a better one?

100. Multiply 9878947 by 9.

101. What would 47 hats cost at 87 cents each?

102. How much would 27 cords of wood come to at $6.18 per cord?

103. Henry Brown bought 16 cows at $17.50 apiece, 4 pairs of oxen at $145 a pair, and 4 horses at $217 apiece. What did the whole cost?

104. Thomas Bowie hired 6 servants 8 months for $12 per month each; he paid $27 per month rent for a farm; bought 117 bushels of corn at 73 cents per bushel; 67 bushels of oats at 43 cents per bushel. At the end of the season he sold 617 bushels of wheat at $2.14 per bushel; 1437 bushels of corn at 93 cents per bushel. Did he gain or lose?

105. What will 1782 sheep cost at $4.78 per head?

106. James Allen sold 479 yards of cloth at $1.97 per yard, and 628 yards at $2.47 per yard. What did he get for both pieces?

107. Which would you prefer, to sell 57 acres of land for $74 per acre, or sell the whole for $4000?

108. Bought 1396 pine-apples for 8 cents apiece; 587 were decayed, and I sold the rest for 12 cents apiece. Did I lose or gain?

109. Which is the most, 13 times 17 or 12 times 18?

110. Which is the most, 14 times 16 or 15 times 15?

111. Which is the most, 11 times 19 or 12 times 18?

112. How many beats of the pulse in a day at 70 per minute?

113. How many in a week?

114. How many in a year of 365 days?

115. How many in a lifetime of 70 years?

116. How many times would a man, 62 years old, breathe at 18 per minute?

117. How many cubic inches of air would he inhale if he inhaled 23 each time?

118. What is the sum of the answers on this page?

Ans. 16780914173.

119. In a book there are 7 vols., each volume containing 347 pages, and 3471 ems on a page. How many ems in the entire work?

120. There are at work on the United States Capitol Extension 1000 men. What will it cost per month (of 30 days) if they are paid $2.57 for each day's labor?

121. What will a granite block weigh that contains 1297 cubic feet, if each cubic foot weighs 63 lbs?

122. William Denny bought 58 marbles, and gave 3 cents for 2; he bought 16 bat-balls at 17 cents a-piece; 19 foot-balls at $2.37 a-piece. What did the whole cost?

123. Samuel Slater bought 47 lbs. of sugar at 14 cents per pound; 19 galls. of molasses at 87 cents per gallon; 138 lbs. of ham at 16 cents per pound; 27 bbls. of flour at $11.86; 236 bus. of wheat at $1.47 cents per bushel. What did the whole cost?

124. How many days from the birth of our Saviour, to Jan. 1st, 1858, counting 365 days in a year, and adding in 446 days for leap years?

125. Suppose there are 214578 persons in Boston and its immediate vicinity; what are they are all worth if each is worth $4178.56?

126. The United States contains about 28987637 persons. Suppose each pays a tax of 7 cents and 9 mills. What would the revenue amount to?

127. James Baldwin bought of Titus Baker,

14	yds.	of cotton cloth,	@	11 cents	=
19	"	of French flannel,	@	63 cents	=
43	"	of black broadcloth,	@	$5.47	=
78	"	of silk barege,	@	78 cents	=
39	"	of Pongee hkf.	@	98 cents	=
137	"	of black lace,	@	$2.37	=
63	"	of Brussels carpet,	@	$3.48	=

What did the bill amount to?

128. What is the sum of the answers on this page?

Ans. 91968764743.

129. What would it cost to forward 1789 bbls. of flour to tah, at $57.86 per barrel?

130. What is the New York and Erie rail road worth at 19787.56 per mile. Length, 444 miles?

131. What was the value of the cotton imported from the Vest Indies during the year 1852, at 7 cents per pound?

132. What was the entire value of all imported?

133. What was paid for tea during the same time, estimating to be worth 27 cents per pound?

134. The Atlantic Submarine Telegraph Cable is composed f 7 copper wires, and 18 bundles, 7 wires each, of iron. The ngth of cable is 3000 miles. How many miles of wire?

135. How much money was paid to South America for coffee alued at $.075 (7 cents 5 mills) per pound?

136. How much was paid to the West Indies?

137. How much was paid to the East Indies?

138. How much to all other parts?

139. What is the sum of the answers above on this page?
Ans. 16187248968.

140. What was the entire value of coffee for 1852?

141. How much was the duty on the coffee at 7 mills per pound?

142. What is the product of 1 × 2 × 3 × 4 × 5?

143. What is the product of 6, 7, 8, 9, 10?

144. What is the product of the answer to the 142d example, ultiplied by the answer to the 143d?

145. Multiply together 11, 12, 13, 14, 15.

146. Multiply together 16, 17, 18, 19, 20, 21, 22, 23, 24, 25.

147. Multiply together 2, 4, 6, 8, 10, 12, 14, 16, 18, 20.

148. Multiply together 1, 3, 5, 7, 9, 11, 13, 15, 17, 19.

149. Multiply together 22, 24, 26, 28, 30.

150. Multiply together 21, 23, 25, 27, 29.

151. Multiply together 707070 by 80808.

152. Multiply together 757575 by 85858.

153. What is the sum of all the answers on this page, from xample 139?
Ans. 120041327107422.

154. Multiply 78426 by 123.
155. Multiply 546326 by 234.
156. Multiply 7846356 by 345.
157. Multiply 549876 by 456.
158. Multiply 6842604 by 567.
159. Multiply 984200876 by 678.
160. Multiply 847689426 by 789.
161. Multiply 42682341 by 8901.
162. Multiply 632621 by 9012.
163. Multiply 326547 by 1234.
164. Multiply 489876 by 2345.
165. Multiply 568763 by 3456.
166. Multiply 8134567 by 4567.
167. Multiply 7842656 by 5678.
168. Multiply 34098764 by 6789.
169. Multiply 47898426 by 78910.
170. What is the sum of the answers above on this page? Ans. 5826436764020.
171. Multiply 4874356 by 91023.
172. Multiply 5437821 by 10203.
173. Multiply 3782154 by 200304.
174. Multiply 632784 by 840004.
175. Multiply 70004007 by 800404.
176. Multiply 8767879 by 79.
177. Multiply 987789 by 7899.
178. Multiply 879874 by 8978.
179. Multiply 4567898 by 7897.
180. Multiply 7438754 by 9879.
181. Multiply 684268 by 7878.
182. Multiply 432781 by 89897.
183. Multiply 51 by 53 by 59.
184. Multiply 27 by 81 by 73.
185. Multiply 62 by 73 by 78.
186. Multiply 83 by 76 by 98.
187. What is the sum of all the answers on this page, from example 170? Ans. 57989022003628.

188. What is the product of 1, 2, 3, 5, 7, 11, 13, 17?
189. What is the product of 19, 23, 29, 31, 37, 41, 43?
190. What is the product of 47, 53, 59, 61, 67, 71, 73?
191. What is the product of 79, 83, 89, 97, 101, 103?
192. What is the product of 107, 109, 113, 127, 131?
193. What is the product of 137, 139, 149, 151, 157?
194. What is the product of 163, 167, 173, 179?
195. What is the product of 181, 191, 193, 197, 199, 211?
196. What is the product of 223, 227, 229, 233?
197. What is the product of 239, 241, 251, 257?
198. What is the product of 263, 269, 271, 277?
199. What is the product of 281, 283, 293, 307?
200. What is the product of 311, 313, 317, 331?
201. What is the product of 337, 347, 349, 353?
202. What is the product of 359, 367, 373, 379?
203. What is the product of 383, 389, 397, 401?
204. What is the product of 409, 419, 421, 431, 433?
205. What is the sum of the answers above on this page?
Ans. 72559292812952
206. What is the product of 439, 443, 449, 457?
207. What is the product of 461, 463, 467, 479?
208. What is the product of 487, 491, 499, 503?
209. What is the product of 509, 521, 523, 541?
210. What is the product of 547, 557, 563, 569?
211. What is the product of 571, 577, 587, 593?
212. What is the product of 599, 601, 607, 613?
213. What is the product of 617, 619, 631, 641?
214. What is the product of 643, 647, 653, 659?
215. What is the product of 661, 673, 677, 683?
216. What is the product of 691, 701, 709, 719?
217. What is the product of 727, 733, 739, 743?
218. What is the product of 751, 757, 761, 769?
219. What is the product of 773, 787, 797, 809?
220. What is the product of 811, 821, 823, 827, 829?
221. What is the sum of the answers on this page from the 205th example? Ans. 378057637303711.

SECTION V.

SUBTRACTION.

37. *Subtraction* is the process of finding the difference between two numbers.

38. The *Minuend* is the larger number.

39. The *Subtrahend* is the less number.

40. The *Difference* is the number of units more in one than in the other.

The minuend and subtrahend must be of the same denomination, *i. e. have the same unit.*

PROB. III.—TO SUBTRACT ONE NUMBER FROM ANOTHER.

RULE.—1. *Write like denominations in the same column.*

2. *Commence at the lowest denomination; subtract, if possible, and write down the difference.*

3. *If not, reduce and add thereto one of the next higher denomination in the minuend; then subtract and write down the difference.*

4. *Call the next number in the minuend one less than it is, whenever you reduce one; or, which will produce the same result, the next in the subtrahend one more than it is.*

ILLUSTRATION.—Subtract 275 from 427.

Minuend,	427
Subtrahend,	275
Difference,	152

First subtract 5 from 7 and write down the difference, 2. Then, as we cannot take 7 from 2, we take 1 from the next denomination, hundreds, and reduce it to tens, add 2 tens and it gives 12 tens; then subtract 7 tens and write the difference, 5 tens. As we have taken 1 from 4, it is now 3, and the difference between it and 2 is 1.

---

37. What is Subtraction? 38. What is a minuend? What does the word minuend mean? 39. What is a subtrahend? What does the word mean? 40. What is a difference? Why not use the word remainder? (*Ans.* See hints to teachers.) Why should both terms be of the same denomination? What is meant by having the same unit? What is first to be done in performing subtraction? Why? What second? What third? What is meant by reduce and add thereto? What fourth? Why call it one less? What is the rule for subtraction?

222. Subtract 21 from 47.
223. Subtract 26 from 53.
224. Subtract 19 from 63.
225. Subtract 39 from 124.
226. Subtract 56 from 97.
227. Subtract 81 from 125.
228. Subtract 17 from 243.
229. Subtract 93 from 482.
230. Subtract 521 from 4374.
231. Subtract 426 from 2861.
232. Subtract 324 from 3810.
233. From 826321 subtract 384679.
234. From 80004 subtract 34267.
235. From 8426 + 7844 subtract 3784 + 3671.
236. From 846836 + 76326 subtract 7626 + 87326.
237. From 568784 — 326821 subtract 56764 + 22371.
238. From 468768426 + 278416 subtract 70808709.
239. From 54300407 — 6842637 subtract 70000 — 23265.
240. From 6842637 — 78463 subtract 7080708 — 698426.

241. How much more coffee did we import from Brazil than from the entire West Indies?

242. What is the difference between the amount imported from Brazil and that from all other sources?

243. How much more tea was received from China than from all other sources?

244. To which did we pay the most money, and how much, to China for tea or Brazil for coffee?

245. Which has the most church property, according to the seventh census, Massachusetts, or Connecticut, Rhode Island and Vermont together, and what is the difference?

246. Which is the most, the native population of Maine or the entire foreign population of New England, and how much?

247. Which has the most population, the New England or the Middle States, and how much?

248. What is the sum of the answers on this page?

Ans. 945205027.

SECTION VI.

DIVISION.

41. Division is a short process of finding how many times one number may be taken from another.

42. The *Dividend* is the number to be divided, and *may be concrete* (23).

43. The *Divisor* is the number to divide by, and *must always be considered abstract* (23).

44. The *Quotient* is the result of a complete division.

45. A *Remainder* is a number left when the division is not complete, and must be of the same denomination as the dividend.

Prob. IV.—To divide one number by another.

Rule—1. *Commence at the highest denomination, and, if necessary, reduce it to the next lower, adding it to the number of this lower denomination in the dividend, and this result to the second denomination, and so on, till the result will contain the divisor once or more.*

2. *Ascertain how many times the divisor is contained in this result, and write it in the quotient.*

3. *Then multiply the divisor by this number, and subtract the product from the number of the dividend taken.*

4. *Reduce the remainder, and proceed as before.*

Note.—When the divisor is small, most of the work may be done mentally. This is called Short Division.

Illustration 1.—Divide 2685 by 5.

Divisor. Dividend.
5) 2 6 8 5
Quotient, 5 3 7

As 2 will not contain 5, it is reduced to the next lower denomination, hundreds; and adding 6 hundreds, we have 26 hun-

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41. What is division? What is the meaning of the word process? 42. What is a dividend? 43. What is a divisor? 44. What is a quotient? 45. What is a remainder? Why not call it a difference? What is meant by the division not being complete? Can you have a remainder when the divisor may be contained in the dividend an exact number of times? Ans. Yes. How is that?

dreds; dividing by 5, it gives 5 hundreds, and a remainder, 1; thus, reduce and add, and it gives 18, which contains 5, 3 times, and 3 remainder; reduce, and add, making 35, which contains 5, 7 times.

ILLUSTRATION 2.—Divide 468104 by 76.

$$\begin{array}{lll}
 & \text{Divisor. Dividend. Quotient.} & \\
 & 76)\,468104(6159\tfrac{20}{76} & \\
 & \quad\;\; \underline{456} & = 6\times 76 \\
\text{1st Rem.} & \quad\;\; 121 & \text{(See last question, p. 35.)} \\
 & \quad\;\;\; \underline{76} & = 1\times 76 \\
\text{2d Rem.} & \quad\;\;\; 450 & \\
 & \quad\;\;\; \underline{380} & = 5\times 76 \\
\text{3d Rem.} & \quad\;\;\;\; 704 & \\
 & \quad\;\;\;\; \underline{684} & = 9\times 76 \\
\text{Last Rem.} & \quad\;\;\;\;\; 20 & \text{Rem.}
\end{array}$$

First reduce 4 hundred thousands to ten thousands, and add in 6 ten thousands, and this gives 46 ten thousands. This will not contain the divisor. Then reduce this result, 46, to thousands, add in the 8 thousands, and we have 468 thousands. This will contain the divisor 6 times. Therefore, multiply 76 by 6, write the product under the 468, and subtract. This leaves a remainder of 12 thousand; reduce to hundreds, and add 1, making 121 hundred. This contains the divisor 1. Multiply, subtract, and there is a remainder of 45 hundred. This reduced to tens, and added to 0 tens in the dividend, gives 450 tens. This contains the divisor 5 times. Multiply, subtract, reduce, add, and we have 704. This contains the divisor 9 times, and leaves a remainder, 20.

Multiply, subtract, reduce, add—remember these four words.

249. Divide 363 by 3.
250. Divide 2684 by 4.
251. Divide 4567 by 6.
252. Divide 6426 by 9.
253. Divide 2468 by 5.
254. Divide 2135 by 7.
255. Divide 3268 by 8.
256. Divide 4567 by 3.
257. Divide 7326 by 8.
258. Divide 8200 by 9.
259. Divide 7821 by 7.
260. What is the sum of the remainders on this page?

Ans. 18.

Which is the most difficult part of this illustration? Why remember the words multiply, subtract, reduce, add?

261. Divide 764826 by 11.
262. Divide 642684 by 12.
263. Divide 568426 by 31.
264. Divide 567842 by 23.
265. Divide 1536426 by 231.
266. Divide 8426364 by 247.
267. Divide 8004004 by 723.
268. Divide 4283264 by 845.
269. Divide 8577824 by 327.
270. Divide 3684263 by 987.
271. Divide 3141595 by 482.
272. Divide 426826 by 403.
273. Divide 568263 by 4032.
274. Divide 263782601 by 68207.
275. Divide 768479421 by 468204.
276. Divide 367842613 by 84687007.
277. Divide 682636421 by 8000846.
278. Divide 68426375 by 826326.
279. Divide 542687423 by 4781.
280. Divide 437894267 by 84840.
281. Divide 78426012 by 56870.
282. Divide 87346842 by 42868.
283. Divide 36784684 by 32675.
284. Divide 68426364 by 9879.
285. Divide 23468426 by 8760.
286. Divide 7878782 by 8781.
287. Divide 8989893 by 9789.
288. Divide 342684 by 7898.
289. Divide 364687 by 78456.
290. Divide 5768263 by 98478.
291. Divide 787898 by 32684.
292. Divide 42638476 by 984684.
293. Divide 4268261 by 3845826.
294 What is the sum of the remainders on this page?

Ans. 33448614.

295. How many acres of land would each man in Maine receive if the State, containing 32,854 square miles, was equally divided among the native population?

NOTE.—See example 861. Reduce square miles to acres by multiplying by 640.

296. How much would a man have in New Hampshire, its area being 9280 square miles?

297. How much in Vermont, area 10,212 square miles?

298. How much in Massachusetts, area 7800 square miles?

299. How much in Connecticut, area 4674 square miles?

300. How much in Rhode Island, area 1306 square miles?

301. How much if New England was divided in the same manner among its native population?

302. How much if it was divided among its *entire* population?

303. Suppose the church property in Maine was paid for by the "persons accommodated." How much did each individual pay?

304. The conditions being the same, what would be paid in New Hampshire?

305. What in Vermont?

306. What in Massachusetts?

307. What in Rhode Island?

308. What in Connecticut?

309. What in New England?

310. What would each person in New England receive, suppose the church property was equally distributed?

311. How much coffee would each New England person have, supposing one eighth part of all imported into the United States was consumed in New England?

312. How much tea?

313. Divide 1001001 by 10101.

314. What is the sum of the remainders on this page

Ans. 6121898.

MISCELLANEOUS QUESTIONS.

315. I have 3 eagles, 7 half eagles, 6 two and a half dollar pieces, 9 dollars, 16 half dollars, 12 quarter dollars, 8 ten cent pieces, 16 five cent pieces, 13 three cent pieces, and 18 cents. How much have I in all?

316. I wish to pay James Brown $3.18. What is the smallest number of pieces that will pay it?

317. I wish to pay $4.71 for a Dictionary, $3.25 for a Life of Washington, and $13.75 for a Bible. What is the smallest number of pieces that will pay for them?

318. I then spent $27.81 travelling, gave a poor man $3.17, and bought a hat for $3.62. How much money have I left?

319. I paid a tax bill of $27.36, bought a whip for .81, a knife for $1.27, a cane for $7.19, a watch key for $1.80, paid .88 for postage, had ice cream for $1.40, and on my way home lost $1.31. How much change had I left?

320. I have a dollar, a half dollar, a twenty cent piece, and 4 cents, and wish to pay James Brown 31 cents. He has a half dollar, 2 quarters, 3 dimes, a half dime, and a three cent piece. Can we make change? How?

321. James gives 419 sheep, at $2.17 per head, to Andrew for 39 oxen, at $47.81 each. What is the balance due?

322. Lewis exchanges silk with Boardman for broadcloth; price of silk, $1.12 per yard; price of broadcloth, $5.60 per yard. Lewis receives 139 yards of cloth. How much silk does Boardman receive?

323. What was the silk worth?

324. Boardman sells his silk, and gains 27 cents per yard. What does he get for it?

325. Multiply 12345679 by 36 and divide the product by 444.

326. Multiply 12345679 by 63 and divide the product by 777.

327. What is the sum of the answers to the last six examples on this page? Ans. 2272678.

328. Multiply 12345679 by 45 and divide the product by 111.

329. Charles bought 147 marbles, 7 for 5 cents, and sold them for $1.28. Did he make or lose? How much?

330. $9342 + 863 + 79 - 8351$.

331. $7846 \times 623 \div 2743$.

332. Which is most, 71×93 or $9678 \div 87$.

333. What should I get for 2786 hogs at $9.87 each?

334. James sold 817 barrels of flour for $8987. How much was it per barrel?

335. How much will 1942 pairs of boots cost at $4.75 per pair?

336. What will 167 chests of tea cost at $94.78 per chest?

337. How long must a man labor at $1.87 per day to obtain $486.20?

338. What will 18 chronometers cost at $298.76 each?

339. What is the difference between 187×186 and 188×185?

340. What is the difference between $18576 \div 9$ and $37152 \div 18$?

341. What will 377 miles of railroad cost at $17,986.87 per mile?

342. A man sells 67841 lbs. of coal to A, 37842 to B, 98463 to C, 76821 to D, 42687 to E, from 768426 lbs. How much remains?

343. A grocer buys 6784 lbs. of sugar, sells 3842 lbs., buys 68231, sells 42687, buys 56423 lbs., and sells the balance at 11 cents per lb. What does he get for it?

344. Alonzo Bride buys 437 lbs. of tea at 46 cents, 981 lbs. at 54 cents, and sells the whole for 50 cents. How much does he make or lose?

345. A train of cars runs 8 hours, 29 miles per hour, then runs back 3 hours, at 19 miles per hour. How far from the starting point?

346. How long will it take a horse to draw 5000000 bricks, if he draw 20 loads per day of 384 bricks each?

347. How much can the President spend daily to lay up annually $10,945.67 of his salary?

348. What is the sum of the answers on this page?

Ans. 690292464 and 535 Rem.

Now make a thorough review. Why not advance?

CHAPTER II.

SPECIAL RULES FOR MULTIPLICATION AND DIVISION.

[This Chapter may be omitted until a review.]

SECTION I.

MULTIPLICATION.

46. The *Powers* of a *Number* are the several products arising from a continued multiplication of a number by itself.

47. *An Aliquot Part* of a *Number* is that number which exactly measures or divides it; *e. g.* 5 is an aliquot part of 20.

48. *Equi-distant Numbers* are those equally distant from a number; *e. g.* 7 and 13 are equi-distant numbers of 10.

49. A *Mean Number* lies half way between two given numbers; *e. g* 10 is a mean between 6 and 14 or 5 and 15.

PROB. V.—TO MULTIPLY BY TEN OR ANY POWER OF TEN.

RULE.—*Annex the ciphers of the multiplier to the multiplicand.*

EXAMPLES.

349. Multiply 76 by 10.
350. Multiply 784 by 100.
351. Multiply 684 by 1000.
352. Multiply 5678 by 100.
353. Multiply 7684 by 10.
354. Multiply 5679 by 10000.
355. Multiply 7800 by 100000.
356. Multiply 8260 by 100.
357. Multiply 76923 by 1000.
358. Multiply 42682 by 10.
359. Multiply 35647 by 1000000.
360. What is the sum of the answers on this page?
Ans. 36563373620.

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46. What are powers of a number? What is a product? 47. What is an aliquot part? What does aliquot mean? When does one number measure another? 48. What is an equi-distant number? 49. What is a mean number? What rule to multiply by 10 or any power of 10?

PROB. VI.—TO MULTIPLY BY THE ALIQUOT PARTS OF TEN OR ANY POWER OF TEN.

RULE.—*Divide the multiplicand with the ciphers (in the power of ten used) annexed, by the number of aliquot parts.*

ILLUSTRATION.—Multiply 67 by 25.

By trial we find 25 to be an aliquot part of 100. Therefore annex *two* ciphers to 67, thus: 6700. Now divide 6700 by the number of aliquot parts, 4, thus: $6700 \div 4 = 1675$. Ans.

EXAMPLES.

361. Multiply 76842 by 5. $(10 \div 5 = 2.)$
362. Multiply 64263 by $3\frac{1}{3}$. $(10 \div 3\frac{1}{3} = 3.)$
363. Multiply 84262 by $2\frac{1}{2}$. $(10 \div 2\frac{1}{2} = 4.)$
364. Multiply 56841 by $1\frac{2}{3}$. $(10 \div 1\frac{2}{3} = 6.)$
365. Multiply 37681 by $1\frac{3}{7}$. $(10 \div 1\frac{3}{7} = 7.)$
366. Multiply 26840 by $1\frac{1}{4}$. $(10 \div 1\frac{1}{4} = 8.)$
367. Multiply 56844 by $1\frac{1}{9}$. $(10 \div 1\frac{1}{9} = 9.)$
368. Multiply 37845 by 50.
369. Multiply 568425 by $33\frac{1}{3}$. $(100 \div 33\frac{1}{3} = 3.)$
370. Multiply 378456 by 25
371. Multiply 8486940 by $16\frac{2}{3}$. $(100 \div 16\frac{2}{3} = 6.)$
372. Multiply 7840826 by $14\frac{2}{7}$. $(100 \div 14\frac{2}{7} = 7.)$
373. Multiply 56870 by 20.
374. Multiply 568426 by $12\frac{1}{2}$.
375. Multiply 378459 by $11\frac{1}{9}$. $(100 \div 11\frac{1}{9} = 9.)$
376. Multiply 45687 by $66\frac{2}{3}$. $(1000 \div 66\frac{2}{3} = 15.)$
377. Multiply 894687 by $83\frac{1}{3}$. $(1000 \div 83\frac{1}{3} = 12.)$
378. Multiply 426384 by $6\frac{1}{4}$.
379. Multiply 568764 by $8\frac{1}{3}$.
380. Multiply 5684789 by 625.
381. Multiply 378426 by $62\frac{1}{2}$.
382. Multiply 4268427 by $166\frac{2}{3}$.
383. What is the sum of the answers on this page?
Ans. 4670321025.

What is the rule to multiply by the aliquot parts of ten or any power of ten.

PROB. VII.—TO MULTIPLY BY THE FACTORS OF A NUMBER.

RULE.—*Multiply by one factor, and the product arising by another, and in the same manner by all the factors of the multipliers.*

EXAMPLES.

384. Multiply 84263 by 12.
385. Multiply 56842 by 16.
386. Multiply 784268 by 18.
387. Multiply 56827 by 27.
388. Multiply 56878 by 48.
389. Multiply 96823 by 64.
390. Multiply 56874 by 92.
391. Multiply 56843 by 42.
392. Multiply 210042 by 66.
393. Multiply 368784 by 72.
394. Multiply 256784 by 144.
395. Multiply 234567 by 216.
396. Multiply 784235 by 81.
397. Multiply 568704 by 128.
398. Multiply 678426 by 343.
399. Multiply 368468 by 121.
400. Multiply 876542 by 108.
401. Multiply 7845684 by 272.
402. Multiply 5684267 by 108.
403. Multiply 3684568 by 416.
404. Multiply 7856847 by 396.
405. Multiply 56846841 by 404.
406. Multiply 568790 by 728.
407. Multiply 426832 by 84.
408. Multiply 3684687 by 96.
409. Multiply 356876 by 24.
410. Multiply 567843 by 2162.
411. Multiply 7842631 by 1056.
412. What is the sum of the answers on this page?

Ans. 41350310662.

What are factors? Ans. (34.) What is the rule to multiply by factors?

PROB. VIII.—TO MULTIPLY WHERE ONE PART OF THE MULTIPLIER IS AN ALIQUOT PART OF SOME OTHER PORTION.

RULE —*Multiply by the aliquot part, then that product by the number of aliquot parts, and other figures, if any, as usual. Be careful to write like denominations in the same columns.*

ILLUSTRATION 1.—Multiply 4321 by 284.

We perceive 4, one part of the multiplier, is an aliquot part of 28, another portion. Therefore proceed and multiply by 4. We have now 4 times the multiplicand, and if that is multiplied by 70, the number of aliquot parts, we have 1209880, or 280 times the multiplicand. This added gives the correct result.

$$\begin{array}{r l} 4321 & \\ 284 & \\ \hline 17284 & \times 70 \\ 120988 & \\ \hline 1227164 & \end{array}$$

ILLUSTRATION 2.—Multiply 4321 by 428.

$$\begin{array}{r l} 4321 & \\ 428 & \\ \hline 17284 & \\ 120988 & = 17284 \times 7 \\ \hline 1849388 & \text{Ans.} \end{array}$$

EXAMPLES.

413. Multiply 347742 by 168.
414. Multiply 782631 by 1684.
415. Multiply 4268347 by 3284.
416. Multiply 384264 by 42217.
417. Multiply 3467421 by 64168.
418. Multiply 3568471 by 128648.
419. Multiply 426784 by 256328.
420. Multiply 3268278 by 1024256328.
421. Multiply 4267845 by 7283287.
422. Multiply 1237847 by 963284.
423. What is the sum of the answers on this page?

Ans. 3380653348292431.

What is the rule to multiply when one part of the multiplier is an aliquot part of some other portion?

PROB. IX.—TO MULTIPLY BY USING MEAN AND EQUI-DISTANT NUMBERS.

RULE.—1. *From the square of the mean number subtract the square of the distance.*

2. *To the product of the equi-distant numbers add the square of the distance.*

ILLUSTRATION 1.—Multiply 33 by 27.

Square 30, the mean between 33 and 27, and we have 900. From this subtract the square of the distance, 3. Ans. 891.

ILLUSTRATION 2.—Multiply 35 by 35.

The product of the equi-distant numbers, 30 and 40, is 1200, to which add the square of the distance, 5. Ans. 1225.

NOTE.—*This problem is designed for mental exercise.*

EXAMPLES.

424. Multiply 18 by 22.
425. Multiply 17 by 23.
426. Multiply 16 by 24.
427. Multiply 27 by 33.
428. Multiply 45 by 55.
429. Multiply 62 by 78.
430. Multiply 61 by 79.
431. Multiply 52 by 68.
432. Multiply 49 by 71.
433. Multiply 50 by 50.
434. Multiply 26 by 24.
435. Multiply 42 by 42.
436. Multiply 37 by 37.
437. Multiply 38 by 38.
438. Multiply 75 by 85.
439. Multiply 67 by 53.
440. Multiply 68 by 68.
441. Multiply 123 by 117.
442. What is the sum of the answers on this page?

Ans. 57849.

What is the rule to multiply by mean numbers? What to multiply by equi-distant numbers?

PROB. X.—TO MULTIPLY WITHOUT WRITING THE PARTIAL PRODUCTS.

RULE.—*Multiply those numbers that will produce units, then all that will produce tens, all that will produce hundreds, and thus continue until the entire product is obtained.*

ILLUSTRATION 1.—Multiply 635 by 325.

635	5×5	= 25 units.
325	$5 \times 3 + 2 \times 5$	= 25 tens.
206375 Ans.	$5 \times 6 + 3 \times 5 + 3 \times 2$	= 51 hundreds.
	$2 \times 6 + 3 \times 3$	= 21 thousands.
	3×6	= 18 ten thousands.

NOTE.—*This problem is designed for mental exercise.*

EXAMPLES.

443. Multiply 684 by 326.
444. Multiply 784 by 348.
445. Multiply 531 by 427
446. Multiply 28 by 82.
447. Multiply 56 by 57.
448. Multiply 28 by 38.
449. Multiply 568 by 567.
450. Multiply 5687 by 8267.
451. Multiply 8426 by 321.
452. Multiply 3678 by 6842.
453. Multiply 384 by 368.
454. Multiply 745 by 326.
455. Multiply 504 by 326.
456. Multiply 3268 by 5687.
457. Multiply 784 by 556.
458. Multiply 456 by 263.
459. Multiply 326 by 223.
460. Multiply 284253 by 567842.
461. What is the sum of the answers on this page?

Ans. 161506489370.

What is the rule to multiply without writing the partial products.

SECTION II.

DIVISION.

PROB. XI.—TO DIVIDE BY TEN OR ANY POWER OF TEN.

RULE.—1. *Take for a remainder, from the right of the dividend, as many places as there are ciphers in the divisor.*

2. *The remaining figures of the dividend will be the whole numbers of the quotient.*

EXAMPLES.

462. Divide 78426 by 10.
463. Divide 37968 by 100.
464. Divide 4276831 by 1000.
465. Divide 4263700 by 10.
466. Divide 863200000 by 10000.
467. Divide 2984625 by 100.
468. Divide 8768426371 by 10000000.

PROB. XII. — TO DIVIDE BY THE ALIQUOT PARTS OF TEN OR ANY POWER OF TEN.

RULE.—1. *Multiply the dividend by the number of aliquot parts.*

2. *As many places, at the right of this product, as there are ciphers in the power of ten used, will be the remainder: the remaining figures will be the whole numbers of the quotient.*

EXAMPLES.

469. Divide 78426 by 25.
470. Divide 327684 by 125.
471. Divide 3278 by $12\frac{1}{2}$.
472. Divide 56826 by $16\frac{2}{3}$.
473. Divide 32784 by $11\frac{1}{9}$.
474. Divide 56878 by $33\frac{1}{3}$.
475. Divide 68782 by $66\frac{2}{3}$.
476. Divide 46826 by $8\frac{1}{3}$.
477. Divide 326847 by 625.
478. What is the sum of the remainders? Ans. 8438241.

What is the rule to divide by ten or any power of ten? What is the rule to divide by the aliquot parts of ten or any power of ten?

S[illegible]

D[illegible]

PROB. XI.—To [illegible]

RULE.—1. *Take* [illegible] *dividend, as man*[illegible]

2. *The remain*[illegible] *numbers of the quo*[illegible]

462. Divi[illegible] 7[illegible]
463. Divi[illegible] 37[illegible]
464. Divi[illegible] 42[illegible]
465. Divi[illegible] 42[illegible]
466. Divi[illegible] [illegible]
467. Divide 20[illegible]
468. Divi[illegible] [illegible]

PROB. XII.—[illegible] **ANY POWER OF** [illegible]

RULE.—1. *M*[illegible]

2. *As many* [illegible] *ciphers in the powe*[illegible] *remaining figures* [illegible]

469. Divide 7842[illegible]
470. Divide 8276[illegible]
471. Divide 827[illegible]
472. Divide 56[illegible]
473. Divide [illegible]
474. Divide [illegible]
475. Divide [illegible]
476. Divide [illegible]
477. Divide [illegible]
478. What is [illegible]

32019.

the process by

PROB. XIII.—TO DIVIDE BY THE FACTORS OF THE DIVISOR.

RULE.—1. *Divide the dividend by one factor of the divisor, and continue to divide successive quotients by successive factors, till all the factors of the divisor are used.*

2. *Multiply each partial remainder by all the preceding divisors, except the last: the sum of these products will be the true remainder.*

ILLUSTRATION.—Divide 7856774 by 2520.

OPERATION.

$$2520 = 3\times5\times6\times4\times7 \text{ the factors.}$$

$$\begin{array}{r|rlr}
3 & 7856774 & & \\
\hline
5 & 2618924 - 2 = & & 2 \\
\hline
6 & 523784 - 4 = & 4\times3 & 12 \\
\hline
4 & 87297 - 2 = & 2\times5\times3 & 30 \\
\hline
7 & 21824 - 1 = & 1\times6\times5\times3 \quad = & 90 \\
\hline
 & 3117 - 5 = & 5\times4\times6\times5\times3 = & 1800 \\
\hline
 & & & 1934
\end{array}$$

Ans. $3117\frac{1934}{2520}$.

EXAMPLES.

479. Divide 784268 by 16.
480. Divide 846826 by 24.
481. Divide 84626 by 32.
482. Divide 378462 by 48.
483. Divide 32684 by 64.
484. Divide 268427 by 72.
485. Divide 687894 by 84.
486. Divide 426347 by 128.
487. Divide 368476 by 144.
488. Divide 568426 by 2520.
489. Divide 3687821 by 504.
490. Divide 567878 by 360.
491. Divide 9368784 by 840.
492. What is the sum of the remainders? Ans. 2275.

What is the rule to divide by the factors of a divisor? What is meant by the divisors that produced it? Explain why that will give the true remainder.

PROB. XIV.—TO DIVIDE BY CANCELLING FACTORS.

RULE.—*Separate the dividend and divisor into their prime factors, and cancel those common. Then divide the product of those remaining in the dividend by the product of those remaining in the divisor.*

EXAMPLES.

493. Divide 5040 by 720.
494. Divide 15120 by 2160.
495. Divide 5040 by 144.
496. Divide 1092 by 91.
497. Divide 26092 by 341.
498. Divide 16536 by 39.
499. Divide 3751 by 682.
500. Divide 41261 by 451.
501. Divide 453871 by 253.

PROB. XV.—TO SHORTEN THE PROCESS OF DIVISION BY ARRANGEMENT.

RULE.—*Divide as in the General Rule, except omitting to bring down the figures in the dividend.*

ILLUSTRATION.—Divide 7853792 by 234.

OPERATION.

$$234\,)\,7\,8\,5\,3\,7\,9\,2\,(\,33563\tfrac{50}{234}\ \text{Ans.}$$

$$\begin{array}{rlcr}
\underline{7^1 0^1 2^1 2^2 0^3 4^4 2^5} & \text{1st Subtrahend,} & & 702 \\
8\,3\,1\,7\,5\,0^6 & \text{2d} & \text{“} & 702 \\
\underline{7^2 0^2 7^3 0^4 0^5} & \text{3d} & \text{“} & 1170 \\
1\,3\,4\,7\,5^6 & \text{4th} & \text{“} & 1404 \\
\underline{1^3 1^3 4^4 7^5} & \text{5th} & \text{“} & 702 \\
1^4 & \text{6th} & \text{“} & 50
\end{array}$$

EXAMPLES.

502. Divide 7845682 by 8462.
503. Divide 8426327 by 3784.
504. Divide 8468782 by 56842.
505. What is the sum of the remainders? Ans. 32019.

How do you divide by cancelling? How can you shorten the process by arrangement?

PROB. XVI.—TO SHORTEN DIVISION BY A MENTAL PROCESS.

RULE.—*Perform the usual operations mentally, except writing down the remainders over the dividend.*

ILLUSTRATION.—Divide 3474325 by 102.

$$\begin{array}{r} 4\ 62 \\ 1630 \\ 102)\underline{3474325} \\ 34062\tfrac{1}{102}\ \text{Ans.} \end{array}$$

EXAMPLES.

506. Divide 368787 by 252.
507. Divide 327845 by 326.
508. Divide 784268 by 103.
509. Divide 368247 by 105.
510. Divide 234684 by 201.
511. Divide 2784567 by 311.
512. Divide 368278 by 221.
513. Divide 326846 by 323.
514. Divide 847689 by 406.
515. Divide 897887 by 807.
516. Divide 3684569 by 306.
517. Divide 826388 by 504.
518. Divide 568498 by 802.
519. Divide 368745 by 3201.
520. Divide 934257 by 8602.
521. Divide 4437821 by 3681.
522. Divide 9134578 by 913421.
523. Divide 2357942 by 23572.
524. Divide 3756824 by 37471.
525. What is the sum of the remainders? Ans. 21878.

Explain the process of dividing when the remainders only are written.

CHAPTER III.

PROPERTIES OF NUMBERS.

SECTION I.

DIVISIBILITY OF NUMBERS.

50. An *Integral Number* is a unit or a collection of like units.

51. A *Prime Number* has no factors greater than unity.

52. A *Composite Number* is the product of two or more prime numbers.

53. A *Prime Factor* of a *Number*, is a prime number that will exactly divide it. All other factors are composite factors.

54. A *Number is said to be divisible* when its factors are whole numbers greater than unity.

55. A *Number to be divisible* by a given number, must be divisible by each factor of the given number.

56. No number can have a prime factor greater than its square root.

57. A composite number is divisible by any one of its factors.

58. A number that will divide two or more numbers, will divide their *sum*, *difference* or *product*.

59. Numbers are prime to each other when they have no common factor.

60. The difference between every number and the sum of its digits, is divisible by nine.

61. If a number is divisible by two or more prime numbers, it is divisible by their product.

---

50. What is an integral number? 51. What is a prime number? 52. What is a composite number? 53. What is a prime factor? A composite factor? 54. When is a number divisible? 55. How can we know that a number is divisible by a given number? 56. What is the largest prime number that a given number can have? 57. What will divide any composite number? 58. What will divide the sum, difference or product of two or more numbers? 59. When are numbers prime to each other? 60. What is the difference between any number and the sum of its digits divisible by —? 61. When is a number divisible by the product of two or more prime numbers?

62. A number is separated into factors when the factors are obtained that will produce it.

63. A number is separated into its prime factors when it is divided into its greatest possible number of factors.

64. Every prime number, except 1 and 2, is one greater or one less than some multiple of 4.

65. Every prime number, except 1, 2 and 3, is one greater or one less than some multiple of 6.

66. Two is the only even prime number.

67. An even number is one that is divisible by 2.

68. An odd number is one that is not divisible by 2.

69. The sum of two or more even numbers, is even.

70. The difference of two even or two odd numbers, is even.

71. The sum of an even number of odd numbers, is even.

72. The sum of an odd number of odd numbers, is odd.

73. The sum or difference of an even and odd number, is odd.

74. The product of an even and odd number, is even.

75. The quotient of an even dividend by an odd number, is even.

76. The product of any number of factors is even, if one of them is even.

77. The product of any number of odd numbers, is odd.

78. If an even number be divided by an odd, its half may be.

79. The product of two square numbers, is a square.

80. The quotient of a square, divided by a square, is a square.

81. The difference between a cube and its root, is divisible by 6.

82. *Two will divide* any even number.

---

62. When is a number separated into factors? 63. When is a number separated into its prime factors? Repeat principle 64, 65. 66. What is the only even prime number? 67. What is an even number? 68. What is an odd number? 69. Is the sum of two or more even numbers, odd or even? 70. How is their difference? 71. How is the sum of an even number of odd numbers? 72. How is it with an odd number of odd numbers? 78. Is the sum or difference of an odd and even number odd or even? 74. What is the product of an odd and an even number? 75. The quotient of an even divided by an odd? 76. The product of any number of factors if one is even? 77. The product of any number of odd numbers? Repeat prin. 79, 80, 81. 82. What numbers will 2 divide?

83. *Three will divide* any number the sum of whose digits is divisible by three.

84. *Four will divide* any number whose two right hand digits are divisible by four.

85. *Five will divide* any number with a five or a cipher in the unit's place

86. *Six will divide* any even number that is divisible by three.

87. *Eight will divide* any number whose three right hand digits are divisible by eight.

88. *Nine will divide* any number the sum of whose digits is divisible by nine.

89. *Ten will divide* any number with a 0 in the unit's place.

90. *Eleven will divide* any number the sums of whose alternate digits are equal, or differ by a multiple of eleven.

91. *Twelve will divide* any number divisible by three and four.

92. *Fifteen will divide* any number divisible by three and five.

PROB. XVII.—TO FIND SOME OF THE SMALLER PRIME NUMBERS.

RULE.—1. *Write an indefinite series of odd numbers, commencing with one. Then counting from three, mark every third number; counting from five, mark every fifth number; and so continue counting from every prime number.*

2. *The numbers not marked are prime.*

NOTE.—This rule was discovered by Eratosthenes, and is called "Eratosthenes' Sieve." For a list of prime numbers to 829, see page 34.

ILLUSTRATION.—1, 3, 5, 7, ~~9~~, 11, 13, ~~15~~, 17, 19, ~~21~~, 23, ~~25~~, ~~27~~, 29, 31, ~~33~~, ~~35~~, 37, ~~39~~, 41, 43, ~~45~~, 47, ~~49~~, ~~51~~, 53, ~~55~~, ~~57~~, 59, 61, ~~63~~, &c. The unmarked numbers are prime.

EXAMPLES.

526. Find the prime numbers from the 18th to the 25th.

527. From 25th to 35th.
528. From 35th to 45th.
529. From 45th to 55th.
530. From 55th to 65th.
531. From 65th to 75th.
532. From 75th to 85th.
533. From 85th to 95th.
534. From 95th to 100th.
535. From 100th to 110th.
536. From 110th to 120th.

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83. What numbers will 3 divide? 84. Four? 85. Five? 86. Six? 87 Eight? 88. Nine? 89. Ten? 90. Eleven? 91. Twelve? 92. Fifteen? Repeat Eratosthenes' Sieve.

PROB. XVIII.—TO ASCERTAIN WHETHER A NUMBER IS PRIME OR NOT.

RULE.—1. *Ascertain if the number is divisible by any prime number from 1 to its square root.*

2. *If it is not divisible, it is prime.*

3. *If it is divisible, it is composite, and the several divisors, with the last quotient, will be its prime factors.*

ILLUSTRATION 1.—Examine 48.

This is divisible by 2 four times. The number is therefore composite, and its prime factors are 2, 2, 2, 2, 3.

2	48
2	24
2	12
2	6
	3

ILLUSTRATION 2.—Examine 2805.

By sections 82 to 92 we perceive that it is not divisible by 2, and is by 3; dividing, we find the quotient is divisible by 5; we then try that quotient by 7, and find it not divisible, but 11 divides and gives a prime quotient, 17. The number is composite, and its prime factors are 3, 5, 11, 17.

3	2805
5	935
11	187
	17

ILLUSTRATION 3.—Examine 191.

As before, we examine it by the numbers of the prime series, and find it is not divisible by 2, 3, 5, 7, 11 or 13; therefore it is prime.

EXAMPLES.

537. Examine 432 and 233.
538. Examine 6851 and 435.
539. Examine 8456 and 627.
540. Examine 8784 and 381.
541. Examine 7984 and 383.
542. Examine 3681 and 443.
543. Examine 326 and 4515.
544. Examine 4768 and 9009.
545. Examine 3256 and 4816.
546. Examine 278 and 4214.
547. Examine 439, 707, 9191.
548. Examine 407, 8008, 7676.
549. How many prime factors are found in the last 12 examples? Ans. 93.

How can you find whether a number is prime or composite?

PROB. XIX.—TO DIVIDE TWO OR MORE NUMBERS INTO THEIR PRIME FACTORS.

RULE.—1. *Arrange the numbers in a line.*

2. *Divide all those that can be exactly divided, by the numbers in the prime series successively, and bring down the undivided numbers in a line with the quotients.*

3. *Continue dividing and bringing down till all are prime.*

4. *The prime factors of each number will consist of its quotient and its prime divisors.*

NOTE.—The principle of Problems XVIII. and XIX. are identical.

ILLUSTRATION.—Separate 12, 16, 20, 24, 30, 42, 48 and 50 into prime factors.

2	12	16	30	24	30	42	48	50
2	6	8	15	12	15	21	24	25
2	3	4	15	6	15	21	12	25
3	3	2	15	3	15	21	6	25
5	1	2	5	1	5	7	2	25
	1	2	1	1	1	7	2	5

The prime factors of 12 are 2, 2, 3; of 16—2, 2, 2, 2; of 30—2, 3, 5; of 24—2, 2, 2, 3; of 30—2, 3, 5; of 42—2, 3, 7; of 48—2, 2, 2, 2, 3; of 50—2, 5, 5.

EXAMPLES.

550. Separate 81, 42, 68 and 27 into prime factors.
551. Separate 39, 45, 78 and 96 into prime factors.
552. Separate 56, 86, 92 and 128 into prime factors.
553. Separate 32, 47, 53, 61 and 73 into prime factors.
554. Separate 12, 18, 28, 34 and 54 into prime factors.
555. Separate 16, 18, 27, 32, 57 and 82 into prime factors.
556. Separate 13, 17, 19, 49, 79, 47 and 91 into prime factors
557. Separate 126, 138, 156, 183 and 12 into prime factors.
558. Separate 246, 482 and 912 into prime factors.
559. How many prime divisors in the last 9 examples? Ans. 42.

What is first to be done to separate numbers into their prime factors? Second? Third? Fourth? Give the rule.

SECTION II.

MEASURES OR DIVISORS.

93. *A Measure* or *Divisor* of a Number, is a number that will divide it without a remainder.

94. *A Common Measure* or *Divisor* of two or more Numbers, is any number that will divide them without a remainder.

95. *The Greatest Common Measure* of several Numbers, is the largest number that will measure them.

PROB. XX.—TO FIND ALL THE INTEGRAL DIVISORS OF A NUMBER.

RULE.—1. *Separate the number into all its prime factors.*

2. *From these prime factors form as many products as possible, and these, with the prime factors themselves, will be all the divisors of the given number.*

ILLUSTRATION.—Find all the divisors of 48.

48 = 1, 2, 2, 2, 2, 3, the number of prime factors. From these we find the following products, which, with the factors themselves, make the divisors required—1, 2, 3, 4, 6, 8, 12, 16, 24, 48—ten divisors.

EXAMPLES.

560. What are the divisors of 72? 96? 144?
561. What are the divisors of 27? 38? 56?
562. What are the divisors of 84? 636? 545?

PROB. XXI.—TO FIND ALL THE COMMON DIVISORS OF SEVERAL NUMBERS.

RULE.—1. *Find all the common prime factors.*

2. *From these prime factors form as many products as possible.*

EXAMPLES.

563. What are the common divisors of 48, 64 and 144?
564. What are the common divisors of 60, 72, 96 and 80?
565. How many divisors? Ans. 87.

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93. What is a measure or divisor? 94. What is a common measure? 95. What is the greatest common measure? Repeat Prob. XX. Rule. Prob. XXI. Rule. What is the difference between them?

PROB. XXII.—TO FIND THE GREATEST COMMON MEASURE OF SEVERAL NUMBERS EASILY SEPARATED INTO THEIR PRIME FACTORS.

RULE.—1*st Method.—Separate the numbers into their prime factors; the product of their common factors will be their greatest common measure.*

Or 2d Method.—1. *Separate the difference between any two of the given numbers into its prime factors.*

2. *The product of such of these prime factors as will divide all the given numbers, is the greatest common divisor.*

Or 3d Method.—Separate one of the given numbers into its prime factors, and then proceed as in 2d Method.

NOTE.—If there are no common factors, the numbers are prime to each other.

EXAMPLES.

566. Find the greatest common measure of 24, 32 and 48.
567. " " 628 and 84.
568. " " 682 and 796.
569. " " 524 and 284.
570. " " 630 and 729.
571. " " 327, 426 and 49.
572. " " 1566, 326 and 48.
573. " " 786, 321 and 312.
574. " " 327, 360 and 1872.
575. " " 612, 512 and 684.
576. " " 427, 627 and 841.
577. " " 381, 81 and 921.
578. " " 24, 624 and 912.
579. " " 34, 786 and 1028.
580. " " 972, 818, 636 and 66.
581. " " 800, 398 and 1184.
582. " " 327, 317 and 93.
583. What is the sum of the answers? Ans. 72.

What is Prob. XXII? Repeat the rule for the 1st method. 2d. 3d. What class of questions can best be solved by the 1st method? Why? 2d? Why? 3d? Why? By which method would you solve Ex. 578? Ex. 573? Why?

PROB. XXIII.—4TH METHOD.—TO FIND THE GREATEST COMMON MEASURE OF SEVERAL NUMBERS NOT EASILY SEPARATED INTO THEIR PRIME FACTORS.

RULE.—1. *Divide one number by another, and the last divisor by the last remainder, until nothing remains.*

2. *Proceed in the same manner with the last divisor and a third number, and so on till all the numbers are used.*

3. *The last divisor will be the G. C. Measure.*

NOTE.—The principle involved in the 4th Method is identical with, and is fully exhibited in Problem XXII.

ILLUSTRATION.—Find the G. C. M. of 221, 91, 273 and 546.

```
91)221(2          or          221)546(2
   182                            442
   ---                            ---
    39)91(2                       104)221(2
       78                             208
       --                             ---
       13)39(3                         13)104(8
          39                              104
          --                              ---
```

$273 \div 13 = 21$

$546 \div 13 = 42$

$91 \div 13 = 7$

Ans. 13 in each case.

EXAMPLES.

584. Find the G. C. M. of 826, 790 and 846.
585. Find the G. C. M. of 324, 768 and 972.
586. Find the G. C. M. of 332, 648 and 1088.
587. Find the G. C. M. of 144, 672 and 948.
588. Find the G. C. M. of 288, 3360 and 1152.
589. Find the G. C. M. of 561, 346 and 851.
590. Find the G. C. M. of 327, 7236 and 8121.
591. Find the G. C. M. of 28, 777 and 2226.
592. What is the sum of all the answers? Ans. 128.

What is Prob. XXIII? What first to be done? Second? Third? Repeat the rule.

SECTION III.

MULTIPLES OR DIVIDENDS.

96. When any number is divisible by one given number, it is a *Multiple of that Number.*

97. When any number is divisible by two or more given numbers, it is a *Common Multiple of those Numbers.*

98. The smallest number that is divisible by two or more given numbers, is the *Least Common Multiple of those Numbers.*

99. From the above it follows, that a Multiple of a number must contain the prime factors of that number.

100. Also, that the Least Common Multiple of several numbers must contain all their prime factors, and no more.

PROB. XXIV.—TO FIND THE LEAST COMMON MULTIPLE OF SEVERAL NUMBERS EASILY SEPARATED INTO PRIME FACTORS.

RULE.—1. *Separate the numbers into their prime factors.*

2. *The product of the different factors, using each the greatest number of times it occurs in any given number, will be the least common multiple.*

ILLUSTRATION.—Find the L. C. M. of 27, 36 and 48.

27 = 3, 3, 3.
86 = 2, 2, 3, 3.
48 = 2, 2, 2, 2, 3.

We here find the prime factors 2 and 3. 2 is found 4 times in 48, and 3 is found 3 times in 27. Therefore the product of 2, 2, 2, 2, and 3, 3, 3, gives the L. C. M., 432.

EXAMPLES.

593. Find the L. C. M. of 12, 36, 84, 56 and 62.
594. Find the L. C. M. of 17, 34, 42, 21 and 84.
595. Find the L. C. M. of 19, 28, 42, 14, 86 and 96.
596. Find the L. C. M. of 144, 36, 72, 48, 24 and 18.
597. Find the L. C. M. of 272, 846, 728 and 128.
598. What is the sum of the answers? Ans. 84326988.

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96. What is a multiple of a number? 97. What is a common multiple of a number? 98. What is the least common multiple of a number? 99. What constitutes a multiple? 100. What constitutes a least common multiple? What is first to be done to find a L. C. M.? Second? What is the rule?

PROB. XXV.—2D METHOD.—TO FIND THE L. C. MULTIPLE WHEN THE NUMBERS ARE EASILY SEPARATED INTO PRIME FACTORS.

RULE.—1. *Cancel any number that will measure another.*

2. *Separate the remaining numbers into their prime factors.*

3. *Proceed as in 2d part of Rule, Prob. XXIV.*

ILLUSTRATION.—Find the L. C. M. of 17, 34, 42, 21 and 84.

As 17 will measure 34, and as 21 and 42 will measure 84, we reject them, and separate 34 and 84 into prime factors, as in Prob. XXIV., thus: 34 = 2, 17.
84 = 2, 2, 3, 7. The L. C. M. then is 2, 2, 3, 7: 17 = 1428 Ans.

EXAMPLES.

599. Find the L. C. M. of 328, 64, 32, 45 and 180.
600. Find the L. C. M. of 421, 327, 918 and 306.
601. Find the L. C. M. of 468, 234, 117 and 921.
602. Find the L. C. M. of 72, 48, 36, 54 and 27.
603. Find the L. C. M. of 186, 31, 279 and 585.
604. Find the L. C. M. of 128, 256, 1024, 72 and 36.
605. Find the L. C. M. of 812, 406, 2031 and 56.
606. Find the L. C. M. of 972, 162, 84 and 132.
607. Find the L. C. M. of 544, 658, 329 and 78.
608. Find the L. C. M. of 42, 84, 26, 38, 58 and 12.
609. Find the L. C. M. of 8, 10, 12, 14, 16, 18 and 20.
610. Find the L. C. M. of 3, 5, 7, 9, 11, 13, 15, 17, 19 and 21.
611. Find the L. C. M. of 22, 24, 26, 28, 44, 46 and 48.
612. Find the L. C. M. of 56, 58, 68, 78, 98 and 108.
613. Find the L. C. M. of 12, 38, 42, 84, 76 and 152.
614. Find the L. C. M. of 30, 40, 50, 60, 70, 80, 90 and 100.
615. Find the L. C. M. of 12, 28, 36, 82, 47 and 94.
616. Find the L. C. M. of 78, 36, 84, 56 and 68.
617. Find the L. C. M. of 73, 52, 86, 43, 23 and 13.
618. Find the L. C. M. of 75, 55, 45, 15, 25 and 155.
619. What is the sum of the answers? Ans. 138040884.

What is to be done in the 2d method first? Second? Third? What is the entire rule.

PROB. XXVI.—TO FIND THE L. C. MULTIPLE WHEN THE NUMBERS ARE NOT EASILY SEPARATED INTO THEIR PRIME FACTORS.

RULE.—*Divide one of the numbers by their greatest common measure. Multiply this quotient by the other.*

ILLUSTRATION.—Find the L. C. M. of 7842 and 8426.

First find their greatest common measure, thus:—

```
7842)8426(1
     7842
      584)7842(13
           584
           2002
           1752
      584( 250
      500   2
       84)250(2
          168
       84( 82
       82   1
        2)82
          41
```

With this greatest common measure, 2, we divide 8426, thus, and multiply this result by the other number, 7842.

```
2)8426
  4213
```

```
     7842
     4213
    23526
    7842
  15684
 31368
```

The least common multiple, 33038346 Ans.

EXAMPLES.

620. Find the L. C. M. of 8432 and 7858.
621. Find the L. C. M. of 3466 and 4828.
622. Find the L. C. M. of 4284 and 3672.
623. Find the L. C. M. of 8516 and 496.
624. Find the L. C. M. of 578 and 786.
625. Find the L. C. M. of 4428 and 3874.
626. Find the L. C. M. of 386 and 3867.
627. Find the L. C. M. of 26844 and 48788.
628. Find the L. C. M. of 9512 and 4728.
629. Find the L. C. M. of 868 and 8632.
630. What is the sum of the answers? Ans. 388785796.

What is the rule to find the least common multiple when the numbers are not easily separated into prime factors?

SECTION IV.

PROBLEMS UNDER THE PRECEDING RULES.

Prob. XXVII.—The sum of two numbers, and one of them given, to find the other.

Rule.—*Subtract the given number from the sum.*

Prob. XXVIII.—The sum of three numbers, and two of them given, to find the third.

Rule.—*From the given sum subtract the sum of the two numbers.*

Prob. XXIX.—The difference and less of two numbers given, to find the greater.

Rule.—*Add the difference to the less.*

Prob. XXX.—The difference and greater of two numbers given, to find the less.

Rule.—*Subtract the difference from the greater.*

Prob. XXXI.—The sum and difference of two numbers given, to find the two numbers.

Rule.—*Subtract the difference from the sum, and divide the remainder by 2. The quotient will be the smaller number.*

Prob. XXXII.—The product of two numbers, and one of them given, to find the other.

Rule.—*Divide the product by the given number.*

Prob. XXXIII.—The product of three numbers, and two of them given, to find the third.

Rule.—*Divide the given product by the product of the two numbers.*

Prob. XXXIV.—The divisor and quotient given, to find the dividend.

Rule.—*Multiply them together.*

Prob. XXXV.—The dividend and quotient given, to find the divisor.

Rule.—*Divide the dividend by the quotient.*

What is Prob. XXVII? Give the rule. What is Prob. XXVIII? Rule. What is Prob. XXIX? Rule. What is Prob. XXX? Rule. What is Prob. XXXI? Rule. How do you find the other number? What is Prob. XXXII? Rule. What is Prob. XXXIII? Rule. What is Prob. XXXIV? Rule. What is Prob XXXV? Rule.

EXAMPLES.

631. Lucius Smith has $7586 on deposit in the Banks of Washington and Metropolis; $3478 is in the Bank of Metropolis. How much is in the Bank of Washington?

632. From Boston to New York is 234 miles. From Boston to Hartford, 106 miles. How far from Hartford to New York?

633. James is worth as much as John, William, and George. James is worth $743625, William $94137, and George $357221. How much is John worth?

634. How much are they all worth?

635. If Charles should give his marbles to Joseph, he would then have as many as Thomas. Charles has 153, and Joseph 287. How many has Thomas?

636. How many have they all?

637. Samuel Brown is worth as much as the difference between Daniel Frink's and George Belden's property. Brown is worth $2841, and Frink, the wealthiest, $17981. What is Belden worth?

638. What are they all worth?

639. How many barrels of flour can I buy with $17,876.25, paying $11.25 per barrel?

640. How many boxes of shoes can I buy with $9450, each box containing 50 pairs worth $1.50 per pair?

641. The product of three numbers is 6822, and two of the numbers are 6 and 3. What is the third?

642. The divisor is 57 and quotient 31. What is the dividend?

643. The divisor 72 and dividend 78408. What is the quotient?

644. The multiplicand is 7845, the product 54915. What is the multiplier?

645. The subtrahend is 784, the difference 5432. What is the minuend?

646. The terms are 5, 6, 7, 8, 9 and 10. What is the amount?

647. The factors are 5, 6, 7, 8, 9 and 10. What is the product?

648. The quotient is $42\frac{11}{72}$. What is the dividend and divisor?

649. What is the sum of the answers? **Ans. 1998593.**

SECTION V.

CANCELLATION.

101. Cancelling is suppressing or rejecting equal *factors.*

102. The principle involved is, that to multiply a quantity, and divide the product *by the same number*, does not change its value.

103. Cancelling is applicable to all quantities which may be considered in the relation of *Divisor* and *Dividend.*

Note.—To be expert in cancelling, it is necessary to have a thorough knowledge of the preceding part of this chapter, especially pp. 53, 54 and 55.

Prob. XXXVI.—To cancel numbers.

Rule.—*Find all the common factors in the divisor and dividend, and suppress them.*

Illustration.—How much is 48 multiplied by 27, and divided by 18?

$$\begin{array}{r|l} \text{Divisors.} & \text{Dividends.} \\ & 48 = \cancel{2}\times 24 \\ \cancel{2}\times\cancel{9} = 18 & 27 = 3\times\ \cancel{9} \\ \hline & 3\times 24 = 72 \text{ Ans.} \end{array} \qquad \text{or} \qquad \begin{array}{r|r} & \cancel{48}\ \ 24 \\ \cancel{18} & \cancel{27}\ \ \ 3 \\ \hline & 72 \text{ Ans.} \end{array}$$

EXAMPLES.

650. How much is 72 multiplied by 48 and 32, and divided by 16 and 64?

651. How much is 72 × 68 × 24 × 84 ÷ 144?

652. How much is 128, 7, 9, 72 and 96, and divided by 4, 16, 32 and 84?

653. Multiply 13, 7, 9, 11, 17 and 21, and divide the product by 63, 187 and 3.

654. Multiply 27, 16, 18, 36, 54 and 31, and divide by 108, 93 and 144.

655. What is the sum of the answers? Ans. 69391.

---

101. What is cancelling? 102. What is the principle involved? 103. To what is it applicable? What should the student be familiar with? Repeat the 53d, 54th and 55th pages.

MISCELLANEOUS QUESTIONS.

656. A pound of Avoirdupois weight contains 7000 grains; a pound of Troy contains 5760. How many grains in 6 pounds Troy and 4 pounds Avoirdupois?

657. How many more grains in 27 pounds Avoirdupois than in 31 pounds Troy?

658. How many pounds Troy in 97920 grains?

659. The human heart usually beats 70 times in a minute. How many times in an hour?

660. A year is 365 days and 6 hours. How many hours is that?

661. How many times does a healthy human heart beat in 71 years?

662. A propeller and a clipper went to sea at the same time, on the same course. The propeller averaged 8 miles per hour, losing 4 hours for repairs. The clipper, 13 miles per hour for 2 days, 5 miles per hour for 3 days, was becalmed 18 hours, and the rest of the time averaged 9 miles per hour. Which was in advance, and how far, at the end of 20 days?

663. Each number of Harper's Magazine contains 144 pages. If 178,000 copies are printed, how many pages of reading matter does it make?

664. Each copy weighs half a pound. Suppose a man could load 75 pounds into a cart in a minute, how many minutes would it take him to load 178,000?

665. Three men formed a hunting party, and travelled through Arkansas, Texas and Louisiana. They were gone 2 months, September and October. They travelled one-half the time at the rate of 26 miles, and the other half 30 miles per day. Started Monday, camped every Sunday and every other Saturday. How many miles did they pass over?

666. What is the sum of the answers? Ans. 2639741995.

How many grains in a pound Avoirdupois? How many in a pound Troy? How many beats of a healthy human heart in a minute? How many inspirations in a minute? How many beats of the heart to one inspiration?

667. The average speed of express trains on American railroads, is 40 miles per hour. Accommodation trains 25 miles per hour. The distance from Boston to Worcester is 44 miles, to Springfield 56, to Albany 100, to Schenectady 17, to Utica 78, to Syracuse 53, to Rochester 81, to Buffalo 69. How long would it take an express train to pass over the entire road, allowing 1 hour stoppage at each named place?

668. How long for an accommodation train, allowing 1 hour as above, and 5 minutes every ten miles?

669. How long would it take a horse, allowing him to rest Sundays, and travel 57 miles daily?

670. A man can walk 40 miles daily. How long would it take a man to walk from Boston to Buffalo?

671. The population of the United States in 1850 was 23,256,972; number of slaves, 3,198,324; number of free colored, 428,515. How many white persons?

672. The population in 1790 was 3,929,827. What was the increase to 1850?

673. The population of New York, in 1850, was 3,097,394; Virginia, 1,421,661; Tennessee, 1,002,625; Ohio, 1,980,408. How many more inhabitants have Virginia, Tennessee, and Ohio together, than New York?

674. 1. Washington was elected President of the United States at the age of 57, and died at 68 years.

2. John Adams at the age of 62, and died at 91 years.
3. Thomas Jefferson " 58, " 83 "
4. James Madison " 58, " 85 "
5. James Monroe " 58, " 72 "
6. John Quincy Adams " 58, " 80 "

What is the average speed of an express train on American railroads? Of an accommodation train? What was the population of the United States in 1850? What in 1790? Of New York? Virginia? Tennessee? Ohio? At what age were the first 6 Presidents elected? At what age did they die? Which lived to be the oldest?

Give the rule to Prob. XXVI, XXV, XXIV, XXIII, XXII, XXI, XX, XIX, XVIII, XVII, XVI, XV, XIV, XIII, XII, XI, X, IX, VIII, VII, VI, V, IV, III, II, I. Now give the text of the problems.

7. Andrew Jackson at the age of 62, and died at 78.
8. Martin Van Buren " 55.
9. William Henry Harrison " 68, and died at 68.
10. John Tyler " 51.
11. James K. Polk " 49, and died at 54.
12. Zachary Taylor " 65, and died at 66.
13. Millard Fillmore " 50.
14. Franklin Pierce " 49.
15. James Buchanan " 00.

How long did those deceased, live after their accession to the Presidency?

675. What is the value of $187 + 768 + 684 - 726 - 786 + 846 - 786 + 8246 - 1728$?

676. What is the value of $8 \times 64 \times 72 + 8 \times 16 - 87 \times 5 + 11 \times 87 - 68$?

677. What is the value of 8, 8, 71, 65, 57, 57?

678. What is the value of $7 \times 56 \times 7 \times 8 \times 7 \div 521$?

679. What is the value of $(\overline{8 + 6} \times 6 - \overline{12 \times 2} + 6 + 91 \times 3) \div 23$?

680. What is the value of XII + XVIII + XXXVIII — XLIV?

681. What is the value of C + D + MD + DM + IV + XC?

682. What will 86 lbs. of meat cost at 8¼ cents per lb.?

683. James bought 282 oranges at 6¼ cents for 3, and sold them for $6.00. What was his profit?

684. Dick Brown bought 100 marbles at 2 for a cent, and 300 more at 3 for a cent. He then sold them all at 5 for a cent. Did he make or lose? How much?

685. Cæsar is said to have fought 50 pitched battles, in which 1,920,000 persons were killed. He was assassinated in the 56th year of his age. Suppose he commenced his public career at 21. What number of persons did he cause to be killed annually?

What were the ages of the last 9 Presidents on their accession to the Presidency? Which died in office? Which died the youngest? How many, and who, are now living? Where is their place of residence? How many battles did Cæsar fight? How old was he when he was killed? What were the circumstances of his death?

DIVISION II.

FRACTIONS

GENERAL PRINCIPLES.

104. When a dividend contains a divisor a certain number of times without a remainder, the division is called *exact division.*

105. When there is a remainder, and it is left undivided, the division is called *incomplete division.*

106. When the entire dividend is divided, or the division indicated, it is called *complete division.*

107. From indicated or unexecuted division, arises a new class of numerical expressions, called fractions. Thus, if I wish to divide 6 by 7, I find that six entire units cannot be divided into 7 equal parts, and that the division must be indicated. This expression is called a fraction, and is read six-sevenths, written thus : $\frac{6}{7}$.

108. This fraction, $\frac{6}{7}$, means that 6 is to be divided by 7. To ascertain its value, we must suppose a unit to have been divided into 7 equal parts, and that as many of those parts were taken in the fraction as the dividend expresses, viz. 6.

109. The idea of a fraction, then, involves two units. 1st, the unit to be divided; and, 2d, one of the equal parts after division. The unit to be divided is called the unit of the fraction. One of the equal parts is called the fractional unit. Thus, in the expression, $\frac{5}{8}$ of an apple. The "*unit of the fraction*" is a *whole apple.* The *fractional unit* is one piece of an apple cut into 8 equal parts.

110. A *Fraction* is the numerical expression of an aliquot part of unity, or a number of such parts.

---

104. What is exact division? 105. What is incomplete division? 106. What is complete division? 107. What gives rise to fractions? What two ways will express the division of 6 apples among 3 boys? How can you indicate division? What is an indicated division? 108. What does $\frac{6}{7}$ mean? In $\frac{6}{7}$ which is the dividend? Which the divisor? 109. What does the idea of a fraction involve first? Second? What is the unit of a fraction? What is a fractional unit? Explain the difference. 110. What is a fraction? What do you mean by a numerical expression? What is an aliquot part?

111. It must necessarily be expressed by two numbers. One denoting the number of parts into which the unit is divided, and the other, the number of such parts taken.

112. The *denominator* or *divisor* shows the number of parts into which the unit is divided, and is *below* the line.

113. The *numerator* or *dividend* shows the number of parts taken in the fraction, and is *above* the line.

114. The value of a fraction is the quotient of the numerator divided by the denominator.

115. The numerator and denominator of a fraction, taken together, are called the terms of the fraction.

116. The following propositions are evident, and constitute *the basis of all rules in fractions.*

PROP. 1.—IF THE NUMERATOR OF A FRACTION BE INCREASED, THE VALUE OF THE FRACTION WILL BE INCREASED.

1*st. The numerator may be increased by addition.*

ILLUSTRATION.—John has $\frac{2}{6}$ of an apple, and Jane gives him $\frac{1}{6}$ more. How many sixths has he now? $\frac{2}{6}+\frac{1}{6}=\frac{3}{6}$ Ans.

EXAMPLES.

686. How much is $\frac{3}{5}+\frac{6}{5}+\frac{2}{5}+\frac{3}{5}$? Ans. $\frac{14}{5}$.

687. Add $\frac{4}{7}$, $\frac{6}{7}$, $\frac{8}{7}$, $\frac{10}{7}$ and 1 or $\frac{7}{7}$ together. Ans. $\frac{35}{7}$.

688. Add $\frac{15}{16}$, $\frac{11}{16}$, $\frac{9}{16}$ and 2 together. Ans. $\frac{67}{16}$.

689. Add $\frac{3}{17}$, $\frac{9}{17}$, $\frac{11}{17}$, $\frac{19}{17}$ and $3\frac{3}{17}$. Ans. $\frac{96}{17}$.

2*d. The numerator may be increased by multiplication.*

ILLUSTRATION.—John has $\frac{2}{6}$ of an apple, but Jane has 3 times as much. How much has Jane? $\frac{2}{6}\times 3=\frac{6}{6}$.

EXAMPLES.

690. Multiply $\frac{3}{12}$ by 2. Ans. $\frac{6}{12}$.

691. Multiply $\frac{4}{19}$ by 6. Ans. $\frac{24}{19}$.

---

111. Why must a fraction be expressed by two numbers? What does each number represent? 112. What is a denominator? 113. What is a numerator? 114. How can you find the value of a fraction? What is Prop. 1? How many ways can the numerator be increased? What does the first imply? Second? 115. What are the terms of a fraction?

PROP. 2.—IF THE NUMERATOR OF A FRACTION BE DIMINISHED, THE VALUE OF A FRACTION WILL BE DIMINISHED.

1*st. The numerator may be diminished by subtraction.*

ILLUSTRATION.—John has $\frac{5}{6}$ of an apple, and gives Jane $\frac{2}{6}$. How much has John left? $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$ Ans.

EXAMPLES.

691. Allen Brown owned $\frac{9}{13}$ of an acre, and sold $\frac{2}{13}$. How much left?

692. From $\frac{9}{17}$ take $\frac{2}{17}$.

693. Subtract $\frac{11}{21}$ from $\frac{13}{21}$.

694. Subtract $\frac{41}{84}$ from $\frac{91}{84}$.

695. Subtract $\frac{11}{29}$ from 1 or $\frac{29}{29}$.

696. Subtract $\frac{13}{17}$ from 2.

697. What is the difference between $\frac{87}{95}$ and $\frac{23}{95}$?

698. What is the difference between $\frac{14}{33}$ and $\frac{17}{33}$?

699. Which is the most, $\frac{11}{17} - \frac{6}{17}$ or $\frac{23}{17} - \frac{18}{17}$?

700. Take $\frac{13}{19}$ from 5.

2*d. The numerator may be diminished by division.*

ILLUSTRATION.—John divided $\frac{12}{3}$ of an apple among his 3 brothers. How many thirds did each have? $\frac{12}{3} \div 3 = \frac{4}{3}$ Ans.

EXAMPLES.

701. Divide $\frac{24}{7}$ by 6.

702. Divide $\frac{36}{17}$ by 9.

703. Divide $\frac{28}{13}$ by 7.

704. Divide $\frac{72}{128}$ by 36.

705. Divide $\frac{64}{363}$ by 32.

706. Divide $\frac{94}{27} + \frac{28}{27}$ by 61.

707. Divide $\frac{81}{411} + \frac{311}{411}$ by 196.

708. Divide $\frac{79}{144} + \frac{168}{144}$ by 247.

709. Divide $\frac{83}{1440} + \frac{178}{1440} - \frac{11}{1440}$ by 50.

What is Prop. 2d? How may the numerator be diminished first? How second? How may a fraction be divided? How may one fraction be subtracted from another? How may a fraction be multiplied? How may two fractions be added?

Prop. 3.—IF THE DENOMINATOR OF A FRACTION BE MULTIPLIED, THE VALUE OF THE FRACTION WILL BE DIVIDED BY THE SAME NUMBER.

Illustration.—John has $\frac{3}{4}$ of an apple, and he divides it between his 2 brothers. What part of an apple does each have? $\frac{3}{4} \div 5 = \frac{3}{8}$ Ans.

EXAMPLES.

710. Divide $\frac{3}{4}$ of a dollar equally among 7 persons. Ans. $\frac{3}{28}$.
711. Divide $\frac{7}{10}$ of a rod into 8 equal parts. Ans. $\frac{7}{80}$.
712. Divide $\frac{19}{27}$ of a bushel into 7 equal parts.
713. Divide $\frac{36}{47}$ of a gallon into 93 equal parts.
714. Divide $\frac{82}{17}$ by 12.
715. Divide $\frac{41}{391}$ by 17.
716. Divide $\frac{84}{1784}$ by 96.
717. Divide $\frac{874}{238}$ by 928.
718. Divide $\frac{1387}{3487}$ by 8426.

Prop. 4.—IF THE DENOMINATOR OF A FRACTION BE DIVIDED, THE VALUE OF THE FRACTION WILL BE MULTIPLIED BY THE SAME NUMBER.

Illustration.—John has $\frac{4}{12}$ of an apple, but Jane has twice as much. How much has Jane? $\frac{4}{12} \times 2 = \frac{4}{6}$ Ans.

EXAMPLES.

719. Multiply $\frac{18}{48}$ by 8.
720. Multiply $\frac{512}{672}$ by 12.
721. Multiply $\frac{88}{52}$ by 26.
722. Multiply $\frac{9}{36}$ by 6.
723. Multiply $\frac{42}{88}$ by 11.
724. Multiply $\frac{31}{56}$ by 8.
725. Multiply $\frac{37}{108}$ by 12.
726. Multiply $\frac{19}{72}$ by 24.
727. Multiply $\frac{128}{1728}$ by 144.
728. Multiply $\frac{368}{4682}$ by 2341.
729. Multiply $\frac{946}{826}$ by 413.
730. What is the sum of the denominators of the answers on this page? Ans. 29785114.

What is Prop. 3d? How may the denominator of a fraction be increased? How can you divide a fraction? What other way to divide a fraction? Repeat Prop 4. How may the denominator be diminished? How would you multiply a fraction? What other method to multiply a fraction?

PROP. 5.—IF BOTH THE NUMERATOR AND DENOMINATOR BE MULTIPLIED BY THE SAME NUMBER, THE VALUE OF THE FRACTION WILL REMAIN THE SAME.

ILLUSTRATION.—A half of an apple is equal to two-fourths, thus: $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$.

EXAMPLES.

731. Multiply both terms of $\frac{3}{6}$ by 4.
732. Multiply both terms of $\frac{4}{7}$ by 7.
733. Multiply both terms of $\frac{19}{29}$ by 37.
734. Multiply both terms of $\frac{17}{53}$ by 18.
735. Multiply both terms of $\frac{11}{29}$ by 31.
736. Multiply both terms of $\frac{7}{13}$ by 41.
737. Multiply both terms of $\frac{53}{59}$ by 55.
738. Multiply both terms of $\frac{181}{191}$ by 171.

PROP. 6.—IF BOTH THE NUMERATOR AND DENOMINATOR BE DIVIDED BY THE SAME NUMBER, THE VALUE WILL REMAIN THE SAME.

ILLUSTRATION.—Six-twelfths is the same as one-half, thus: $\frac{6}{12} \div \frac{6}{6} = \frac{1}{2}$.

EXAMPLES.

739. Divide both terms of $\frac{12}{24}$ by 4. Ans. $\frac{3}{6}$.
740. Divide both terms of $\frac{28}{49}$ by 7. Ans. $\frac{4}{7}$.
741. Reduce $\frac{63}{77}$ to lower terms, or to *less terms*.
742. Reduce $\frac{187}{176}$ to lower terms.
743. Reduce $\frac{95}{240}$ to lower terms.
744. Reduce $\frac{76}{98}$ to lower terms.
745. Reduce $\frac{25}{55}$ to lower terms.
746. Reduce $\frac{517}{880}$ to lower terms.
747. Reduce $\frac{96}{129}$ to lower terms.
748. What is the sum of the numerators of the answers?

Ans. 35710.

What is Prop. 5? What is the effect of multiplying the numerator? What is the effect of multiplying the denominator? What effect to multiply both at once? Repeat Prop. 6. What effect to divide a numerator? To divide the denominator? To divide both at once? What is it to reduce a fraction to lower terms?

CLASSIFICATION OF FRACTIONS.

117. It will be perceived in the foregoing fractions that the denominator may be any number whatever, as 2, 3, 8. Thus making the fractional unit of any value, as a half, third, eighth, &c.

118. Hence the fractional unit does not increase and decrease in a decimal ratio.

119. But if the denominator be ten, or any power of ten, the decimal system will be preserved in the increase and decrease of the fractional unit.

120. There is also a third method of expressing fractions, where the fractional unit increases and decreases in a twelve-fold or duodecimal ratio.

121. Fractions therefore are expressed by three methods.

1st. *The Decimal*[1] *System of Fractions,* — in which the fractional unit increases and decreases in a ten-fold or decimal ratio.

2d. *The Duodecimal*[2] *System of Fractions,* — in which the fractional unit increases and decreases in a twelve fold or duodecimal ratio.

3d. *The Multecimal*[3] *System, or Common Fractions,* — in which the fractional unit increases and decreases in any ratio.

122. The pupil should always make the proper distinction between the *unit of the fraction* and the *fractional unit,* and remember that a *fraction cannot exist without both.* (109.)

What is a fractional unit? What is the unit of a fraction? 117. What value may the fractional have? 118. How does this fractional unit increase? 119. What advantage by making it ten, or any power of ten? 120. What is the third method of expressing fractions? What is a duodecimal ratio? 121. What is the decimal system of fractions? What is the duodecimal system? What is the multecimal system? 122. What distinction should the pupil always make in the study of fractions? Why? Why cannot a fraction exist without the *two units?* Repeat the 6 propositions. Are these applicable to one system of fractions or to all?

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[1] DECIMAL — Decem. Ten.

[2] DUODECIMAL — Duo and Decem. Two and ten.

[3] MULTECIMAL—Multi and Decem. Many and ten.

CHAPTER I.

DECIMAL FRACTIONS.

SECTION I.

NOTATION AND NUMERATION.

123. Decimal Fractions are divided into two classes, *Finite* and *Infinite*.

124. A *Finite decimal* is one whose entire value can be expressed in the decimal form.

125. An *Infinite decimal* is one whose entire value cannot be expressed in a decimal form.

126. The denominator of a decimal fraction is usually omitted.

127. Decimal fractions express tenths, hundredths, thousandths, &c., of anything.

128. We therefore have a series of numbers below unity, governed by the same laws as those above, and the orders in notation may be extended from unity to the right indefinitely.

129. Unity is the starting point in both decimals and whole numbers, and is always referred to in writing a number. Thus, two hundred is written by writing the figure 2, and filling up the other places to *the point* with ciphers in this manner — 200.

130. In a similar way decimals are written to the right of *the point; e. g.* two thousandths is written by writing 2 in the thousandths place, and filling up the other places to the point with ciphers, thus — .002.

131. A *decimal point* is a dot to mark the place in a series where *whole numbers end* and *fractional numbers begin*.

---

123. Into what classes are decimal fractions divided? 124. What is a finite decimal? 125. What is an infinite decimal? 126. Is the denominator usually written? 127. What do decimal fractions express? 128. How does the series of fractional numbers compare with the series of whole numbers? What is the notation table, page 14? 129. What is the starting point in the series of whole numbers? 130. What is the starting point in a decimal? 131. What is a decimal point?

132. *No number can be expressed without a decimal point.* After whole numbers it is usually understood, but *must always be expressed before a decimal fraction.*

133. A decimal fraction is written by writing the numerator and prefixing ciphers, if necessary, until the number of places equals the number of ciphers in the denominator, and prefixing the decimal point.

134. The denominator of a written decimal may be determined by taking 1 with as many ciphers as there are places in the numerator *from the decimal point.*

135. A decimal fraction written in full takes the form of a common fraction.

136. A decimal fraction is read by reading the numerator and denominator the same as any fraction.

137. Prefixing a cipher to the *numerator* of a decimal fraction, *decreases* its value ten times, by removing the decimal point.

Prefixing a cipher to the *denominator*, does not alter its value.

Annexing a cipher to the *numerator*, does not alter its value.

Annexing a cipher to the *denominator, decreases* its value ten times.

138. The usual manner of expressing a decimal is elliptical, and hence many forms are ambiguous. Thus, "nine hundred thousandths" may mean $\frac{900}{1000}$ or $\frac{9}{100000}$, we cannot determine which. As a division, if the latter value is intended, it would be 9 *divided* by 100000; but if the former, 900 by 1000. The word "divided" *being left out,* its place should be supplied by a comma, "nine, hundred thousandths," or "nine hundred, thousandths."

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132. Can a number exist without a decimal point? Is it always written with a whole number? Can a fraction be written without it? Why not? 133. How is a decimal fraction written? 134. How may the denominator of a written decimal be known? 135. What form does a decimal take if written in full? 136. How is a decimal fraction read? 137. What effect to prefix or annex ciphers? 138. What is the usual method of expressing a decimal? What word is omitted? What should supply its place? What causes ambiguity? Why?

139. NUMERATION TABLE. (Continued from page 18.)

Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hund. Thousandths.	Millionths.	T. Millionths.	H. Millionths.	Billionths.	T. Billionths.	H. Billionths.	Trillionths.	T. Trillionths.	H. Trillionths.	Quadrillionths	T. Quadrillionths.	H. Quadrillionths.	Quintillionths.	T. Quintillionths.	H. Quintillionths.	Sextillionths.	T. Sextillionths.	&c. &c. &c.
.	7	8	4	3	2	7	6	3	5	6	8	9	4	0	3	2	1	6	8	4	2	3	

EXAMPLES FOR WRITING DECIMAL FRACTIONS.

749. Write five, hundredths.
750. Write seven hundred and fifty-seven, thousandths.
751. Write two, hundred thousandths.
752. Write two hundred, thousandths.
753. Write two hundred thousandths. (132, 138.)
754. Write six hundred and five, ten thousandths.
755. Write seven hundred and three, thousandths.
756. Write forty hundred and eleven, millionths.
757. Write six, hundred thousandths.
758. Write seventy-five, ten thousandths.
759. Write three hundred and fifty-four, ten thousandths.
760. Write five hundred and eighty-seven, hundred millionths.
761. Write two hundred and sixty-one, hundred thousandths.
762. Write sixty-five, hundredths.
763. "Write seven hundred thousandths." (?)
764. Write five, hundred thousandths.
765. Write seventeen hundred, millionths.
766. Write seven hundred thousand and twenty, millionths.
767. Write twenty-seven, thousandths.
768. Write fifty-six, ten thousandths.
769. Write nine hundred and fifty-eight, ten thousandths.
770. Write seven, ten millionths.

---

139. Repeat the names of the several orders in Decimal Notation. How many places for thousandths? Millionths? Tenths? Ten thousandths? Ten millionths? Hundredths? Hundred thousandths? Thousandths? Hundred millionths? Tenths? What is the name of the first place? Second? Fifth? Seventh? Fourth? Second? First? Eighth? Sixth? Fourth? Third? First? Fifth? Sixth? Tenth?

EXAMPLES FOR READING DECIMAL FRACTIONS.

771. The ratio of the diameter to the circumference, is commonly written 3.14159, and is called by mathematicians π. (Pi.)

772. To be more accurate, π is 3.14159265 +, and some mathematicians have carried this ratio as far as the 200th place. Vieta carried the approximation to 10 places, Van Ceulen to 36 places, Abraham Sharp to 72 places, and De Lagny to 128 places.

773. One fourth of π .785398163397+, or more commonly .7854, is of frequent use in mensuration.

774. The square root of 3.14159+ is 1.7724538+.

775. The square of 3.141592+ is 9.8696+.

776. The length of one degree of a circle, is .008726 part of the diameter.

777. One half the square root of π is .88623+; or, to be more accurate, .886227+.

778. The square root of .5 is .707+, or .7071+, or .707106+.

779. The ratio of the side of an inscribed square to the circumference, is .225+, or .22507+, or .225079+.

780. The analysis of the oyster shell gives 98.34 of carbonate of lime, 1.26 phosphate of lime, and 0.5 insoluble matter.

781. In a mean Gregorian year the Earth revolves on its axis 365.2425 times.

782. The length of a degree of Latitude, computed by Delambre, is 362.809 English feet. According to Col. Lambton and Capt. Everest, 363.04 feet.

783. The length of a degree of Longitude, at the equator, is 365.152 English feet. At Lat. 40°, 280.106 English feet.

784. The diameter of the Earth is 7924.87 miles at the equator. At the poles it is 7898.63 miles.

What is π? How much is one-fourth of π? What is the square root of π? What is the square root of 5? What part of the oyster shell is carbonate of lime? How long is a Gregorian year? What is the length of a deg. of Latitude? What the length of a deg. of Longitude at Lat. 40°? What is the longest diameter of the Earth? What, the shortest?

SECTION II.

ADDITION OF DECIMAL FRACTIONS.

140. Addition of decimal fractions differs in no respect from that of whole numbers. (See Prob. 1, p. 22.)

EXAMPLES.

785. Add together seventeen, hundredths; nine, tenths; five hundred seventy-one, thousandths; and three hundredths.

786. According to Prof. Struve, sixteen ounces of mineral water from Ems, Germany, contains the following amount of solid matter, viz: carbonate of soda, ten, and seventy-five, hundredths grains; muriate of soda, seven, and six hundred thirty-four, thousandths grains; sulphate of potash, fifty-four, hundredths grains; hydrochlorate of potash, forty-five, thousandths grains; carbonate of lime, one, and fourteen hundred seven, ten thousandths grains; fluate of lime, one hundred ninety-two, hundred thousandths grains; carbonate of magnesia, seven thousand eight hundred eighty-seven, ten thousandths grains. What was the amount of solid matter found?

787. The following is the result of Dr. Prout's analysis of the finest loaf sugar, in a dry and pure state. There were found forty-two, and eighty-five, hundredths ounces of carbon; six, and three hundred fifty, thousandths ounces of hydrogen; and fifty, and eight, tenths ounces of oxygen. How much in the whole?

788. What is the amount of seventeen, thousandths; sixty nine, ten thousandths; twenty, millionths; four, hundredths; three hundred, and fifty-nine, hundred thousandths; twelve, hundredths; seventy, thousandths; and seven hundred, and six, tenths?

789. What is the sum of the answers? Ans. 1123.42583.

Who analyzed the mineral water from Ems? What was the amount of solid matter found? Who has given us an analysis of loaf sugar? Which ingredient is the largest?

SECTION III.

MULTIPLICATION OF DECIMAL FRACTIONS.

PROB. XXXVII.—TO MULTIPLY DECIMAL FRACTIONS.

RULE.—*Multiply the numerators together for the numerator of the product, and the denominators together for the denominator of the product.*

ILLUSTRATION 1.—Multiply .37 by .5.

1st Numerator,	37	1st Denominator,	100
2d Numerator,	5	2d Denominator,	10
Numerator,	185	Denominator,	1000

Ans. $\frac{185}{1000}$ or .185.

The process of multiplying the denominators may always be *mental.* As the number of places in a decimal, is always equal to the number of ciphers in the denominator, the number of places in the product, will always be equal to those in the multiplier and multiplicand counted together.

ILLUSTRATION 2.—Multiply .0071 by .005.

1st Numerator,	.0071	1st Denominator,	10000
2d "	.005	2d "	1000
	.0000355		10000000

EXAMPLES.

790. How many days in 21.5 years? (See Ex. 781.)

791. What is the length of 17.25 deg. of Long. in Lat. 40°?

792. In a synodic revolution of the moon there are 29.53059 days. How many days in 12.5 revolutions?

793. An Irish mile is 1.273 statute United States miles. How many United States miles in 27.75 Irish miles?

794. A Hungarian mile is 5.178 United States miles. How many United States miles in 46.875 Hungarian miles?

795. What is the sum of the answers? Ans. 13331.719125.

What is the rule for the multiplication of decimals? How can you always know how to point off? What is the use of a decimal point? How many days in a year? How long is a deg. of Long. at Lat. 40°? How long is an Irish mile? How long a Hungarian mile?

796. It takes Mercury 87.969258 days to perform a sidereal revolution. How many days difference between 22.5 years of Mercury and 7 Gregorian years?

797. A chronometer is rated at 17.375 sec. loss daily. How much would it lose in a Gregorian year?

798. Every chronometer for the U. S. Navy is put on trial at the National Observatory at Washington, D. C., for one year. If its variation exceeds 8 seconds, it is rejected. Suppose a chronometer gain .07 seconds daily, what would it gain in a Gregorian year?

799. The standard Avoirdupois pound of the United States is the weight of 27.7015 cubic inches of distilled water, at the temperature of the maximum density. How many cubic inches in 12 such pounds?

800. The length of an equinoctial or solar year is 365.2422414 mean solar days. How many days in 21 such years?

801. Multiply 768.253 by 70.07.

802. Multiply 982.76072 by 90.426.

803. Multiply 7878.74 by .0008426.

804. Multiply 98.47 by 93.784.

805. How many feet in 175 loads of wood, if each load contains 98.7687 feet?

806. What are 183 bushels of corn worth at $.9375?

807. What are 346 bushels of wheat worth at $2.0625?

808. A drover with 847 sheep was offered $4.1875 apiece for them, but refused, and sold the entire flock for $3500. Did he lose or gain? How much?

809. What is the sum of the answers on this page?

Ans. 185108.526375744.

How long is a year of Mercury? How long is a Gregorian year? How long is a solar year? How many cubic inches of water in a standard pound? How many grains in a pound? Where are all navy chronometers put on trial? How long on trial? Which are received? Which rejected? What is distilled water? What is maximum density? What is a chronometer?

SECTION IV.

SUBTRACTION OF DECIMAL FRACTIONS.

141. Subtraction of Decimal Fractions differs in no respect from that of whole numbers. (See Prob. III, p. 35.)

EXAMPLES.

810. What is the difference between forty, and eighty-seven, hundredths and 20.35 ?

811. What is the difference between 86.742 and .21 ?

812. What is the difference between a Solar and a Gregorian year ?

813. How much shorter is a degree of Longitude at Latitude 40°, than at the Equator ?

814. What is the difference between 4 Irish miles and 1 Hungarian mile ?

815. Which is the longest, 5.55 United States miles, or 1.002 Hungarian miles ?

816. A link of a Gunter's surveyor's chain is 7.92 inches. What is the difference between 75 links and 5000 inches ?

817. A nautical or geographical mile is 1.1527 statute miles. From Boston to Liverpool, via Halifax, is 2550 nautical miles. How many statute miles ?

818. From Washington to Athens is 5450 nautical miles. How many statute miles ?

819. How many statute miles further from Washington to Athens, than from Boston to Liverpool ?

820. Subtract 8.764 from 18.6.

821. Subtract 91.0467 from 106.7.

822. Subtract 84.638 from 213.2712.

823. What is the sum of the answers ? Ans. 17317.1120052.

How long is a link of a Gunter's chain? How long is a geographical mile? How far from Boston to Liverpool? How far is Washington from Athens? Where is Athens? Where is Liverpool? On what river is it? On what river is Washington? Can you sail from Washington to Athens?

SECTION V.

DIVISION OF DECIMAL FRACTIONS.

PROB. XXXVIII.—TO DIVIDE DECIMAL FRACTIONS.

RULE.—*Divide the numerator of the dividend by the numerator of the divisor, and the denominator of the dividend by denominator of the divisor.*

When the denominator of the divisor is greater than the denominator of the dividend, the division will be impossible. In this case, before division, the denominators must be made alike.

ILLUSTRATION 1.—Divide .35 by .7.

Divisor. Dividend. Ans.

$\frac{7}{10}$) $\frac{35}{100}$ ($\frac{5}{10}$ or .5. Again: .7).35

.5 Ans. as before.

In the usual way the division of the denominator is a *mental* process.

ILLUSTRATION 2.—Divide .75 by .025.

Here the denominator of the dividend is smaller than the denominator of the divisor. Therefore we must multiply both numerator and denominator of the dividend by 10, to make division possible.

Divisor. Dividend. Quotient.

$\frac{25}{1000}$) $\frac{750}{1000}$ ($\frac{30}{1}$ or .025).750(30

750

It will be observed that the number of decimal places in the quotient will be as many *as those in the dividend minus those in the divisor.*

EXAMPLES.

824. Divide seventy-five, hundredths by five tenths.

825. Divide seventy-eight, thousandths by nine hundredths.

NOTE.—It is generally sufficient to carry the quotient to the sixth decimal place.

What is the rule for division of decimals? How many do you point off in the quotient? In what cases is the division impossible? How can it be made possible? How many decimal places in the quotient, when the divisor and dividend have the same number?

826. Divide 18 by seventeen, hundredths.

827. Divide eighteen, hundredths by seventeen, hundredths.

828. Divide 21 by ten, thousandths.

829. Divide three, tenths by six, thousandths.

830. Divide two by three.

831. Divide 3 by 2.

832. Divide 3.1415926 by four, tenths.

833. Divide one by one million.

834. Divide ten by one hundred and ten.

835. Divide 86 by 96.

836. Bought 27 loads of wood for $266.87. How much was it a load?

837. Bought 1128 pounds of butter for $257.68; paid $14.395 for freight; other expenses, $7.56. What did the butter cost per pound?

838. How much should I sell it for, per pound, to gain $50.00?

839. If Capt. M. Meigs pays out $9500.55, every Saturday night, to the workmen on the Capitol of the United States, how many men are employed if each receives 1.583425 per day?

840. Divide seventy-six, and eight hundred forty-two thousandths by ninety-three, and eighty-seven hundredths.

841. Divide seven hundred sixty-eight by seven, and eighty-six hundredths.

842. Divide ninety-two, and six hundred eighty-four thousand seven hundred sixty-three, millionths by forty-seven thousand six hundred eighty-two, hundred thousandths.

843. Divide four thousand six hundred eighty-two by nine hundred thirty-one thousandths.

844. Divide 6 by 17.
845. Divide 1 by 17.
846. Divide 11 by 17. } *Carry out these answers* 16 *places.*
847. Divide 5 by 17.
848. Divide 9 by 17.

849. What is the sum of the answers?

Ans. 13602.1657509411764704.

SECTION VI.

TO CHANGE THE DECIMAL FORM TO THE DUODECIMAL OR COMMON FORM.

PROB. XXXIX.—TO CHANGE A DECIMAL FRACTION TO A DUODECIMAL.

RULE.—*Multiply the decimal by* 12, *and the product will be twelfths. Multiply the decimal part of this result by* 12, *and the product will be* 144*ths; and so continue to multiply each decimal part by* 12, *till the desired result is found.*

ILLUSTRATION.—Change 7.271 to a duodecimal.

7.271
12

3.252 Twelfths. Primes, or $'$.
12

3.024 144ths. Seconds, or $''$.
12

0.288 1728ths. Thirds, or $'''$.
12

3.456 20736ths. Fourths, or ^{iv}.

Ans. $7-3'-3''-0'''-3.456^{iv}$.

EXAMPLES.

850. Change .75 to a duodecimal.
851. Change nineteen, hundredths to a duodecimal.
852. Change seventeen, and five, hundredths to a duodecimal.
853. Change ninety seven, thousandths to a duodecimal.
854. Change seven hundred eighty-four, thousandths.
855. Change 84.26826.
856. Change 764.263.
857. Change 7.46265.
858. Change .84636.
859. What is the sum of the thirds in the answers? Ans. 62.

What is meant by a change of the decimal form? How many forms may fractions have? What are they? How do you change decimals to duodecimals? Is it the same as to multiply by 12, and then express a division by 12? Does that affect the value?

PROB. XL.—TO CHANGE A DECIMAL TO A COMMON FRACTION.

RULE.—*Write the denominator under the numerator and omit the point.*

ILLUSTRATION.—Change .19 to a common fraction. Write denominator, 100, under the numerator, 19. Ans. $\frac{19}{100}$.

EXAMPLES.

860. Change .7, .76, .842, .5678 and 11.32 = $11\frac{32}{100}$.
861. Change 3.57, 8.426, 57.826.
862. Change .76, .321, .6, .06.
863. Change .0006, 8.00006, 20.02.
864. Change 42.6007, 426.007.
865. Change seven hundred twenty-eight, and forty-six, hundredths; two thousand six hundred eighty-one, ten thousandths; and eight hundred twenty-six, ten thousandths to common fractions.

SECTION VII.

INFINITE DECIMALS.

142. For all practical purposes, Infinite Decimals may be treated as finite decimals, being carried out 6 places, and the remainder dropped as valueless.

ILLUSTRATION.—The Infinite Decimal 2.333333 is as correct for practical purposes as though it was continued twenty places.

143. Addition, Subtraction, Multiplication, and Division of Infinite Decimals, will differ in no way from that of Finite Decimals.

NOTE.—See Infinite Decimals, p. 283.

How do you change a decimal to a common fraction? What are Infinite Decimals? How do they differ from Finite Decimals? What rules for adding, multiplying, subtracting, and dividing them?

SECTION VIII.

SPECIAL RULES FOR CONTRACTING THE MULTIPLICATION AND DIVISION OF DECIMAL FRACTIONS.

Prob. XLI.—To shorten the multiplication of decimals.

Rule.—1. *Write the Multiplier under the Multiplicand.*

2. *Commence to multiply with the left hand figure of the multiplier, and proceed as in general multiplication.*

3. *Multiply only so far as to obtain the required number of decimals.*

Illustration.—Multiply .823415 by .786426, and obtain a product to 5 decimal places.

.82341	5
.78642	6
.576390	5
65873	20
4940	490
329	4660
16	47930
4	940490
647555	076790

As those at the right of the line are rejected, it is needless to produce them.

Note.—The position of the decimal point may be determined from the product of the first two decimal places.

EXAMPLES.

866. Multiply 7.8464268 by 8.426842.
867. Multiply 4.268426 by 3.268794.
868. Multiply 3.784569 by 2.682004.
869. Multiply 4.084004 by 7.589307.
870. Multiply 7.000078 by 4.687682.
871. Multiply 4.0000468 by .468260.
872. What is the sum of the answers on this page?

Ans. 155.905388.

Repeat the rule for shortening the multiplication of decimals. How may you know where the decimal point is? How much labor is generally saved by this process?

PROB. XLII. — TO SHORTEN THE PROCESS OF DIVISION OF DECIMAL FRACTIONS.

RULE. — 1. *Commence the division as usual, and continue until the number of decimal places remaining to be found is two less than the number of figures in the divisor.*

2. *Then reject the right hand figure of the divisor and divide, rejecting one figure at the right of the divisor at each division, until the divisor is gone.*

ILLUSTRATION. — Divide 78.4264 by 63.784.

Common Method.		Short Method.
63.784)78.4264(1.229		6|3|.7|8|4)78.4264(1.229
6378	4	6378
1464	24	1464
1275	68	1275
188	560	188
127	568	127
60	9920	60
57	4056	57
3	5864	3
61 figures used.		38 figures used.

The method explains itself by a careful inspection.

EXAMPLES.

873. Divide .876426 by .78426.
874. Divide 8.246 by .326845.
875. Divide .78468 by 68.7567.
876. Divide 4.2632 by 8.2687.
877. Divide 56.872 by 4.2684.
878. Divide 368.42 by 56.842.
879. Divide 36.874 by 36.8263.
880. What is the sum of the answers? Ans. 47.680329.

How is the process of dividing by decimals shortened? How can you tell where the decimal point will be? Why is not the value affected by cutting off the right hand figures of the divisor?

CHAPTER II.

DUODECIMAL FRACTIONS.

SECTION I.

NOTATION AND NUMERATION.

144. Duodecimal Fractions usually have their denominators abbreviated by accents, thus $\frac{6}{12}$ is written $6'$; $\frac{5}{144}$ is written $5''$.

145. The accents at the right denote the number of twelves or power of twelve in the denominator.

146. These *powers* or *denominations* are called primes, seconds, thirds, fourths, fifths, &c.

147. They are marked by accents; *primes* by *one accent*, *seconds* by *two*, *thirds* by *three*, &c., forming a series of fractional units increasing and decreasing in a *twelve fold ratio.*

148. It is evident, therefore, that in changing from one denomination to another, we must *multiply* or *divide* by 12, as 12 seconds make one prime, 12 primes make one unit.

149. A duodecimal is written thus: $4\ \ 6'\ \ 7''\ \ 4'''\ \ 3^{iv}\ \ 8^{v}$.

150. A duodecimal expression is read by naming the numerator, and then the denomination or number of twelves in the denominator. Thus, the expression in Art. 149 is read 4 units, 6 primes, 7 seconds, 4 thirds, 3 fourths, and 8 fifths.

SECTION II.

151. Addition of Duodecimal Fractions differs in no respect from the addition of whole numbers. (See Prob. I, p. 22.)

ILLUSTRATION.—Add $7\ \ 6'\ \ 8''$, $3\ \ 4'\ \ 7''$.

7	$6'$	$8''$	
3	$4'$	$7''$	The 1st column amounts to 15, divide by 12.
10	$11'$	$3''$	Ans.

---

144. How are Duodecimals usually written? 145. What do the accents denote? 146. What are they called? 147. How are these denominations marked? How does a duodecimal series of fractional units increase and decrease. 148. How change from one denomination to another? 149. How is a duodecimal written? 150. How read? 151. Rule for addition?

EXAMPLES.

881. Add $6\ \ 4'\ \ 5''\ \ 7'''\ \ 3^{iv}$, $6'\ \ 8''$, $4'\ \ 8'''$ and $9''$.

882. Add $16\ \ 5'\ \ 6''\ \ 7'''$, $71\ \ 3'\ \ 8''\ \ 9^{iv}$, and $4\ \ 7^{v}$.

883. Add $25\ \ 8'\ 9^{iv}$, $17\ \ 8^{v}$, $9'''$, $7''$, $6'\ \ 4'''$ and $11'$.

884. Add $1'\ \ 1''\ \ 2'''$, $14\ \ 7'''\ \ 8^{iv}\ \ 4^{v}$, and $23\ \ 6'$.

885. Add 8.5, $76\ \ 8'$, $8\ \ 6'$, $9\ \ 2'\ \ 8''$ and $7'$.

886. Add 17.05, $17\ \ 5'$, $678\ \ 4'\ \ 7''$, and $6.5'$.

887. Add $6.5'$ and $6'\ \ 6''$.

888. Add 19.8, $16.7'$, $14.8''$, $23.84'$, and $78.26'$.

889. Add $75\ \ 6'$, $82.5'$, $43.84''$, $78\ \ 6'$, and $2\ \ 3'''$.

890. What is the sum of the answers?

Ans. $1128\ \ 9'\ \ 1''\ \ 9'''\ \ 2^{iv}\ \ 6^{v}$.

SECTION III.

MULTIPLICATION OF DUODECIMALS

152. Multiplication of Duodecimals differs in no respect from the multiplication of decimal fractions. (See Prob. XXXVII, p. 81.)

ILLUSTRATION.—Multiply $6'\ \ 6''\ \ 6'''$ by $2'\ \ 3''\ \ 4'''$

		$6'$	$6''$	$6'''$	
		$2'$	$3''$	$4'''$	
		24^{iv}	24^{v}	24^{vi}	The product by $4'''$
	$18'''$	18^{iv}	18^{v}		" " $3''$
$12''$	$12'''$	12^{iv}			" " $2'$
$12''$	$30'''$	54^{iv}	42^{v}	24^{vi}	Entire product.
1	$2''$	$10'''$	9^{iv}	8^{v}.	The same reduced.

1. In this example the *numbers are* the *numerators* and the *accents indicate* the *denominators*.

2. The denominators are multiplied by adding the accents: thus, multiplying $6'''$ by $4''' = \frac{6}{1728}$ by $\frac{4}{1728}$. Multiplying numerators we have 24, and multiplying denominators we have $1728 \times 1728 = 12 \times 12 \times 12 \times 12 \times 12 \times 12$, or 6 twelves or ^{vi}. Ans. 24^{vi}.

---

152. What is the rule for Multiplication of Decimal fractions? What for Duodecimal fractions? Is there any difference? How do you multiply the denominators in decimals? How do you multiply the denominators in duodecimals? How find the denominator of a decimal? Of a duodecimal?

3. The product as first obtained may be reduced, but *the reduction is no part of the multiplication.* It may take place, however, at the same time with the multiplication, and the answer obtained in its reduced form at once.

EXAMPLES.

891. Multiply $6\ \ 8'\ \ 4''\ \ 6'''\ \ 2^{iv}$ by 7.
892. Multiply $6\ \ 8'\ \ 4''\ \ 6'''\ \ 2^{iv}$ by $3'\ \ 2''$
893. Multiply $6\ \ 8'\ \ 4''\ \ 6'''\ \ 2^{iv}$ by $3'\ \ 7''\ \ 8'''\ \ 4^{iv}$.
894. Multiply $17\ \ 9'\ \ 8''\ \ 5'''\ \ 4^{iv}$ by $6'\ \ 3''$.
895. Multiply $9\ \ 8'\ \ 7''\ \ 5'''$ by $6\ \ 7'\ \ 8''\ \ 3'''$.
896. Multiply $8\ \ 7'\ \ 6''\ \ 7'''\ \ 6^{iv}$ by $8'\ \ 7''$.
897. Multiply $16\ \ 5''\ \ 4'$ by $16\ \ 5''\ \ 4'$.
898. Multiply $15\ \ 9'\ \ 7''$ by .5.
899. Multiply 17.6542 by $7\ \ 6'$.
900. Multiply 98.625 by $42\ \ 3'\ \ 4''$.
901. Multiply $8'\ \ 6''$ by $8'\ \ 6''$.

SECTION IV.

SUBTRACTION OF DUODECIMALS.

153. Subtraction of Duodecimals differs in no respect from the subtraction of whole numbers. (See Prob. III, p. 35.)

ILLUSTRATION. — Subtract $9—8'$ from $13—6'$.

$$\begin{array}{rr} 13 & 6' \\ 9 & 8' \\ \hline 3 & 10' \end{array}$$

As $8'$ cannot be taken from $6'$, we take 1 and change to primes, giving 12 added to $6'$ makes $18'$, then subtract $8'$ leaving $10'$, then say 9 from 12 leaves 3. Ans. $3\ \ 10'$.

EXAMPLES.

902. Subtract $3\ \ 4'\ \ 5''\ \ 2'''$ from $6\ \ 7'\ \ 8''\ \ 4'''$.
903. Subtract $1\ \ 9'\ \ 6''\ \ 7'''$ from $4\ \ 8'\ \ 4''\ \ 6'''$.
904. Subtract $3\ \ 4'''\ \ 7^{iv}$ from $7\ \ 8'''\ \ 12^{iv}$.
905. What is the sum of the answers?

Ans. $4720\ \ 2'\ \ 6''\ \ 6'''\ \ 0^{iv}\ \ 11^{v}$.

Can you reduce the product at the same time you multiply? Is it any part of multiplication? 153. What is the rule for subtracting whole numbers? Decimals? Duodecimals?

SECTION V.

DIVISION OF DUODECIMAL FRACTIONS.

Prob. XLIII.—To divide duodecimal fractions.

Rule.—1. *Change Duodecimals to Decimals: then divide. Or Rule 2. See Prob. IV, p. 37.*

Illustration 1.—Divide 78 4′ 4″ by 7.

7)78 4′ 4″(11 2′ 4″ Ans.
77
1 4′=16′ The 1st rem. is 1 4′ reduced gives 16′.
14′ The 2d " 2′ 4″ " 28″.
2′ 4″=28″ The other steps are easily understood.
28″

Illustration 2.—Divide 78 4′ 2″ 3‴ by 7 6′.

7 6′)78 4′ 2″ 3‴(10 5′ 4″ 3‴ 7^{iv}.
75 0′ =7 6′×10.
3 4′ 2″
3 1′ 6″ =7 6′×5′.
2′ 8″ 3‴
2′ 6″ 0‴ =7 6′×4″.
2″ 3‴
1″ 10‴ 6^{iv} =7 6′×3‴.
4‴ 6^{iv}
4‴ 4^{iv} 6^{v} =7 6′×7^{iv}.
1^{iv} 6^{v} Rem.

EXAMPLES.

906 Divide 8 6′ 7″ 4‴ by 2 6′ 8″.

907. Divide 12 7′ 8″ 7‴ by 4 3′ 4″.

908. Divide 144 7′ 8″ 4‴ 3^{iv} by 11 4′ 5″.

909. What is the sum of the answers?

Ans. 19 0′ 3″ 7‴ 2^{iv} 3^{v}.

What is the first rule for the division of duodecimals? The second? In what way does it differ from division of whole numbers? Have you carefully examined every step in illustrations 1 and 2? Can you tell how to obtain every number?

SECTION VI.

CHANGE OF DUODECIMALS TO DECIMALS AND COMMON FRACTIONS.

PROB. XLIV.—TO CHANGE DUODECIMALS TO DECIMALS.

RULE.—1. *Divide the lowest denomination by 12, and add the quotient to the next denomination.*

2. *Divide that sum by 12, and add to the next denomination.*

3. *So continue till you add to units.*

ILLUSTRATION.—Change 6 6′ 6″ 6‴ to a decimal form.

$$\begin{array}{r|l} 12 & 6'''. \\ \hline 12 & 6.5'' \\ \hline 12 & 6.54166'+ \\ \hline & 6.54513833+ \text{ Ans.} \end{array}$$

Dividing 6‴ by 12 it gives 5″, which, added to 6″, makes 6.5″—. This, divided by 12, gives 54166′+, &c.

EXAMPLES.

910. Change 6 8′ 4″ to a decimal form.
911. Change 7 8′ 4″ 5‴ to a decimal form.
912. Change 5 6′ 8″ 4‴ 2^{iv} to a decimal form.
913. Change 78. 4′ 8″ 7‴ 5^{iv} to a decimal form.
914. What is the sum of answers? Ans. 98.342927.

PROB. XLV.—TO CHANGE DUODECIMALS TO COMMON FRACTIONS.

RULE.—*Change all the duodecimals to the same denomination, and write the fraction in full.*

ILLUSTRATION.—Change 6 8′ 2″ to a common fraction.

$$\begin{array}{l} 6 \quad 8' \quad 2'' \\ \underline{12'} \\ 72'+8'=80' \\ \qquad\quad \underline{12'} \\ \qquad\quad 960''+2''=962'' \end{array}$$

Multiplying 6 by 12′ changes it to primes; add the 8′ and multiply by 12′ again, and it gives 960″; add 2″ = 962″ $= \frac{962}{144}$ Ans.

EXAMPLES.

915. Change 7. 8′ 4″ 6‴ to a common fraction.
916. Change 128 4′ 6″ 8‴ to a common fraction.

Rule to change duodecimals to decimals? To common fractions?

CHAPTER III.

COMMON FRACTIONS.

SECTION I.

CLASSIFICATION OF COMMON FRACTIONS.

154. Common Fractions are divided into six classes, *viz.:* Proper, Improper, Simple, Mixed, Compound, and Complex.

155. A *Proper Fraction* is one whose value is less than unity, *e. g.* $\frac{4}{5}$, $\frac{7}{8}$.

156. An *Improper Fraction* is one whose value is equal to or exceeds unity, *e. g.* $\frac{4}{4}$, $\frac{6}{5}$, or $\frac{10}{9}$.

157. A *Simple Fraction* is one that has a whole number for its numerator, and a whole number for its denominator.

158. A *Mixed Fraction* or *Number* is a whole number joined to a fraction, *e. g.* $4\frac{1}{2}$, $5\frac{2}{3}$.

159. A *Compound Fraction* is a fraction expressed by two or more fractional parts, $\frac{3}{4}$ of $\frac{5}{8}$.

160. A *Complex Fraction* is an indicated division of fractional quantities, $\frac{3\frac{1}{2}}{\frac{7}{8}}$, $\frac{\frac{7}{6}}{\frac{3}{4}}$, $\frac{5\frac{1}{2}}{17\frac{3}{4}}$.

SECTION II.

CHANGE IN FORM WITHOUT CHANGE IN VALUE.

Art. 1. To change a fraction to its lowest terms.

Art. 2. To change an improper fraction to a whole or mixed number.

Art. 3. To change a whole or mixed number to an improper fraction.

Art. 4. To change common fractions to decimals.

What is a common fraction? 154. How many classes are there? 155. What is a proper fraction? 156. What is an improper fraction? 157. What is a simple fraction? 158. What is a mixed fraction? 159 What is a compound fraction? 160. What is a complex fraction? Give an example of each. Give a second example of each. How many ways to change the form of a common fraction without changing its value? What is the 1st? 2d? 3d? 4th?

Art. 5. To change common fractions to duodecimals.

Art. 6. To change fractions to a common denominator.

Art. 7. To change fractions to a common numerator.

Art. 8. To change one fraction to another with a given numerator or denominator.

Art. 1.—**Prob.** XLVI.—To CHANGE A FRACTION TO ITS LOWEST TERMS.

Rule.—*Divide the numerator and denominator by their Greatest Common Measure.*

161. A fraction may be reduced to lower terms by dividing by any common factor of the numerator and denominator; this common factor *may often be found by inspection.*

Illustration.—Change $\frac{12}{18}$ to its lowest form; 6 is their G. C. M. Hence $6)\frac{12}{18}=\frac{2}{3}$. Ans.

Illustration.—Change $\frac{231}{297}$ to *lower terms.*

From inspection we find 3 a common measure. $3)\frac{231}{297}=\frac{77}{99}$. and by dividing, have the Ans.

162. Fractions are in their lowest forms when the terms are prime to each other.

EXAMPLES.

917. Change $\frac{15}{24}$, $\frac{18}{27}$, and $\frac{25}{35}$ to their lowest terms.

918. Change $\frac{18}{56}$, $\frac{27}{99}$, and $\frac{66}{144}$ to their lowest terms.

919. Change $\frac{486}{792}$ to its lowest terms.

920. Change $\frac{352}{384}$ to its lowest terms.

921. $\frac{1727}{14710}$.
922. $\frac{253}{341}$.
923. $\frac{869}{1067}$.
924. $\frac{781}{803}$.
925. $\frac{53}{109}$.
926. $\frac{72}{486}$.
927. $\frac{341}{682}$.
928. $\frac{385}{693}$.
929. $\frac{231}{385}$.
930. $\frac{47}{282}$.
931. $\frac{286}{572}$.
932. $\frac{374}{715}$.

933. What is the sum of the numerators of the answers on this page? Ans. 2075.

What is the 5th? 6th? 7th? 8th? Is a decimal fraction a common fraction? Is a common fraction a decimal? 161. How do you change a fraction to its lowest terms? What is the G. C. M. of several numbers? How do you obtain the G. C. M.? How can you reduce certain fractions without getting the G. C. M.? How reduce a fraction to lower terms?

Art. 2.—Prob. XLVII.—To change an improper fraction to its equivalent whole or mixed number.

Rule.—*Perform the indicated division.*

EXAMPLES.

934. Change $\frac{12}{4}$ to a whole number. $12 \div 4 = 3$ Ans.
935. Change $\frac{682}{11}$ to a whole number $682 \div 11 = 62$ Ans.
936. Change $\frac{17}{5}$ to a mixed number.
937. Change $\frac{48}{7}$ to a whole or a mixed number.
938. Change $\frac{583}{22}$ to a whole or a mixed number.
939. Change $\frac{6876}{37}$ to a whole or a mixed number.
940. Change $\frac{84268}{351}$.
941. Change $\frac{8746}{471}$.
942. Change $\frac{78426}{7268}$.
943. Change $\frac{42468}{24684}$.
944. Change $\frac{26874}{5672}$.
945. Change $\frac{7874}{523}$.
946. Change $\frac{4256}{191}$.
947. Change $\frac{1111}{111}$.

Art. 3.—Prob. XLVIII.—To change a whole or mixed number to its equivalent improper fraction.

Rule.—*Multiply the whole number by the denominator, and add the numerator, if any, to the product for a numerator; under which write the given denominator.*

EXAMPLES.

948. Change $5\frac{3}{4}$ to an improper fraction. $5 \times 4 = 20$. $20 + 3 = 23$. $\frac{23}{4}$.
949. Change 8 to an improper fraction. Ans. $\frac{8}{1}$ or $\frac{16}{2}$ or $\frac{24}{3}$, &c.
950. Change $17\frac{5}{7}$ to an improper fraction.
951. Change $177\frac{5}{11}$ to an improper fraction.
952. Change 16 to a fraction with a denominator 7.
953. Change $16\frac{1}{2}$ to an improper fraction.
954. Change 87 to twentieths.
955. Change $2468\frac{17}{31}$ to an improper fraction.
956. Change $486\frac{81}{96}$ to an improper fraction.
957. What is the sum of all the numerators of the answers on this page? Ans. 151745.

How do you change improper fractions to whole or mixed numbers? How do you change whole or mixed numbers to improper fractions?

ART. 4.—PROB. XLIX.—TO CHANGE COMMON FRACTIONS TO DECIMALS.

RULE.—*Annex ciphers to the numerator, and divide by the denominator. The decimal places will equal the number of ciphers annexed.*

EXAMPLES.

958. Change $\frac{4}{5}$ to a decimal fraction. $40 \div 5 = .8.$
959. Change $\frac{19}{20}$ to a decimal fraction.
960. Change $\frac{79}{80}$ to a decimal fraction.
961. Change $\frac{34}{60}$ to a decimal fraction.
962. Change $\frac{17}{41}$ to a decimal fraction.
963. Change $\frac{11}{17}$ to a decimal fraction.
964. Change $4\frac{1}{2}$.
965. Change $19\frac{7}{20}$.
966. Change $84\frac{7}{50}$.
967. Change $\frac{31}{97}$ to a decimal.
968. Change $41\frac{3}{17}$ to a dec.
969. Change $10\frac{111}{331}$ to a dec.

ART. 5.—PROB. L.—TO CHANGE COMMON FRACTIONS TO DUODECIMALS.

RULE.—1. *Multiply the numerator by 12, and divide the product by the denominator; the quotient will be primes.*

2. *Multiply the remainder by 12, and divide the product by the denominator; the quotient will be seconds.*

3. *Continue multiplying and dividing till the required answer is obtained.*

EXAMPLES.

970. Change $\frac{7}{5}$ to a duodecimal fraction.
971. Change $\frac{8}{11}$ to a duodecimal fraction.
972. Change $\frac{9}{13}$ to a duodecimal fraction.
973. Change $\frac{8}{15}$ to a duodecimal fraction.
974. Change $\frac{17}{19}$ to a duodecimal fraction.
975. Change $\frac{11}{21}$ to a duodecimal fraction.
976. Change $\frac{18}{29}$ to a duodecimal fraction.
977. What is the sum of the answers?

Ans. $5\ 4'\ 8''\ 5'''\ 7^{\text{iv}}\ 4^{\text{v}}$.

How do you change common fractions to decimals? What are decimals? How do you change common fractions to duodecimals? What is a duodecimal fraction?

ART. 6.—PROB. LI.—PART 1.—TO CHANGE FRACTIONS TO A COMMON DENOMINATOR.

RULE.—1. *Change all the fractions to the simple form.*

2. *Reduce them to their lowest terms.*

3. *Multiply both the numerator and denominator of each fraction by such a number as will make a common denominator.*

ILLUSTRATION.—Change $\frac{3}{5}$, $\frac{2}{4}$, $\frac{4}{6}$, and $4\frac{1}{2}$, to a common denominator.

1st. $\frac{3}{5}$, $\frac{2}{4}$, $\frac{4}{6}$, $\frac{9}{2}$. All are in the simple form.
2d. $\frac{3}{5}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{9}{2}$. All are in the lowest form.
3d. $\frac{3}{5} \times 6$, $\frac{1}{2} \times 15$, $\frac{2}{3} \times 10$, $\frac{9}{2} \times 15$.
Ans. $\frac{18}{30}$, $\frac{15}{30}$, $\frac{20}{30}$, $\frac{135}{30}$.

The numbers to multiply by, are to be found by inspection.

EXAMPLES.

978. Change $\frac{5}{8}$, $\frac{12}{18}$, $\frac{11}{24}$, and $\frac{9}{15}$ to a common denominator.
979. Change $\frac{7}{10}$, $\frac{9}{16}$, $\frac{4}{20}$, and $\frac{3}{30}$ to a common denominator.
980. Change $4\frac{1}{2}$, $\frac{6}{8}$, $\frac{5}{10}$, and $\frac{5}{15}$ to a common denominator.
981. Change $\frac{25}{30}$, $\frac{42}{49}$, $\frac{28}{35}$, and $\frac{63}{28}$ to a common denominator.
982. Change $3\frac{13}{26}$, $5\frac{17}{51}$, and $14\frac{28}{35}$ to a common denominator.
983. Change 6.5, 7.25, $\frac{3}{4}$, and $\frac{2}{5}$ to a common denominator.
984. Change 6 6′, 5.3, 14.5, and $7\frac{1}{2}$ to a common denom.
985. Change 3 9′, 6 8′, 27.25, and .5 to a com. denom.
986. Change 27.8, 32 6′, and $\frac{27}{45}$ to a common denominator.
987. Change 31.24, $18\frac{4}{5}$, and $31\frac{21}{35}$ to a common denominator.
988. Change $23\frac{3}{5}$, $17\frac{19}{95}$, and 6′ to a common denominator.
989. Change 17.8, 17 8′, and $\frac{17}{85}$ to a common denominator.
990. Change $27\frac{81}{270}$, $37\frac{12}{60}$, and 17 6′ to a common denom.
991. Change $17\frac{8}{12}$, $\frac{9}{18}$, $\frac{33}{44}$, and $\frac{8}{24}$ to a common denominator.
992. Change $83\frac{4}{9}$, $37\frac{7}{18}$, $\frac{12}{54}$, and $\frac{3}{27}$ to a common denom.
993. What is the sum of the denominators? Ans. 804.
994. What is the sum of the numerators? Ans. 11104.

What is the rule for changing fractions to a common denominator? What is to be done, 1st? 2d? 3d? How do you find what number to multiply by?

PROB. LI. — PART 2.—TO CHANGE FRACTIONS TO THE LEAST COMMON DENOMINATOR.

RULE. — *Prepare the fractions as in* 1 *and* 2, Part 1.

3d. *Find the least common multiple of the denominators*

4th. *Take such parts of this multiple for numerators as the several fractions represent.*

ILLUSTRATION.—Change $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{4}{6}$ to a common denom.

3d. $\cancel{2}$ 4 $\cancel{3}$ 5 3 2 . 2 . 3 . 5 = 60. L. C. Multiple.
4th. $\frac{1}{2}$ of 60 = 30. 1st. Numerator.
$\frac{3}{4}$ of 60 = 45. 2d. "
$\frac{2}{3}$ of 60 = 40. 3d. "
$\frac{3}{5}$ of 60 = 36. 4th. "
$\frac{4}{6}$ of 60 = 40. 5th. "

The fractions to the L. C. Denominator are $\frac{30}{60}$, $\frac{45}{60}$, $\frac{40}{60}$, $\frac{36}{60}$, $\frac{40}{60}$.

EXAMPLES.

995. Change $\frac{3}{7}$, $\frac{4}{8}$, $\frac{9}{14}$, $\frac{7}{8}$, to the L. C. Denominator.
996. Change $\frac{5}{9}$, $\frac{6}{10}$, $\frac{9}{13}$, $\frac{8}{26}$, and $\frac{8}{17}$ to L. C. D.
997. Change $\frac{3}{8}$, $\frac{6}{9}$, $\frac{10}{12}$, $\frac{13}{19}$, and $\frac{3}{11}$ to L. C. D.
998. Change $\frac{5}{9}$, $\frac{8}{11}$, $\frac{8}{14}$, and $\frac{9}{15}$ to L. C. D.
999. Change $6\frac{8}{17}$, $9\frac{3}{12}$, and $11\frac{3}{21}$ to L. C. D.
1000. Change $13\frac{8}{12}$, $15\frac{9}{18}$, $17\frac{8}{17}$, and $\frac{23}{92}$ to L. C. D.
1001. Change $17\frac{3}{7}$, $19\frac{13}{28}$, $23\frac{11}{35}$, and $\frac{22}{70}$ to L. C. D.
1002. Change 7 6′, 6.82, and $\frac{27}{54}$ to L. C. D.
1003. Change 3.68, 12.5, 8 7′ 2″ to L. C. D.
1004. Change 6′ 7″, 7.5, and $9\frac{4}{5}$ to L. C. D.
1005. Change 7 6′ 8″, 7.55, and $7\frac{3}{8}$ to L. C. D.
1006. Change $\frac{217}{868}$, $\frac{17}{680}$, $\frac{13}{260}$, $\frac{96}{312}$, and $\frac{3}{7}$ to L. C. D.
1007. Change $\frac{68}{95}$, $\frac{121}{133}$, $\frac{111}{133}$, and $\frac{13}{19}$ to L. C. D.
1008. What is the sum of the numerators of the answers on this page? Ans. 188654.
1009. What is the sum of the denominators of the answers on this page? Ans. 38507.

What is Part 2 to Prob. LI.? What is first to be done? 2d? 3d? 4th? What is the rule to find the L. C. D.? Which rule do you prefer, Part 1. or Part 2.? Why? Which do you think would be most used in practice?

ART. 7.—PROB. LI.—PART 3.—TO CHANGE FRACTIONS TO A COMMON NUMERATOR.

RULE.—1. *Prepare the fractions as in* 1 *and* 2, *Part* 1.

2. *Find the least common multiple of the numerators for a common numerator.*

3. *Divide this common numerator by each numerator, and multiply the quotient by the given denominator for the required denominator.*

ILLUSTRATION.—Change $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{4}{7}$ to a common numerator.

The least common multiple of 3, 5, and 4 is 60. $\frac{60}{3} \times 4 = 80$. $\frac{60}{5} \times 8 = 96$. $\frac{60}{4} \times 7 = 105$. Ans. $\frac{60}{80}$, $\frac{60}{96}$ and $\frac{60}{105}$.

EXAMPLES.

1010. Change $\frac{4}{7}$, $\frac{5}{9}$, $\frac{8}{11}$, and $\frac{3}{8}$ to a common numerator.

1011. Change $\frac{9}{11}$, $\frac{6}{18}$, $\frac{7}{11}$, and $\frac{13}{25}$ to a common numerator.

ART. 8.—PROB. LII.—TO CHANGE A FRACTION TO ONE HAVING A GIVEN NUMERATOR OR DENOMINATOR.

RULE.—1*st. Divide the given fraction by the proposed numerator, and the result will be the required denominator.*

ILL.—Change $\frac{5}{15}$ to a fraction having 6 for a numerator.

1st. $\frac{5}{15} \div 6 = \frac{5}{90} = \frac{1}{18}$, the required denominator. Ans. $6_{\frac{1}{18}}$.

RULE.—2*d. Multiply the given fraction by the proposed denominator, and the result will be the required numerator.*

ILL.—Change $\frac{6}{7}$ to a fraction having 21 for a denominator.

$\frac{6}{7} \times 21 = \frac{126}{7} = 18$. The required fraction then is $\frac{18}{21}$.

EXAMPLES.

1012. Change $\frac{5}{7}$ to a fraction with the numerator 8.

1013. Change $\frac{5}{7}$ to a fraction with the denominator 8.

1014. Change $\frac{11}{15}$ to a fraction with the denominator 31.

1015. Change $\frac{19}{23}$ to a fraction with a denominator 46.

1016. Change $\frac{11}{17}$ to a fraction having a numerator 22.

1017. Change $\frac{9}{5}$ to a fraction having a numerator 5.

What is Prob. LI? Give the rule. What is Prob. LII? Give Rule 1. Give Rule 2.

SECTION III.

ADDITION OF COMMON FRACTIONS.

Prob. LIII.—To add fractions or mixed numbers.

Rule.—1. *Change the fractions to a common denominator.*

2. *Add their numerators.*

3. *Add whole numbers, if any, separately.*

Illustration.—Add $3\frac{1}{2}$, $5\frac{2}{8}$ and $\frac{3}{5}$.

1. $\frac{1}{2}, \frac{2}{8}, \frac{3}{5} = \frac{10}{20}, \frac{5}{20}, \frac{12}{20}$
2. $\frac{10}{20} + \frac{5}{20} + \frac{12}{20} = \frac{27}{20} = 1\frac{7}{20}$.
3. $3 + 5 + 7 + 1\frac{7}{20} = 16\frac{7}{20}$ Ans.

EXAMPLES.

1018. Add $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$, and $\frac{2}{7}$.
1019. Add $2\frac{4}{7}$, $2\frac{5}{7}$, $2\frac{6}{7}$, and $2\frac{2}{7}$.
1020. Add $5\frac{5}{7}$, $6\frac{2}{7}$, $8\frac{3}{7}$, and $9\frac{4}{7}$.
1021. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{3}{8}$, and $\frac{3}{5}$.
1022. Add $4\frac{1}{2}$, $5\frac{2}{3}$, $6\frac{4}{5}$, $9\frac{3}{8}$, and $7\frac{3}{5}$.
1023. Add $8\frac{2}{3}$, $6\frac{4}{5}$, $\frac{17}{19}$, and $\frac{11}{13}$.
1024. Add $\frac{18}{24}$, $\frac{18}{72}$, $\frac{24}{96}$, $\frac{76}{152}$, and $\frac{312}{624}$.
1025. Add $6'$, $\frac{6}{13}$, $\frac{6}{15}$, and 7.
1026. Add $5'$, .15, and $\frac{4}{20}$. Ans. in decimal form.
1027. Add $7'$, .17, and $\frac{17}{34}$. Ans. in decimal form.
1028. Add $8'$, .23, and $\frac{17}{19}$. Ans. in decimal form.
1029. Add $9'$, .8, .17, and $\frac{2}{7}$.
1030. Add 7.8, $5\frac{3}{4}$, and $\frac{9}{17}$.
1031. Add $8\ \ 2'\ \ 7''\ \ 6'''$, and $\frac{31}{67}$. Ans. decimal.
1032. Add 7.568 and $\frac{29}{84}$. Ans. decimal.
1033. Add $8\ \ 7'\ \ 8''\ \ 4'''$, and 7.846. Ans. decimal.
1034. Add $\frac{85}{117}$, $19\frac{7}{8}$, and .876. Ans. decimal.
1035. Add $9\frac{17}{41}$, $7\frac{11}{13}$, $11\frac{17}{23}$, and 18.7. Ans. decimal.
1036. Add $8\frac{18}{19}$, $11\frac{17}{261}$, $84\frac{17}{91}$, and $\frac{13}{29}$. Ans. decimal.
1037. What is the sum of the answers? Ans. 334.099817.

What is the rule for addition of fractions?

SECTION IV.

MULTIPLICATION OF COMMON FRACTIONS.

163. Multiplication of Common Fractions differs in no respect from the multiplication of Decimal Fractions. (See Prob. XXXVII, p. 84.)

EXAMPLES.

1038. Multiply $\frac{5}{6}$ by $\frac{7}{8}$. ($\frac{5}{6} \times \frac{7}{8} = \frac{35}{48}$.)
1039. Multiply $\frac{7}{9}$ by 5. ($\frac{7}{9} \times \frac{5}{1} = \frac{35}{9} = 3\frac{8}{9}$.)
1040. Multiply 13 by $\frac{4}{7}$. ($\frac{13}{1} \times \frac{4}{7} = \frac{52}{7} = 7\frac{3}{7}$.)
1041. Multiply $4\frac{1}{4}$ by $3\frac{3}{5}$. ($\frac{17}{4} \times \frac{18}{5} = \frac{306}{20} = 15\frac{3}{10}$.)
1042. Multiply $\frac{1}{2}$ by $\frac{3}{4}$ by $\frac{7}{8}$ or $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{7}{8}$.
1043. Multiply $\frac{3}{4}$ by $\frac{3}{9}$ by $\frac{9}{16}$.
1044. Multiply $\frac{3}{4}$ by $\frac{7}{9}$ by $\frac{9}{10}$.
1045. Multiply $\frac{3}{4}$ of $\frac{7}{9}$ of $\frac{4}{10}$ by $\frac{3}{6}$.
1046. Multiply $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{5}{7}$ of $\frac{9}{10}$ by $\frac{8}{9}$.
1047. Multiply $\frac{5}{2}$ of $\frac{7}{3}$ of $\frac{8}{4}$ of $\frac{11}{5}$ of $\frac{3}{7}$ by $\frac{3}{5}$.
1048. Multiply $2\frac{1}{2}$ of $2\frac{1}{3}$ of 2 of $2\frac{1}{5}$ of $\frac{3}{7}$ by $\frac{3}{5}$.
1049. Multiply $3\frac{3}{4}$ of $6\frac{7}{8}$ of $\frac{3}{17}$ of $41\frac{5}{9}$ by $\frac{3}{7}$.
1050. Multiply .6 of 6′ of $\frac{1}{2}$ of 3′.
1051. Multiply 6′ of .05 of $\frac{2}{3}$ of $\frac{8}{11}$.

These are sometimes called *Compound Fractions*.

1052. Multiply $7\frac{4}{7}$ by $19\frac{3}{11}$ by $8\frac{7}{13}$. (See Ex. 1041.)
1053. Sum of the numerators of the answers above? Ans. 1256405.
1054. Multiply 6 6′ 7′′ by 6.66. Ans. decimal.
1055. Multiply 7.847 by 6 7′ by $\frac{3}{11}$. Ans. decimal.
1056. Multiply 6 8′ 8′′ by .5 by $\frac{11}{17}$. Ans. decimal.
1057. Multiply 6 8′ 4′′ by 7.56 by $\frac{8}{13}$. Ans. decimal.
1058. Multiply $7\frac{41}{43}$ by $11\frac{13}{87}$. Ans. decimal.
1059. Multiply $19\frac{23}{29}$ by $37\frac{41}{47}$. Ans. decimal.
1060. Multiply $23\frac{11}{31}$ by $17\frac{53}{59}$. Ans. decimal.
1061. What is the sum of all the answers to last seven examples carried to six decimal places? Ans. 1347.320086.

How do you multiply common fractions? How multiply a fraction by a whole number? How multiply two mixed numbers?

SECTION V.

SUBTRACTION OF COMMON FRACTIONS.

PROB. LIV.—PART 1.—TO FIND THE DIFFERENCE BETWEEN TWO FRACTIONS.

RULE.—1. *Change the fractions to a common denominator*
2. *Subtract the numerators.*

EXAMPLES.

1062. Subtract $\frac{1}{4}$ from $\frac{3}{4}$.
1063. Subtract $\frac{3}{8}$ from $\frac{7}{8}$.
1064. Subtract $\frac{2}{5}$ from $\frac{5}{7}$.
1065. Subtract $\frac{5}{11}$ from $\frac{17}{19}$.
1066. Subtract $\frac{16}{18}$ from $\frac{34}{36}$.
1067. Subtract $\frac{2}{3}$ from $9'$—$8''$.

PART 2.—TO FIND THE DIFFERENCE BETWEEN TWO MIXED NUMBERS.

RULE.—1. *Change the fractional parts to a common denominator.*

2. *If the fraction of the minuend is larger than that of the subtrahend, subtract.*

3. *If the fraction of the minuend is smaller, reduce a unit of the whole number, add to the minuend, and then subtract.*

EXAMPLES.

1068. Subtract $3\frac{1}{3}$ from $5\frac{2}{3}$.
1069. Subtract $2\frac{2}{3}$ from $6\frac{1}{3}$. $(6\frac{1}{3} = 5\frac{4}{3}.\quad 5\frac{4}{3} - 2\frac{2}{3} = 3\frac{2}{3}.)$
1070. Subtract $5\frac{4}{6}$ from $9\frac{4}{12}$. $(9\frac{4}{12} = 8\frac{4}{3}.\quad 8\frac{4}{3} - 5\frac{2}{3} = 3\frac{2}{3}.)$
1071. Subtract $5\frac{1}{4}$ from $7\frac{1}{5}$.
1072. Subtract $8\frac{3}{7}$ from $11\frac{7}{131}$.
1073. Subtract $8\frac{3}{11}$ from $19\frac{13}{17}$.
1074. Subtract $\frac{1}{2}$ of $\frac{3}{4}$ from $\frac{10}{11}$.
1075. What is the sum of the answers? Ans. 20.883183.

How do you find the difference between two fractions? How find the common denominator? How find the difference between two mixed numbers? How between a mixed number and a whole number? How between a *whole* and a mixed number?

SECTION VI.

DIVISION OF COMMON FRACTIONS.

Prob. LV.—To divide fractional quantities.

Rule.—1. *Change the quantities to the form of simple fractions.*

2. *Invert the divisor, and proceed according to Problem XXXVII, p.* 81.

EXAMPLES.

1076. Divide $\frac{7}{8}$ by 5. $(\frac{7}{8} \times \frac{1}{5} = \frac{7}{40}$ Ans.)

1077. Divide $\frac{7}{8}$ by $\frac{3}{5}$. $(\frac{7}{8} \times \frac{1}{3} = \frac{7}{24}$. $\frac{7}{24} \times \frac{5}{1} = \frac{35}{24} = 1\frac{11}{24}$ Ans.)

1078. Divide $\frac{7}{8}$ by $\frac{4}{9}$. $(\frac{7}{8} \times \frac{9}{4} = \frac{63}{32} = 1\frac{31}{32}$ Ans.)

1079. Divide $\frac{7}{8}$ by $4\frac{3}{5}$. $(\frac{7}{8} \times \frac{23}{5} = \frac{7}{8} \times \frac{5}{23} = \frac{35}{184}$ Ans.)

1080. Divide $\frac{7}{11}$ by $\frac{9}{13} = \dfrac{\frac{7}{11}}{\frac{9}{13}} = \frac{7}{11} \times \frac{13}{9}$.

1081. Divide $\frac{11}{12}$ by $\frac{7}{8} = \dfrac{\frac{11}{12}}{\frac{7}{8}}$.

1082. Divide $4\frac{1}{2}$ by $\frac{9}{13} = \dfrac{4\frac{1}{2}}{\frac{9}{13}}$.

1083. Divide $7\frac{3}{4}$ by $4\frac{3}{5} = \dfrac{7\frac{3}{4}}{4\frac{3}{5}}$.

1084. Divide $8\frac{7}{8}$ by $5\frac{4}{9} = \dfrac{8\frac{7}{8}}{5\frac{4}{9}}$.

1085. Divide $19\frac{11}{17}$ by $13\frac{3}{4} = \dfrac{19\frac{11}{17}}{13\frac{3}{4}}$.

These are sometimes called *Complex Fractions.*

1086. Divide 6′ by 8′ $= \dfrac{\frac{6}{12}}{\frac{8}{12}}$.

1087. Divide 4′ by 6′ 8″.

1088. Divide 4.75 by $\frac{7}{8}$.

1089. Divide $\frac{7}{8}$ by $4\frac{1}{2}$.

1090. Divide 9.27 by $\frac{3}{4}$ of $\frac{7}{8}$.

1091. Divide .75 by 9‴.

1092. Divide 86.627 by $17\frac{5}{7}$.

1093. Divide $8\frac{7}{91}$ by $3\frac{71}{87}$.

1094. Divide $19\frac{11}{17}$ by $7\frac{1}{13}$.

1095. Divide $76\frac{3}{4}$ by $7\frac{8}{11}$.

1096. Divide $8\frac{3}{4}'$ by $7\frac{5}{7}''$.

1097. What is the sum of the answers? Ans. 211.635753.

What is Prob. LV? The Rule? Why invert the divisor?

SECTION VII.

DIVISORS AND DIVIDENDS OF FRACTIONS.

1. Greatest Common Divisor of Fractions.

PROB. LVI.—TO FIND THE GREATEST COMMON DIVISOR OF TWO OR MORE FRACTIONS.

RULE.—*Change the fractions to their least common denominator, over which, write the greatest common divisor of their numerators.*

ILLUSTRATION.—What is the greatest common divisor of $\frac{3}{5}$, $\frac{7}{8}$, and $5\frac{1}{4}$? $\frac{3}{5}$, $\frac{7}{8}$, and $5\frac{1}{4} = \frac{3}{5}, \frac{7}{8}, \frac{21}{4} = \frac{24}{40}, \frac{35}{40}, \frac{210}{40}$. The greatest common divisor of 24, 35, and 210, is 1. Ans. $\frac{1}{40}$.

EXAMPLES.

1098. What is the greatest common divisor of $\frac{3}{7}$, $\frac{6}{9}$, $\frac{8}{11}$, and $\frac{2}{3}$?
1099. What is the greatest common divisor of $5\frac{1}{2}$ and $6\frac{1}{4}$?
1100. What is the greatest common divisor of $9\frac{7}{8}$ and 10?
1101. What is the greatest common divisor of $14\frac{1}{3}$ and $4\frac{7}{8}$?
1102. What is the greatest common divisor of $9\frac{1}{4}$ and $5\frac{3}{8}$?

2. Least Common Multiple or Dividend.

PROB. LVII.—TO FIND THE LEAST COMMON MULTIPLE OF FRACTIONS.

RULE.—1. *Change the fractions to their lowest terms.*

2. *Write the least common multiple of the numerators over the greatest common measure of the denominators.*

ILLUSTRATION.—What is the least common multiple of $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{8}$. Least common multiple of 2, 4, 6, is 12, and the greatest common measure of 3, 5, and 8, is 1. Ans. $\frac{12}{1} = 12$.

EXAMPLES.

1103. What is the least common multiple of $2\frac{1}{2}$, $3\frac{1}{3}$, and $\frac{3}{7}$?
1104. What is the least common multiple of $5\frac{1}{4}$, $10\frac{2}{7}$, and 3?
1105. What is the least common multiple of $4\frac{1}{2}$ and $5\frac{1}{6}$?
1106. What is the least common multiple of $11\frac{1}{4}$ and $17\frac{1}{3}$?
1107. Sum of the answers? Ans. 3014.045995.

What is Prob. LVI? Repeat the rule. Repeat Prob. LVII. Rule.

REVIEW OF PART I.

MISCELLANEOUS QUESTIONS.

1108. The total population of the United States in 1850, was 23,387,125. This does not include Indians. Native population was 17,737,505. Foreign population, 2,210,828. Nativity not ascertained, 39,014. The remainder were slaves. How many slaves in the United States of America according to the U. S. Census for 1850?

1109. Of the "Natives" 4,112,433 have migrated beyond the States of their birth. How many remain in their native States?

1110. In the New England States 1 person in 64 dies yearly. The population is 2,234,816. How many die annually in that part of the United States?

1111. In the United States 1 person in 73 dies yearly. How many deaths occur annually in the Union?

1112. Received the following packages of gold dust by Adams & Co.'s Express. No. 1 contained twenty, and four hundred seventeen, ten thousandths ounces. No. 2, three hundred seven, and nine hundred ninety seven, millionths ounces. No. 3, seventeen hundred eighty seven, and nineteen, hundredths ounces; and No. 4, nine hundred one, and seventeen, thousandths ounces. What is the whole worth at $16.0625 per oz.?

1113. Adams & Co. charged $.33$\frac{1}{3}$ per ounce freight. The Metropolitan Insurance Co. insured it for .01875 per dollar. What was the entire expense?

NOTE.—What would be the charge for $2? $3? $100? $672?

1114. What is the sum of the answers on this page?

Ans. 17430485.64.

What was the population of the U. S. A. in 1850? How many Native? Foreign? Slaves? Are Indians included? What was the population of New England in 1850? How many die there annually? How many in the Union? Repeat Prob. 1, 2, 3, 4, 5, 6. Repeat Art. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13. Repeat Rules to Prob. 1, 2, 3, 4, 5, 6.

1115. The gold cost me in San Francisco, $14.25 per ounce. What did the whole of it cost in California?

1116. What was my profit?

1117. I have 25 dozen hats, *unsalable*, which cost $4.25 each. Brown has caps at $3.00 that I can sell for 4.12½. I exchange all my hats for caps, giving him 3 hats for 4 caps. Do I make or lose by this trade, and how much?

1118. How much did I get for my hats?

1119. What did they cost Brown?

1120. How much did Brown make or lose by selling the hats at $4.00?

1121. What would have been my loss to have sold them at $4.00?

1122. John Chamberlain, the merchant, left his office to pay certain bills. He took with him $47.50; collected of James Hallet $71.53; of Simon Brown an order on Charles Dike of $13.71. He then paid a debt of $117.23, using Dike's order. James Root paid him $9.17. He gave $7.71 for charitable purposes. Received of Andrew Jackson $73.00 for rent of a house, $33.00 of Lemuel Cook on account, then paid a note of $107.50. How much did he collect?

1123. How much pay out?

1124. How much left?

1125. A train of cars left New York for Albany, 160 miles distant, and travelled at the rate of 30 miles an hour. After it had gone 20 miles, a second train left New York for Albany at the rate of 20 miles per hour. On the arrival of the first train at Albany, a third train left for New York at the rate of 30 miles per hour. How many miles from Albany would it pass the second?

1126. What is the sum of the answers on this page?

Ans. 50307.58.

Repeat Articles 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24 and 25.
Repeat Prob. 7, 8, 9, 10, 11, 12, 13, 14, 15 and 16.
Repeat Rule to Prob. 7, 8, 9, 10, 11, 12, 13, 14, 15 and 16.

1127. Which is the most $\frac{6}{2}$ of 8 or $\frac{8}{2}$ of 6?

1128. How much is 9 times $\frac{13}{3}$?

1129. How much is $4\frac{1}{3}$ times 9?

1130. What is the amount of fifteen, hundredths; one hundred, thousandths; four thousand and nine, hundred thousandths, ninety-one, millionths; ten, ten millionths; and six hundred ninety-seven, ten thousandths?

1131. What is the sum of seventy-one, hundredths; seventy-one, tenths; and thirty-nine, billionths; fifteen million, two hundred and two thousand; and four thousand and nine, ten thousandths?

1132. What is the difference between ten thousand and ten thousandths?

1133. What is the difference between nine hundred, thousandths and nine tenths?

1134. R. B. Dyson bought two house lots in Washington, D. C., for speculation. The lots contained 4412 square feet, and he paid 23 cents per square foot. He was taxed for them as follows: 1856, $7.54, city tax; paving tax, $39.48; school tax, $1.00. In 1857, $5.98 city tax and $1.00 school tax. He also paid $5.00 for a deed, and $3,75 to a lawyer for ascertaining the validity of the title to his lot. In 1858 he was offered 30 cents per foot for his lots and sold them, paying $10.00 more for advertising. How much did he make or lose by his trade, *not allowing interest?*

1135. What were the expenses of his lot, including the original cost?

1136 Charles and Silas bought 20 tubs of butter, weighing 1784 lbs., for $13\frac{1}{2}$ cents per pound. What did the butter cost them?

1137. What is the sum of the answers on this page?

Ans. 15213651.000782039.

Repeat Article 26, 27, 28, 29, 30, 31, 32, 33, 34, 35 and 36.
Repeat Prob. 17, 18, 19, 20, 21, 22, 23, 24, 25.
Repeat Rule to Prob. 17, 18, 19, 20, 21, 22, 23, 24 and 25.

1138. They then hired a carriage and transported the butter 22 miles to a railroad for $1.87. Paid $5.87 R.R. freight, and other expenses $3.96. What was the entire cost and expense of the butter?

1139. Allowing them $5.00 for their own labor, what was the butter worth per pound to them?

1140. They sold 768 lbs. at 22 cents per pound, 78 lbs. at 24 cents, 287 at 20 cents, 198 at 19 cents, 228 at $19\frac{1}{2}$ cents, and the remainder at 24 cents. What did they make by their speculation?

1141. A person performed a journey of 1791 miles, travelling $\frac{1}{2}$ the time at the rate of 87 miles per day, and the other half 112 miles. How many days did it take?

1142. How many tons of coal can you buy at $6.75 per ton with 1800 cords of wood at $5.75 per cord?

1143. Bought at one time $5\frac{7}{8}$ yards of broadcloth at $\$3\frac{3}{5}$ per yard; at another, $27\frac{3}{19}$ yards at $\$5\frac{7}{9}$ per yard. What did he pay for both pieces?

1144. How much more did he pay for one piece than for the other?

1145. A man had $\$587\frac{3}{7}$, and purchased corn at $\$\frac{9}{11}$ per bushel. How many bushels did he buy.

1146. What is the greatest common measure of 971, 791, 179, 719, 917, and 197?

1147. What is the greatest common measure of 221, 247, 299, 377, 403 and 481?

1148. What is the greatest common measure of 677, 899, 1537 and 1189?

1149. What is the greatest common measure of 1633, 2231, 3749 and 4163.

1150. What is the sum of the answers on this page?
Ans. 3016.722188.

Repeat Article 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, and 50.
Repeat Prob. 26, 27, 28, 29 and 30.
Repeat Rule to Prob. 26, 27, 28, 29, and 30.

1151. From a careful estimate by the architect of the Capitol, Thomas U. Walter, L L. D., it is found that the entire weight imposed on each superficial square foot will be but 7100 pounds, about $\frac{1}{157}$ part of the absolute strength of material of which it is composed. Suppose the foundation of the Capitol covers 10000 sq. ft., what would be its weight?

1152. How much would it be capable of sustaining?

1153. The pressure, per square foot, of Girard College, Philadelphia, is 13140. What would be the pressure of the walls of a building 284 feet long, 176 wide, and 4 feet thick, at the same rate?

1154. The entire Diplomatic Corps of the United States, in 1857, consisted of the following grades and salaries, viz:

Two Missions, each $17,500 per annum.
Five Missions, each $12,000 per annum.
Two Missions, each $10,000 per annum.
Seventeen Missions of second grade, $7500 per annum each.
Commissioner to China, $9000 per annum.
Commissioner to Sandwich Islands, $7500 per annum.
Two Secretaries of Legation, each $2625 per annum.
Six Secretaries of Legation, each $1800 per annum.
One Secretary of Legation, $3000 per annum.
One Secretary of Legation, $2000 per annum.
Eighteen Secretaries of Legation, each $1500 per annum.
What did the Diplomatic Corps cost for 1857?

1155. At the Paris election for "Corps Legislatif," June 21st and 22d, 1857, 110,000 votes were thrown for the Government, and 95,000 for the Opposition. The population was 1,400,000 How many persons for each vote?

1156. How much is $\frac{7}{8}$ of $4\frac{3}{5}$ divided by $\frac{3}{5}$ of $7\frac{19}{32}$?

1157. What is the sum of the answers on this page?

Ans. 11265821297.712669.

Repeat Art. 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66. Repeat Prob. 31, 32, 33, 34, 35, 36, 37, 38. Repeat Rule to Prob. 31, 32, 33, 34, 35, 36, 37 and 38.

1158. For sixteen years previous to 1850, the people of India have retained of the bullion sent to that country, $10,000,000 of silver, annually. For the last seven years, ending 1857, they have retained $27,500,000, annually. What amount of silver had India in 1858 from this source?

1159. What will 37 tubs of butter cost, each weighing $57\frac{7}{9}$ pounds, at $23\frac{3}{11}$ cents per pound, allowing each tub to weigh 11 pounds?

1160. Exchanged 750 barrels of flour at $\$7\frac{9}{13}$ per barrel, for 827 casks of nails, each weighing $103\frac{3}{7}$ pounds — cask weighing $5\frac{3}{5}$ pounds each—at $5\frac{3}{7}$ cents per pound, and received the balance in cash. How much?

1161. How much is $1111\frac{111}{1111}$ of $11\frac{11}{111}$ divided by $1\frac{1}{111}$ of $1111\frac{111}{1111}$?

1162. It is estimated that 102 first class cars on the N. York and Erie road will seat 6120 persons. How many is that to each car?

1163. The highest structure in the world is the Pyramid of Cheops in Egypt. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{8}$ of its height make $520\frac{3}{8}$ feet. Required its height.

1164. Pennsylvania, in 1848, had 7731 public schools, and 359564 pupils. What was the average number to each school?

1165. The library of Harvard University at Cambridge, Massachusetts, has 92000 vols. The library of Yale College, New Haven, Connecticut, has 51000 vols. How many must Harvard give to Yale that each may have the same number?

1166. Mt. Choumalarie, in Thibet, Asia, is estimated to be 29000 feet in height. Mt. De Sorato, in Bolivia, South America, 25370 feet in height. How many Bunker Hill Monuments, 220 feet high, placed one above another on Sorato, would make it just equal to Mt. Choumalarie?

1167. What is the sum of the answers on this page?

Ans. 352562834.215501.

Repeat Art. 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79 and 80. Repeat Prob. 39, 40, 41, 42, 43, 44, 45, 46, 47, 48. Repeat Rule to Prob. 39, 40, 41, 42, 43, 44, 45, 46, 47, 48.

1168. Divide eighty-three, thousandths by ninety-seven.

1169. What is a common multiple of 6, 8, 10, 12, 14 and 16?

1170. What is the least common multiple of 10, 12, 14 and 16?

1171. What is a common measure of 300, 400, 500 and 600?

1172. What is the G. C. measure of 300, 400, 500 and 600?

1173. What are the prime factors of 2000?

1174. What are the divisors of 1728?

1175. What is the least common multiple of 11, 13, 17 and 23?

1176. Give a multiple of 11.

1177. Give a multiple of 23.

1178. Find a multiple of 37 greater than a thousand.

1179. Find a multiple of 53 greater than 700.

1180. Find a multiple of $4\frac{1}{2}$ and $5\frac{1}{3}$.

1181. Find three multiples of $6\frac{3}{11}$ greater than 71.

1182. Find a common multiple of 4, 9, and 23.

1183. Find a common multiple of $7\frac{1}{4}$, $8\frac{1}{5}$ and $11\frac{1}{7}$.

1184. Change $\frac{180}{252}$ to its lowest form of expression.

1185. Given the quotient $9\frac{7}{12}$ to find the *dividend* and *divisor*.

1186. Change $\frac{1889}{2063}$ to its lowest form of expression.

1187. The quotient of two numbers is $91\frac{17}{23}$ what are the two numbers?

1188. What is the difference between 6.72 and 6 feet 8′?

1189. Divide \$$71\frac{31}{91}$ equally between 31 persons.

1190. At a certain election 813 votes were thrown. Adams had 406, Brown 207, Cobb 200. How many did Adams want of an election?

1191. At a second trial it appeared that Adams had 408 votes, Brown 207, and Cobb 201. Was Adams elected?

1192. Change $\frac{37\frac{1}{2}}{60}$ to its lowest terms.

1193. Divide $17\frac{3}{5}$ by $19\frac{1}{7}$.

Repeat Article 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115. Repeat Prob, 49, 50, 51, 52, 53, 54, 55. Repeat Rule to Prob. 49, 50, 51, 52, 53, 54, 55.

Analysis of Part II. Denominate Numbers.

CHAPTERS. SECTIONS. PAGE

- I. NOTATION
  - 1. Origin of Denominate Numbers 115
  - 2. Classification 118
  - 3. Notation of U. S. System 122
  - 4. Notation of French System 124
  - 5. Miscellaneous Tables 125
- II. PRIMARY RULES
  - 1. Reduction of Denominate Numbers 130
  - 2. Addition 134
  - 3. Multiplication 136
  - 4. Subtraction 137
  - 5. Division 138

REVIEW OF PARTS I. AND II. 139

PART II.

DENOMINATE SYSTEM OF NUMBERS.

CHAPTER I.

NOTATION.

SECTION I.

ORIGIN OF DENOMINATE NUMBERS.

164. In the Decimal System of Numbers, quantities increase and decrease in a ten fold ratio.

165. In the Denominate System of Numbers, quantities increase and decrease in very different ratios, and hence are called Denominate Numbers.

166. This great irregularity necessarily results from the origin of the system:

167. 1. All denominate numbers had their origin in the different customs of nearly all nations.

168. 2. The constant independent changes made by law and practice in different nations.

169. 3. The entire want of a standard "unit of measure," that is, of a quantity of a given, definite value, by which all quantities of that kind may be measured.

170. 4. It is a selection of measures from those used in all parts and ages of the world.

Of what does Part 1 treat? What Part 2? 164. What is the Decimal System? 165. What the Duodecimal System? What are Denominate numbers? 166. What great irregularity? 167. Where did denominate numbers have their origin? 167. What has increased the number of denominate numbers? 168. What effect did the want of a uniform standard have? 169. What is a unit of measure? 170. From what is this entire system obtained? What does Chap. I., Part II., treat of? Chap. II.?

171. 5. The standard for each class of measures was independent of that for every other class; *e. g.* a foot was the length of the king's foot, and hence must be long or short, as a long-footed or a short-footed king happened to reign.

Henry I., King of England, early in the 12th century declared that the ell or ulna, corresponding to the present yard, should be of the exact length of his own arm. From this all measures of length were derived.

The standard of weight was a grain of wheat gathered out of the middle of the ripe ear.

The standard of land measure was what a yoke of cattle could plow in a day, called an acre.

A scruple was a small stone.

A mile was a thousand steps, &c. &c. Standards, to say the least, of a very indefinite character.

172. 6. In forming tables from these standards of measure, another source of irregularity arose from the ratio taken. Some took multiples of 2; as, 4 farthings make a penny, 8 drams an ounce, 16 ounces a pound, &c. Others took 3 and its multiples; as, 3 scruples 1 dram, 3 feet = 1 yard, &c. Again, the multiples of 2 × 3 were taken together; as, 6 feet = 1 fathom, 12 inches = 1 foot, &c. And still others have the multiple of 2, 3, and 5; as, 60 seconds = 1 minute, 60 minutes = 1 hour or degree, &c., besides innumerable other ratios.

173. England has endeavored, for many years, to perfect a "System of Weights and Measures," but has not yet accomplished it. The "Standard Yard of 1760," of Great Britain, was burned with the Parliament Houses in 1834, and the new "Standard Yard of 1835" is one thousandth of an inch shorter than the "Standard Yard of 1760."

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171. How did the standard unit of one class compare with that of another? What was a standard foot? What objection to it? What the standard of weight? Of land measure? Apothecaries' weight? Of long measure? 172. What ratios were taken? 173. Has England a perfect system? How does the present yard compare with that of 1760? What became of the yard of 1760?

174. It was supposed that France had a *perfect system*, and she seemingly took every precaution to have it accurately based. But, unfortunately, *serious errors have been found*, leaving the system with no fixed basis.

175. The United States copied English standards, before they were improved under the light of the present state of the sciences; but has declined, thus far, to make any change.

176. Individual States, in the absence of National legislation, have instituted and established various weights and measures, *entirely independent of each other*, and the result is, that the United States has an exceedingly complicated system, thus:

A New York gallon contains 221.184 cubic inches.

The United States gallon contains 231. cubic inches.

In many States a ton is 2000 lbs.

The United States ton is 2240 lbs.

The United States is now distributing 50 lb. weights instead of 56, thus making 2000 lbs. in a ton.

177. The tendency now is to gain uniformity in the standard units of measure, and change the ratio to the *Decimal System*.

178. This should be kept before the teacher, and all denominations not *strictly legal* should be dropped.

179. The following classification will materially assist the learner, and convey a correct idea of the formation of denominate numbers, as well as point out the direction in which simplification and improvement may be made.

---

174. What of France? 175. The United States? Of individual States? Why has the United States a most complicated system? 176. How does the New York gallon compare with a gallon of the United States? What is a ton in many States? 177. Does the United States distribute standard weights to verify the "long ton" or the "short ton"? 178. What is the tendency of improvement? Repeat Art. 178. 179. What advantage in the following classification?

SECTION II.

CLASSIFICATION.

180. All quantities capable of measurement must take one of five forms, *viz.:* Abstract numbers, Circles, Lines, Surfaces or Volumes.

181. The *measures* of these quantities therefore must constitute the "Units of Denominate Numbers."

182. These units, given in the order in which they are obtained, are as follows, *viz.:*

1st Class.	1. Abstract Units.
2d Class.	2. Units of Circular Measure.
	3. Units of Time.
3d Class.	4. Linear Units.
4th Class.	5. Superficial Units.
5th Class.	6. Volume Units.
	7. Weight Units
	8. Money Units.

1. Abstract Units.

183. The Abstract Number *One* is the base of all Mathematics. Ten such Abstract Units make a *Unit* of the 2d order, called tens. Ten Abstract Units of the 2d order make one of the 3d order, called hundreds, &c.; thus the series of numbers is formed. (See Notation, Art. 14 and 15.)

2. Units of Circular Measure.

184. The Unit of Circular Measure is arbitrary, being one 360th part of a circle, called a degree.

185. As circles may be indefinite in extent, a degree will have *no definite size*, yet *will* always bear the same relation to the whole, and hence is fixed in value.

---

180. How many forms may quantities take? What are they? 181. What do the measures of these quantities constitute? 182. How many units of the 1st class? 2d? 3d? 4th? 5th? 183. What is an Abstract Number? How is a series of Numbers formed? 184. What is a unit of Circular measure? As circles have no definite size, how can a degree be said to be fixed?

186. All other Units are dependent on those of Abstract Numbers and Circular Measure.

187. It was from the circumference of the Earth that the French derived the value of the "Metre"—the *base* of the French System of Weights and Measures.

188. They actually measured several degrees of Latitude on the surface of the Earth, and from it determined 90 degrees of the circumference of the earth, or the distance of the pole from the equator.

189. *One ten millionth part of this distance was called a metre,* and is very nearly equal to 39.37079 English inches.

190. It is now known that errors were made—hence *a "French Metre" is not one ten millionth part of the distance of the pole from the equator.*

191. From the "French Metre" *an inch may be determined.*

3. Units of Time.

192. The units of Time are derived from Circular Measure.

193. A point on the surface of the earth in one daily revolution completes a circle.

194. The measures of time are that portion of time that elapses while this point is passing through a given part of this circle.

195. An hour is the time that elapses in its passing through one twenty-fourth part of the circle. This hour is subdivided into minutes, and the minutes into seconds.

196. As this circle of the earth is *invariable,* so the second must be a *definite length of time;* and, consequently, must give a definite fixed length to the pendulum.

---

186. What are all other units dependent on? 187. From what did the French determine the length of the Metre? 188. How? 189. What part of the distance of the pole from the equator is a metre? 190. Is that correct? 191. What may be determined from the Metre? 192. From what are units of time derived? 193. What does a point in the surface of the earth complete in one revolution? 194. What are measures of time? 195. What is an hour? 196. Is the circle of the earth invariable?

197. The pendulum therefore becomes the basis of all Linear Measures.

198. The English have determined the length of a pendulum, beating seconds in vacuo, at the level of the sea, at London, Tem. 62° Fahr., Bar. 30 inches, to be 39'.13929 English inches.

199. From this "pendulum" *an inch may be determined.*

4. Linear Units.

200. The Inch, as determined by the English or French method, is the base of all Linear Measures.

5. Superficial Units.

201. A surface one inch long and one inch wide, is a square inch, and is the base of all Superficial Measures.

6. Volume Units.

202. A block one inch long, one inch wide, and one inch deep, is a cubic inch, and is the base of Volume Measures.

203. It is not always convenient to use Volume Measures based on the cubic inch; hence, a new class of Volume Measures is formed, called.

7. Weight Units.

204. A Cubic Foot of distilled water, Tem. 62° Fahr., Bar. 30 in., weighs 997.17 avoirdupois ounces. Sixteen such ounces is one pound, and contains 7000 grains. One of these grains is the base of all Weight and Money Measures.

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197. What is the basis of all Linear Measures? Is a second always of the same length of time? 198. How long is a pendulum? Who determined its length? 199. What may be determined from the pendulum? 200. What is the base of Linear Measures? 201. What of Superficial Measures? 202. What is a Volume Unit? 203. Why is it not convenient to use the cubic inch as a measure always? 204. What is the method of finding the value of a grain?

8. Money Units.

205. Money Measures are certain weights of various precious metals determined by law.

206. Standard Gold of the United States Mint is an alloy, containing 900 parts of pure gold, 50 parts of pure silver, and 50 parts of pure copper.

207. Standard Silver is an alloy, containing 900 parts of pure silver, and 100 parts of pure copper.

208. An Eagle is a coin containing 258 grains of Standard Gold.

209. A Silver Dollar is a coin containing 412½ grains of Standard Silver.

210. The Old Cent is a coin containing 168 grains of pure copper.

211. The New Cent is a coin containing .88 parts pure copper, and .12 of nickel and weighs 72 grains.

212. The various Gold and Silver coins contain the same proportion of Standard Metal, according to value.

NOTE. — The student should, before proceeding farther, be certain that he understands this series of Unities, their entire dependence one on another, the order in which they are developed, and their actual value.

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205. What are Money Measures? 206. What is Standard Gold? 207. What Standard Silver? 208. What is an eagle? 209. What is a silver dollar? 210. A copper cent? 211. Do smaller or larger coins contain like proportions of standard metals? 212. What should the student do before proceeding farther? Why?

SECTION III.

NOTATION OF DENOMINATE NUMBERS. UNITED STATES SYSTEM.

Angular Measures.

1. CIRCULAR MEASURES.

60 Seconds (″) = 1 Minute (′)
60 Minutes = 1 Degree (°)
30 Degrees = 1 Sign (S.)
12 Signs = 1 Circle (C.)

2. TIME MEASURES

60 Seconds (sec.) = 1 Minute (m.)
60 Minutes = 1 hour (h.)
24 Hours = 1 Day (da.)
365¼ Days = 1 Year (y.)

Linear Measures.

1. LAND MEASURES.

12 *Inches* (in.) = 1 Foot (ft.)
3 Feet = 1 Yard (yd.)
5½ Yards = 1 Rod (r.)
40 Rods = 1 Furlong (F.)
8 Furlongs = 1 Mile (M.)

2. SEA MEASURES.

12 *Inches* = 1 Foot.
5.11625 Feet = 1 Log Fathom.
10 Log Fath. = 1 Knot or Mile
3 Knots(K.) = 1 League (L.)
20 Leagues = 1 Degree.

3. SURVEYOR'S LONG MEASURE.

7.92 *Inches* = 1 Link (l.)
25 Links = 1 Rod.
4 Rods = 1 Chain, Gunter.
10 Chains(ch.) = 1 Furlong.
8 Furlongs = 1 Mile.

4. CLOTH MEASURES.

2¼ *Inches* = 1 Nail (na.)
4 Nails = 1 Quarter (qr.)
4 Quarters = 1 Yard.

Superficial Measures.

1. SQUARE MEASURES.

144 *Square Inches* = 1 Square Foot (sq. f.)
9 Square Feet = 1 Square Yard (sq. yd.)
30.25 Square Yards = 1 Square Rod (sq. r.)
40 Square Rods = 1 Rood (R.)
4 Roods = 1 Acre (A.).
640 Acres = 1 Section or Sq. Mile.
36 Sections = 1 Township (T.)

2. SURVEYOR'S SQUARE MEASURES.

62.726 *Square Inches* = 1 Square Link.
625 Square Links = 1 Square Rod.
16 Square Rods = 1 Square Chain (sq. ch.)
10 Square Chains = 1 Acre.
640 Acres = 1 Section (S.)
36 Sections = 1 Township.

Volume Measures.

DIVISION I.

1. Solid Measures.

1728 *Cubic Inches* = 1 Cubic Foot.
27 Cubic Feet = 1 Cubic Yard.

2. Wood Measures.

1728 *Cubic Inches* = 1 Cubic Foot.
16 Cubic Feet = 1 Cord Foot.
8 Cord Feet = 1 Cord.

3. Dry Measures.

2150.4 *Cubic Inches* = 1 Bushel.
¼ Bushel (bu.) = 1 Peck.
½ Peck (pk.) = 1 Gallon.
¼ Gallon (gal.) = 1 Quart.
½ Quart (qu.) = 1 Pint.

4. Liquid Measures.

231 *Cubic Inches* = 1 Gallon.
¼ Gallon = 1 Quart.
½ Quart = 1 Pint.
¼ Pint = 1 Gill.

DIVISION II.

1. Avoirdupois Measures.

7000 *Grains* (gr.) = 1 Pound (lb.)
$\frac{1}{16}$ Pound = 1 Ounce (oz.)

2. Troy Measures.

5760 *Grains* = 1 Pound.
$\frac{1}{12}$ Pound = 1 Ounce.

3. Apothecaries' Measures.

5760 *Grains* = 1 Pound (℔).
$\frac{1}{12}$ Pound = 1 Ounce (℥).
$\frac{1}{8}$ Ounce = 1 Drachm (ʒ).
$\frac{1}{3}$ Drachm = 1 Scruple (℈).
$\frac{1}{20}$ Scruple = 1 Grain (gr.)

4. Money Measures.

412½ *Grains*, Silver = 1 Dollar ($)
$\frac{1}{10}$ Dollar = 1 Dime (d.)
$\frac{1}{10}$ Dime = 1 Cent (ct.)
$\frac{1}{10}$ Cent = 1 Mill (m.)

213. Besides the above tables, there are innumerable other weights and measures used in different parts of the country. For a list of *some* of them see Miscellaneous Tables, p. 125.

214. The simplest form that a System of Weights and Measures can take, is, to have one table, each, for Circular Motion, Time, Length, and Surfaces, and two for Volumes, using *a decimal ratio.*

Repeat the Tables for Angular Measure. What are the denominations of Circular Measures? What is Circular Measure used for? Repeat the denominations of the Time table. Its use. How many tables for Linear Measures? Their use? Name the denominations of the 1st? 2d? 3d? 4th? How many tables for surface? Their use? Give the denominations to the 1st? 2d? How many tables for the Volume Measures? Repeat the 1st? 2d? 3d? 4th? 5th? 6th? 7th? 8th? Give the use of each? 214. What is the simplest form that a System of Weights and Measures can take?

SECTION IV.

FRENCH SYSTEM.

215. In all the tables of this System the Latin prefixes, "Deci," "Centi," "Milli," &c., denote less; but the Greek prefixes, "Deca," "Hecto," "Kilo," "Myria," &c., denote more than the unit of value.

Angular Measures.

CIRCLE.

10 Milligrades = 1 Centigrade
10 Centigrades = 1 Decigrade.
10 Decigrades = 1 *Grade.*
10 Grades = 1 Decagrade.
10 Decagrades = 1 Quadrant.
4 Quadrants = 1 Circle.

TIME.

The same as in England and the United States.

Linear Measures.

10 Millimetres = 1 Centimetre.
10 Centimetres = 1 Decimetre.
10 Decimetres = 1 *Metre.*
10 Metres = 1 Decametre.
10 Decametres = 1 Hectometre.
10 Hectometres = 1 Kilometre.
10 Kilometres = 1 Myriametre.
A Metre is 3.281741 U. S. feet.

Surface Measures.

10 Milliares = 1 Centiare.
10 Centiares = 1 Deciare.
10 Deciares = 1 *Are.*
10 Ares = 1 Decare.
10 Decares = 1 Hectare.
10 Hectares = 1 Kilare.
10 Kilares = 1 Myriare.
An Are is 119.6648496 U. S. sq. yds.

Volume Measures.

CUBIC MEASURE.

10 Decisteres = 1 *Stere.*
10 Steres = 1 Decastere.
10 Decasteres = 1 Hectostere.
10 Hectosteres = 1 Kilostere.
10 Kilosteres = 1 Myriastere.
A Stere is 353,4384 U. S. cubic ft.

WEIGHT MEASURE.

10 Decigrammes = 1 *Gramme.*
10 Grammes = 1 Decagramme.
10 Decagrammes = 1 Hectogramme
10 Hectogrammes = 1 Kilogramme.
10 Kilogrammes = 1 Myriagramme
A Gramme is 15.433159 Troy grs.

Dry and Liquid Measure.

10 Decilitres = 1 *Litre.*
10 Litres = 1 Decalitre.
10 Decalitres = 1 Hectolitre.
10 Hectolitres = 1 Kilolitre.
10 Kilolitres = 1 Myrialitre.
A Litre is 2.6439. U. S. Liq. Gal.

Money Measures.

10 Centimes = 1 Decime.
10 Decimes = 1 *Franc.*
A Franc is .186 U. S. Act 1843.

SECTION V.

MISCELLANEOUS TABLES FOR REFERENCE.

216. Time Measures.

A *Civil Year* is 365 days, except Leap Year, which is 366.

A *Solar Year* is 365 days, 5 hours, 48 minutes, 49.65696 seconds, or 365.2422414 days.

A *Sidereal Year* is 365 days, 6 hours, 9 minutes, 11.5344 seconds, or 365.2563835 days.

Leap Year is every fourth year, except hundreds; and has one more day, which is given to February.

Days of the Months.
Thirty days hath September,
April, June, and November;
February twenty-eight,
Thirty-one the others rate.

217. Linear Measures.

3 Barleycorns make an Inch; 18 Inches make 1 Cubit.
3 STATUTE MILES DO NOT make a League.
3 Nautical Miles make a League.
5280 Feet make 1 Mile.
6120 Feet make a Marine, Nautical, or Geographical Mile.
6139.5 Feet make a " " according to U. S. Coast Sur.
6 Feet make a Fathom in depth.
3 Quarters make 1 Ell Flemish; 5 qrs. 1 Ell English.
6 Quarters make 1 Ell French; 4 qrs. $1\frac{1}{5}$ in. 1 Ell Scotch.

A Gunter's Chain is 66 feet long, and is divided into 100 links generally, sometimes into 80 links; and a Half Gunter's Chain is 50 or 40 links, and 33 feet long. In public service, an "Engineer's Chain" is 100 feet long, and divided into 120 links.

Pendulum.
6 Points make 1 Line; 12 Lines make 1 Inch.
39.101688 Inches make a Pendulum at N. York.
39.13908 " " " London.
39.12843 " " " Paris.

Height of Animals.
3 Inches make 1 Palm; 4 Inches make 1 Hand.
9 Inches make 1 Span.

218. Surface Measures.

Size of Paper.	Cap,	In. 13 by 16.	Columbia,	23 by 33¾.
	Demy,	15½ by 19½.	Atlas,	26 by 33.
	Medium,	18 by 22.	Theorem,	28 by 34.
	Royal,	19 by 24.	Dou. Elephant,	26 by 40.
	Super Royal,	19 by 27.	Antiquarian,	31 by 52.
	Imperial,	21¼ by 29.	Emperal,	40 by 60.
	Elephant,	22¼ by 27¾.	Uncle Sam,	48 by 120.

100 Square Feet is called a "Square," by Plasterers and other artificers.

100 Acres was called a Hide by the Ancients.

219. Volume Measures.

50 Cubic feet of hewn timber, *stows* 1 Ton in a ship.

40 Cubic feet of round timber, *i. e.* as much round timber as will make 40 feet hewn, will *stow* 1 Ton in a ship.

24¾ Cubic feet of stone make 1 Perch.

57¼ Cubic feet make 1 Chaldron.

1 Cubic foot of hard coal weighs about 58 lbs.; soft coal, about 50 lbs.; charcoal, hard wood, 18.5 lbs.; soft wood, 18 lbs.

1 Cord of Virginia pine weighs about 2700 lbs.

1 " " Southern pine " " 3300 lbs.

1 Ton of coals is 28 bu. 5 pecks = 43.56 cubic feet.

1 Ton of meadow hay contains 10 cu. yds.; clover, 11.5 yds.

1 Ton is 23 cubic feet of sand, 18, of earth, or 17, of clay.

18 Cubic feet of gravel make 27, when dug.

1 Cubic foot of loose earth or sand weighs 97 lbs.; clay, 135; common soil, 124; brick, 135; stone, 160; cork, 15; tallow, 59; cast iron, 450; wrought iron, 486; steel, 483; lead, 708; salt water, 64.3; fresh water, 62.8 lbs.

1 Barrel of cement contains 300 lbs.; flour, 196; gunpowder and raisins, each 112; anchovies, 30; soap, 256; beef, pork, salmon, and shad, each 200 lbs.

1 Barrel of cider or beer contains 32 gallons.

1 Barrel of corn is 5 bushels on the ear.

1 Hogshead of salt contains 8 bu. in ships, 7½, on shore.

1 Gallon of oil weighs 7½ lbs.; molasses, 11 lbs.

1 Stone of meat weighs 8 lbs.; iron, 14 lbs.

1 Tod weighs 28 lbs.

1 Firkin of butter weighs 56 lbs.; soap, 94 lbs.

1 Weigh is 182 lbs.; a Lock, 364 lbs.; a Last, 4368 lbs.; and a Fother, 19½ cwt.

1 Chaldron of coal yields 1000 cubic feet of gas.

31½ Gallons make 1 Barrel; 42, 1 Tierce; 63, 1 Hogshead.

2 Hogsheads make 1 Pipe; 2 Pipes, 1 Tun.

282 Cu. in. make 1 Beer Gallon; 9 Beer Gallons, 1 Firkin.

A Hogshead is any large cask, containing 50 gallons or upwards.

2 Firkins, 1 Kilderkin; 3 Kilderkins, 1 Hogshead.

54 B. Gallons, 1 Hogshead; 84 B. Gallons, 1 Puncheon.

Troy Weight.
24 Grains make 1 Pennyweight.
20 Pennyweights make 1 Ounce.

Avoirdupois Weight.—16 Drams make 1 Ounce.

A Ton is 2000 lbs. in Me., N. H., Vt., Mass., Conn., R. I., N. Y., N. J., Md., Ky., Ind., Ill., Wis., Mo., Cal., Iowa, O., Penn., Min., and Wash. In the other States, and at the Custom House, it is 2240 lbs.

220. Miscellaneous Measures.

20 Things make 1 Score.
12 Things " 1 Dozen.
12 Dozen make 1 Gross.
12 Gross " 1 Great Gross.

Diamonds.
16 Parts make 1 Grain = .8 Troy Grains.
4 Grains " 1 Carat = 3.2 "

Coopers.
3 Hoops make 1 Cast.
40 Cast " 1 Hundred.
10 Hundred " 1 Thousand.

Paper.
24 Sheets make 1 Quire.
20 Quires " 1 Ream.
2 Reams make 1 Bundle.
5 Bundles " 1 Bale.

Books.
1 sheet in 2 leaves is Folio.
1 " 4 " Quarto.
1 " 8 " Octavo.
1 sheet in 12 leaves is Duod.
1 " 18 " 18mo.
1 " 24 " 24mo.

[From Report of Superintendant of Weights and Measures, U. S., from Jan. 1848 to Dec. 31, 1856, by A. D. Bache. Published 1857.]

221. *Table showing the weight of a bushel of different grains and other substances, as established by law, in the different States — in pounds, avoirdupois.* 1857.

STATES.	Rye.	Wheat.	Indian corn.	Buckwheat.	Barley.	Oats.	Flax seed.	Clover seed.	Bran.	Potatoes.	Beans.	Timothy seed.	Hemp seed.	Blue grass seed.	Corn meal.	Onions.	Salt.	Dried apples.	Dried peaches.	Castor beans.	Mineral coal.	Stone coal.	Corn on cob.	Hair.	Rye meal.	Ruta-baga.	Sugar beet.	Mangel wurzel.
Maine	...	...	...	...	...	30	...	...	...	60	...	...	...	...	50	...	...	...	...	...	...	...	...	11	50	64	64	64
New Hampshire	...	...	...	...	...	30	...	...	...	60	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Vermont	56	60	56	46	46	32	...	...	...	60	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Massachusetts	56	60	56	48	48	32	...	...	...	...	...	...	...	...	50	52	...	...	...	...	...	...	...	...	50	...	...	...
Rhode Island	...	...	...	...	...	...	...	...	...	60	...	...	...	...	50	50	...	...	...	...	...	...	...	...	50	...	...	...
Connecticut	56	56	56	45	...	28	...	...	...	60	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
New York	56	60	56	...	48	32	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
New Jersey	56	60	56	50	48	30	55	64	...	60	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Pennsylvania	56	60	56	48	47	32	...	...	...	...	...	...	...	...	...	...	*80 †70 ‡62	...	...	...	...	...	...	...	...	...	...	...
Delaware	...	60	56	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Louisiana	32	60	56	...	32	32	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Kentucky	56	60	56	52	48	33⅓	56	56	20	56	60	45	44	14	50	57	50	...	...	...	...	§80 ‖70	...	...	...	...	...	...
Ohio	56	60	56	...	48	32	56	60	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Michigan	56	60	56	42	48	32	...	60	...	...	...	...	...	...	...	...	...	28	28	...	...	...	...	...	...	...	...	...
North Carolina ¶	...	60	54	50	48	30	55	...	...	...	...	...	...	...	45	...	...	...	...	...	...	...	...	...	...	...	...	...
Indiana	56	60	56	50	48	32	56	60	...	60	60	45	44	14	50	48	50	25	33	46	70	...	68	...	...	...	...	...
Illinois	56	60	56	52	48	32	56	60	20	60	60	45	44	14	48	57	50 ‡55	24	33	46	...	80	70	8	...	...	...	...
Wisconsin	56	60	56	42	48	32	56	60	...	60	...	46	...	...	...	...	...	28	28	...	...	...	...	...	...	...	...	...
Iowa	56	60	56	52	48	35	56	60	20	60	60	45	44	14	...	57	50	24	33	46	...	...	70	...	...	...	...	...
Missouri	56	60	56	52	48	35	56	60	20	60	60	45	44	14	...	57	50	24	33	46	80	...	...	...	...	...	...	...
California	54	60	52	40	50	32	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Minnesota Territory	56	60	56	42	48	32	...	60	...	...	...	...	...	...	...	...	...	28	28	...	...	...	...	...	...	...	...	...
Washington Territory	56	60	56	42	45	36	...	60	...	60	...	...	...	...	...	60	...	28	28	...	...	...	...	...	...	...	...	...

* Coarse. † Ground. ‡ Fine. § Bituminous. ‖ Cannel. ¶ Not officially communicated.

NOTE. — No laws specifying the weight to the bushel of grain or other commodities have been passed by the States of Maryland, Virginia, South Carolina, Georgia, Florida, Alabama, Mississippi, Tennessee, Arkansas, or Texas.

1. *U. S. Standard Yard Measure with Matrix 36 inches long.*

2. *Table of U. S. Standard Liquid Measures.*

These are brass with ground glass covers.

1 Gallon Measure containing 231 cubic inches.

1 Half Gallon, 1 Quart, 1 Pint, and 1 Half Pint Measure.

3. A U. S. Standard Half Bushel, with ground glass cover, containing 1075.21 cubic inches.

4. Table of U. S. Standard Troy Weights.

Thirteen Brass Weights.

	Oz.		Oz.
viz.: One,	12	Pound.	
"	10		
"	6		
"	5	and One,	.5
"	4	"	.4
"	3	"	.3
"	2	"	.2
"	1	"	.1

Fifteen Silver Wire Weights.

These are made of three sizes of wire — one for hundredths, one for thousandths, and one for ten thousandths — thus ≡.

Oz.	Oz.	Oz.	Form.
One, .05	One, .005	One, .0005	⬠
" .04	" .004	" .0004	□
" .03	" .003	" .0003	▽
" .02	" .002	" .0002	∨
" .01	" .001	" .0001	—

5. Table of U. S. Standard Avoirdupois Weights.

Eighteen Brass Weights.

	lb.		Oz.
viz.: One,	50	One,	8
"	25	"	4
"	20	"	2
"	10	"	1
"	5	"	.5
"	4	"	.4
"	3	"	.3
"	2	"	.2
"	1	"	.1

Fifteen Silver Wire Weights.

These are made in the same manner as in Troy Weight, of three sizes of wire, and in five forms — the number of sides indicating their value.

One, .05	One, .005	One, .0005	⬠
" .04	" .004	" .0004	□
" .03	" .003	" .0003	△
" .02	" .002	" .0002	∨
" .01	" .001	" .0001	—

Note. — The above list contains all the Standard Weights and Measures of the United States.

Note.—See Rep. Sup. of Weights and Measures, Ex. Doc. No. 27, Sen., 34th Congress, 3d Session.

CHAPTER II.

PRIMARY RULES.

SECTION I.

REDUCTION OF DENOMINATE NUMBERS.

222. Reduction shows the method of changing the form of an expression of a quantity, without changing its value.

Case 1. Reduction Descending.

Prob. LVIII.—To reduce a number from greater denominations to a less.

Rule.—1. *Commence with the greatest denomination, and multiply it by as many as it takes of a less denomination to make one of this.*

2. *Add to the product the number in the less denomination, if any.*

3. *Proceed in the same manner through the denominations necessary to the one required.*

EXAMPLES.

1194. Change 6 gallons to quarts.
1195. Change 8 gallons 3 quarts to pints.
1196. Change 16 cwt. to ounces. (100 lbs. = 1 cwt.
1197. Change 17 cwt. 5 lbs. 7 oz. to ounces.
1198. Change 7 mi. 5 fur. 39 rds. 3 yds. 2 ft. 4 in. to inches.
1199. Change 7 cwt. 3 lbs. to grains.
1200. Change 7 lbs. 9 oz. 16 pwt. 17 grs. to grains.
1201. Change 3 gal. 3 qts. to cubic inches.
1202. Change 12 mi. 4 furlongs to inches.
1203. Change 4¾ gallons to pints.
1204. What is the sum of the answers? Ans. 1874004.25.

Note.—In cases like this, the answers in denominate numbers will be considered abstract.

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222. What is Reduction? What is Reduction Descending? What is Prob. LVIII? What to be done first in the rule? Second? Third? What is the entire rule? How do you reduce gals. to qts.? Why multiply? What do you multiply by to reduce gals. to inches? How many inches in a gal.? What table is this? What is a cwt.?

1205. There were about 35 tons (2240 lbs.) of silver received at the Exhibition, Crystal Palace, London. How many dollars?

1206. The great Montreal bell weighs 13 tons (2240 lbs.) and 280 lbs. How many men can lift it, if each lifts $175\frac{1}{2}$ pounds?

Ans. 168.

1207. There were 1751 lbs. troy of gold received at Crystal Palace Exhibition, London. How many dollars?

1208. How much money in both gold and silver?

1209. What would it weigh in lbs. if it was all gold?

1210. How much if all silver?

1211. Change $16\frac{7}{8}$ cwt. to ounces.

1212. Change $\frac{7}{13}$ cwt. to ounces.

1213. Change 7 mi. 5 fur. and 3 yds. to inches.

1214. Change $7\frac{3}{5}$ mi. $5\frac{2}{8}$ fur. and $7\frac{9}{27}$ yds. to inches.

1215. Change the same to feet.

1216. Change the same to yards.

1217. Change $\frac{7}{19}$ miles to rods.

1218. Change $71\frac{13}{23}$ miles to furlongs.

1219. Change $\frac{9}{17}$ degrees to miles.

1220. "St. Bernard's Tun," a celebrated wine vat, at Chervaux, France, holds 201600 galls. What would this full of wine be worth at $\$\frac{1}{3}$ per pint?

1221. The Heidelburg wine vat, built in 1343, now contains 67000 gallons. How many casks, if each contains $341\frac{3}{7}$ pints?

1222. Change 4 y. 17 d. 12 h. 13 m. 9 sec. to seconds.

1223. Change $4\frac{1}{2}$ y. $17\frac{3}{5}$ d. $12\frac{3}{9}$ h. $13\frac{3}{11}$ m. to minutes.

1224. Change $7\frac{3}{8}$ yrs. $19\frac{4}{11}$ days to hours.

1225. Change 4 yds. 3 qrs. 3 na. to inches.

1226. Change 7 lbs. 3 oz. Avoirdupois to lbs. Troy.

1227. What will 75 gal. 3 qts. 1 pt. of wine cost at $57\frac{3}{5}$ cents per pint?

1228. How many lbs. of silver coin in that amount?

1229. What is the sum of the answers on this page?

Ans. 135385639.07.

How many grains in a silver dollar? How many in a gold dollar? Where is Heidelburg? What is the difference between a "Ton" and "Tun"?

Case 2. Reduction Ascending.

Prob. LIX.—TO REDUCE A NUMBER FROM A LESS DENOMINATION TO GREATER DENOMINATIONS.

Rule.—1. *Divide the given number by as many as makes one of a greater denomination.*

2. *Set aside the remainder, if any.*

3. *Continue the process through the required denominations.*

4. *If required to reduce a less denomination to a greater denomination, complete each division.*

Ill. 1.—How many days, hours, min. and sec. in 627843 sec.

60)627843
60)10464 + 3
24)174 + 24
7 + 6 Ans. 7 d. 6 h. 24 m. 3 sec.

Ill. 2.—How many days in 627843 seconds?

60)627843
60)1046.405
24)174.40083+
Ans. 7.2667 days.

The answer might have been found in a common fraction.

EXAMPLES.

1230. Change 12096 seconds to minutes.
1231. Change 12096 seconds to hours (*i. e.* fraction of an hour).
1232. Change 2096842 seconds to years.
1233. How many gallons in 756825 pints?
1234. How many years in 7684 d. 18 h. 48 m. and 27 sec.?
1235. Change 75842 grains to lbs.
1236. Change 75842 grains to lbs. Troy.
1237. Change 46826 grains to tons.
1238. Change 46826 grains to cwts.
1239. Change 46826 grains to lbs.
1240. What is the sum of the answers? Ans. 94690.508039.

What is Reduction Ascending? What is Prob. LIX? What is the first part of rule? Second part? Third? How reduce to a greater *denomination?* What is a complete division?

EXAMPLES.

1241. Change 57846 grains to pounds.

1242. Change 57846 grains to higher denominations.

1243. How many sidereal years in 468268463 seconds?

1244. How many solar years in 468268463 seconds?

1245. What part of a cord is 7468 cubic inches?

1246. Change 6842674 grains to lbs. Troy.

1247. What is the difference in number of lbs.?

1248. Change 70000 inches to miles.

1249. Change 700 inches to miles (*i. e.* fraction of a mile).

1250. Change $\frac{7}{10}$ of an inch to a fraction of a mile.

1251. Change $\frac{7}{8}$ of a rod to a fraction of a mile.

1252. Change $\frac{4}{5}$ of a grain to the fraction of a ton.

1253. Change $\frac{4}{5}$ of a grain to sevenths of a ton.

1254. Which is most, 7 lbs. 14 oz. Avoirdupois, or 9 lbs. 3 oz. Troy, and how much?

1255. A man buys coal in Penn. to sell in Washington, and pays $5.12 per ton at the mines, including transportation to his coal yard. For what must he sell it per ton to get back his money?

1256. How many inches difference between 712 gals. 2 qts. and 1 pint at New York, and the same number of gallons, quarts, and pints at Georgetown, D. C.?

1257. A man has a flock of 710 sheep, and at a shearing he took off, on an average, 3 lbs. $4\frac{1}{2}$ oz. from each sheep. It cost 10 cents each to shear the sheep, and 4 cents each to market the wool. What had he, after paying expenses, if he sold his wool at $41\frac{1}{3}$ cents per pound?

1258. How many dollars in 7842 grains of silver?

1259. How many dollars in 5786 ounces of gold?

1260. The Capitol of the United States covers about $4\frac{1}{2}$ acres. The foundations press 7572 lbs. per square foot. How many pounds would that be per acre?

1261. What is the sum of the answers of the last six examples?

Ans. 329951849.90.

SECTION II.

ADDITION OF DENOMINATE NUMBERS.

223. Addition of Denominate Numbers differs in no respect from the Addition of Decimal Numbers. (See Prob. I, p. 22, and Prob. LIII, p. 102.)

ILLUSTRATION. — What is the sum of 5 da. 4 hr. 27 m., 17 da. 8 hr. 23 m., 19 da. 13 hr. 6 m., and 73 days 13 minutes.

	D.	H.	M.
	5	4	27
	17	8	23
	19	13	6
	73	0	13
Ans.	115	2	9

Adding the min. we have 69 m. = 1 h. 9 m. Adding the hours we have 25 + 1 = 26 h. = 1 d. 2 h. Adding the days we have 114 + 1 = 115 d.

EXAMPLES.

1262. Add 4 lbs. 8.5 oz., 17 lbs. 12 oz., 47 lbs. .97 oz., 138 lbs. $11\frac{3}{5}$ oz., and 19 lbs. 11 oz. Ans. 227 lbs. 12.07 oz.

1263. Add $5\frac{1}{2}$ lbs. $6\frac{1}{2}$ oz, $9\frac{3}{4}$ lbs., and $10\frac{3}{8}$ oz.
Ans. 16 lbs. $4\frac{7}{8}$ oz.

1264. Add $7\frac{3}{4}$ days, $9\frac{5}{7}$ days, and $8\frac{5}{11}$ hrs. Ans. 17d. $19\frac{46}{77}$h.

1265. I have bonded 4 boxes of sugar at Custom House, Charleston, net weight as follows :

No. 1.	4 cwt.	3 qrs.	21 lbs.	6 oz.
No. 2.	3 cwt.	2 qrs.	27 lbs.	11 oz.
No. 3.	5 cwt.	1 qr.	3 lbs.	8 oz.
No. 4.	4 cwt.	3 qrs.	17 lbs.	9 oz.

How many pounds?

1266. Add 25 lbs. Avoirdupois, 13 lbs. Troy. What is the amount in Troy pounds?

1267. What is the amount in Avoirdupois pounds?

1268. A ship sails $13\frac{1}{2}$ knots per hour for $17\frac{1}{2}$ hours and meets another ship that has sailed $12\frac{7}{9}$ knots per hour for $23\frac{1}{3}$ hours. What distance have both sailed?

1269. What is the sum of the answers on this page?
Ans. 2508.602234.

What is the rule for the addition of Denominate Numbers?

1270. Add 47 A. 3 R. 27 sq. r. 6 sq. yds., 104 A. 2 R. 17 sq. rd. 21 sq. yds. and 231 A. 1 R. 17 sq. rd. 5 sq. yds.

1271. Add 7° 39′ 27″, 35° 27′ 31″, 81° 27″, 19° 18′, 36° 18′ 41″, and 78° 39″.

1272. Add 7 da. 14 hrs. 49 min. 17 sec., 24 d. 3 m. 8 sec., 127 d. 8 h., 179 d. 37 sec., 18 d. 47 m., and 327 d. 12 h.

Ans. 683 da. 11 hr. 40 min. 2 sec.

1273. Add 14 dollars 7 cents, 9 dollars 7 mills, 300 dollars 1 cent, 48 dollars 1 dime.

1274. Add 1 eagle, 1 half eagle, 1 quarter eagle, 1 dollar, 1 half dollar, 1 dime, 1 half dime, 1 three-cent piece, and 1 cent.

1275. Add 7 mi. 4 fur. 31 rds. 7 yds. 2 ft. 8 in., 19 mi. 6 fur. 3 yds. 12 ft. 18 in., 7 miles $8\frac{3}{4}$ fur. $27\frac{1}{7}$ ft. and $8\frac{1}{4}$ inches.

Ans. 35 mi. 3 fur. 25 rds. 2 yds. 1 ft. $11\frac{27}{28}$ in.

1276. Add 68 A. 3 R. 19 sq. rds. 21 sq. ft. 31 sq. in., 43 A. 17 R. 91 sq. rds. 73 sq. ft. 827 sq. in.

1277. Add 8 cords 27 cu. ft., 12 c. 12 cd. ft., 17 c. 4 cu. ft. 2 cu. ft., 822 c. 9 cd. ft. 128 cu. ft., and 237 c.

Ans. 1099 cords 7 cd. ft. 1 cu. ft.

1278. Add 122 gall. 3 qt. 1 pt., 118 gall. 1 qt. $1\frac{1}{2}$ pt., 97 gall. 3 qt. 1 pt.

1279. Add 18 lbs. 12 oz., 192 lbs. 13 oz., 157 lbs. 9 oz., 271 lbs. 9 oz., 231 lbs. 7 oz, 98 lbs. 5 oz., 93 lbs. 11 oz., 37 lbs. 6 oz., and 51 lbs.

1280. Add 7 knots 6 deg., 8 k. 19 deg., 12 k. 28 deg., 19 k. 238 deg., 39 k. and 7 leagues.

1281. Add 18 da. 12 hr. 17 m. 53 sec., 25 da. 19 hr. 47 m. 56 sec., 91 da. 18 m. 12 sec., and 35 da. 12 hr. 37 m. 47 sec.

1282. Add 17 cwt. 21 lbs. 15 oz., 3 tons 12 cwt. 1 qr. 37 lbs. 13 oz., and 32 tons 6 cwt. 9 oz.

1283. What is the sum of 6℔ 9℥ 3ʒ 1℈ 13 grs., 17℔ 7℥ 7ʒ 2℈ 19 grs., and 17℔ 11℥ 1℈.

In adding up ounces what do you divide by? Why? What table? How can you find how many inches there are in any given number of quarts? Can you trace any relation between a dollar and a pint? How are each connected with a pendulum?

SECTION III.

MULTIPLICATION OF DENOMINATE NUMBERS.

224. Multiplication of Denominate Numbers differs in no way from Multiplication of Decimal Numbers. (See Prob. II, p. 28, and Prob. XXXVII, p. 81.)

Illustration.—Multiply 5 lbs. 8 oz. by 3.

lbs.	oz.
5	8
	3
16 lbs.	8 oz. Ans.

Multiply 8 oz. by 3, and it gives 24 oz. = 1 lb. 8 oz. Multiply 5 lb. by 3 gives 15 lbs. + 1 = 16 lbs.

EXAMPLES.

1284. Multiply 4 cwt. 1 qr. 5 lbs. 8 oz. by 5. (430.5 × 5 = 2152.5 Ans.)

1285. Multiply 4 cwt. 1 qr. 5 lbs. 8 oz. by $5\frac{1}{2}$.

1286. Multiply 4 cwt. 1 qr. 5 lbs. 8 oz. by 5.7.

1287. Multiply 4 cwt. 1 qr. 5 lbs. 8 oz. by 3.67.

1288. Multiply 4 cwt. 1 qr. 5 lbs. 8 oz. by 3 6′.

1289. Multiply 4 cwt. 1 qr. 5 lbs. 8 oz. by $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{9}$.

1290. Multiply 4 cwt. 1 qr. 5 lbs. 8 oz. by $4\frac{3}{4}$ of $7\frac{3}{19}$.

1291. Multiply 4 cwt. 1 qr. 5 lbs. 8 oz. by $4\frac{1}{2}$.

1292. Sum of the above answers? Ans. 24626.035 lbs.

1293. Multiply 7° 8′ 9″ by $4\frac{3}{5}$.

1294. Multiply 4 m. 3 fur. 5 rds. 3 yds. 1 ft. 6 in. by 21.

1295. Multiply 2 m. 2 fur. 5 rds. 3 ft. 7 in. by 93.

1296. Multiply 3 m. 3 fur. 7 ch. 12 links by 41.

1297. Multiply 5373 M. 210 A. 6 sq. ch. 34 sq. lks. by 43.

1298. Multiply 41 cu. yds. 13 cu. ft. 1311 cu. in. by 91.

1299. Multiply 17 cords 7 cd. ft. 13 cu. ft. 1329 cu. in. by 83.

1300. Multiply 23 bu. 3 pks. 2 gals. 3 qts. 1 pt. by 47.

1301. Multiply 217 gals. 3 qts. 1 pt. by $17\frac{3}{5}$.

1302. Multiply 21 lbs. 13 oz. by $9\frac{1}{2}$ of $\frac{3}{11}$.

1303. Multiply 27 yds. $3\frac{3}{5}$ qrs. by $23\frac{1}{17}$.

What is Multiplication of Denominate Numbers? What is the Rule? Repeat Article 106, 107, 108, 109, 110, 111, 112, 113, 114, 115.

SECTION IV.

SUBTRACTION OF DENOMINATE NUMBERS.

225. Subtraction of Denominate Numbers differs in no way from the Subtraction of Decimal Numbers. (See Prob III, p. 35, and Prob. LIV, p. 104.)

Illustration.—Subtract 7 lbs. 9 oz. from 27 lbs. 7 oz.

lbs.	oz.	
27	7	
7	9	
19	14	Ans.

We cannot subtract 9 from 7, therefore take 1 lb. = 16 oz., and add to 7 oz. = 23 oz. Now 23 oz. — 9 oz. = 14 oz. Then 7 from 26 = 19.

EXAMPLES.

1304. Subtract 23 lbs. 11 oz. from 29 lbs. 9 oz.
1305. Subtract 11℔ 11℥ 1ʒ 2℈ 16 grs. from 31℔ 6℥.
1306. Subtract 7° 31′ 57″ from 31° 43′ 12″.
1307. Subtract 12 d. 13 h. 11 m. 29 s. from 37 d. 9 h. 16 sec.
1308. Subtract 7 bu. 3 pks. 5 qts. 1 pt. from 23 bu. 1 pk. 3 qts.
1309. How long from June 6th, 1857, to June 30th, 1857?
1310. How long from June 6th to Aug. 3d? Ans. 58 da.
1311. How many days from Jun. 6th, 1857, to Sep. 25th, 1858?
1312. How long from June 6th, 6 o'clock, A. M., 1857, to Sept. 25th, 6 P. M., 1858? Ans. 1 yr. 111 da. 12 hr.
1313. Subtract April 29th, 1855, from Nov. 17th, 1859.
1314. Subtract Jan. 1st, 1857, from Dec. 31st, 1857.
1315. Subtract Dec. 31st, 1856, from Jan. 1st, 1857.
1316. Subtract Dec. 30th, 1852, from Dec. 1st, 1857.
1317. Answers of the last 4 examples? Ans. 10 y. 173 d.
1318. Subtract 17 A. 3 R. 16 sq. rd. from 39 A. 1 R.
1319. Subtract 81 A. 2 R. 13 sq. rd. from 432 A. 31 sq. rd.
1320. Subtract 17 m. 3 fur. 17 rd. 8 yd. from 93 m, 1 yd.
1321. Subtract 71 cords 7 cd. ft. from 128 c. 3 cd. ft.

What is Subtraction of Denominate Numbers? What is Subtraction of Decimal Numbers? What Rule? Repeat Prob. III. and LIII. What is the difficulty with the 1269th example? Repeat Art. 116, 117, 118, 119.

SECTION V.

DIVISION OF DENOMINATE NUMBERS.

226. Division of Denominate Numbers differs in no way from the Division of Decimal Numbers. (See Prob. IV, p. 37, and Prob. LV, p. 105.)

Illustration. — Divide 27 lbs. 9 oz. by 2.

	lbs.	oz.
2)	27	9
	13	12.5 Ans.

Dividing by 2 there is 1 lb. left = 16 oz. Then 16 oz. + 9 oz = 25 oz. This divided gives 12.5 oz. Ans.

EXAMPLES.

1322. Divide 18 lbs. 14 oz. by 3.
1323. Divide 18 lbs. 14 oz. by 4.
1324. Divide 18 lbs. 14 oz. by 3.5.
1325. Divide 18 lbs. 14 oz. by 27.

} 17 lbs. 1.62 oz.

1326. Divide 13 lbs. 11 oz. 17 pwt. 12 grs. by 13.
1327. Divide 13 lbs. 9℥ 5ʒ 2℈ 7 grs. by 17.
1328. Divide 17 bu. 3 pk. 1 gal. 3 qts. by 11.
1329. Divide 4 m. 3 fur. 21 rds. 7 ft. by 23.
1330. Divide 7 m. 6 ch. 24 lks. by 31.
1331. Divide 8 yds. 3 qrs. 2 n. 2 in. by 17.
1332. Divide 19 lea. 7 k. by 24. Ans. 2.66.
1333. Divide 7 s. 19° 18′ 23″ by 31.
1334. Divide 61 T. 17 S. 231 A. 1 R. 19 sq. rd. by 211.
1335. Divide $47\frac{3}{4}$ yds. by 17. Ans. 2 yds. 3.23 qrs.
1336. Divide $31\frac{3}{11}$ cu. yds. 19 cu. ft. by 49.
1337. Divide $\frac{7}{8}$ of a mile by 39. Ans. in less denomination.
1338. Divide $\frac{7}{9}$ of a ton by 43. Ans. in less denomination.
1339. Divide $4\frac{1}{2}$ pounds equally among 5 men.
1340. Divide $\frac{5}{7}$ of $3\frac{1}{3}$ cords of wood into 7 piles.
1341. Divide 7 years 12 da. 17 min. 19 sec. by 391.
1342. Divide 81 gall. 3 qts. 1 pt. 2 gills by 19.
1343. Divide 10 A. 6 R. 37 rods by 16.

What is Division of Decimal Numbers? Rule for it? What is Division of Denominate Numbers? Rule? Repeat Art. 120, 121, 122, 123, 124.

REVIEW OF PARTS I. AND II.

MISCELLANEOUS QUESTIONS.

1344. In 1841 the first guano was used in Great Britain, and 2881 tons were imported. From the end of that year to the end of 1856, 1500000 tons had been imported into Great Britain. What is the average amount imported annually?

1345. According to Hon. Henry Barnard, 52,970,231 acres of land had been given by the government of the United States for educational purposes prior to January 1, 1854. What will this amount to at $1.25 per acre?

1346. How many more acres than the New England States contain?

1347. It is proposed to tunnel the Straits of Dover between England and France, and the estimated cost is $20,000,000. How many pounds of U. S. silver coin?

1348. How many pounds of U. S. standard gold?

1349. It is estimated that by this *submarine* railway the number of passengers between England and France, now 200,000 annually, will be greatly increased. Suppose it to be increased one fourth, and that the price of passage be $1.50 each way, deducting fifty-two Sundays, what would be the daily income to the road from that source?

1350. By experiment on board the U. S. Frigate Mississippi, on her passages from Spezzia to Leghorn, in 1849, '50, and '51, under Chief Engineer Jesse E. Gray, it was found that with 21 paddles on each wheel, she made during the trip 37½ miles with 3011 revolutions. Then ten paddles were removed, and she made the same passage with 11 paddles in 3536 revolutions. How many feet did 1 revolution with 21 paddles drive the ship?

1351. How many feet did 1 revolution with 11 paddles move the ship?

1352. What is the sum of the answers on this page?

Ans. 78479504.587425.

1353. The telegraphs for the year 1856 consumed $60,000 worth of zinc, $10,000 worth of nitric acid, $30,000 worth of mercury, besides other sums for sulphuric acid, &c. What was the daily expense of the above items, it being Leap Year?

1354. At the Fair of the Royal Agricultural Society, at Carlisle, England, a brick machine was exhibited by Messrs. Porter and Co., capable of turning out 18000 bricks per hour. How many would it make from April 7th, 1857, to October 6th, 1857, working 6 days per week and 10 hours per day?

1355. What would they be worth at $\$9\frac{7}{8}$ per M.?

1356. According to Mr. Airey's experiment, the time occupied for the electric current to go from Greenwich, England, to Brussels, was 0.109″ (being the result of 2616 observations); distance, 270 miles. What is the velocity of the electric current per second?

1357. The following is the amount of copper from the Lake Superior Region for 1855: Ontonagon District, 2190 tons; Kewenaw District, 2225 tons; Portage Districts, $345\frac{1}{2}$ tons. What is that worth at $13\frac{1}{4}$ cents per pound?

1358. The quantity of shell-lac on hand in England, in 1854, was 2358750 lbs. How many tons?

1359. The whole number of patents granted by the United States to the commencement of the year 1855, in the classes named, was as follows: Grain and Grass Harvesters, 111; Plows, 372; Straw Cutters, 153; Smut Machines, 140; Winnowing Machines, 163; Thrashing Machines, 378; Stoves, 1160; Air Engines, not one of which is in use, 21; Steam Boilers, 148; India Rubber Goods, 42; Sewing Machines, 60; Water Wheels, 327; and Washing Machines, 309. What would the cost of procuring these amount to at $\$113\frac{3}{8}$ each?

1360. What is the sum of the answers on this page?

Ans. 30003542.163263.

How many grains in a pound Avoirdupois? What is the weight of a U. S. silver dollar? Repeat Art. 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148.

1361. A person owning $\frac{3}{7}$ of a ship, sold $\frac{5}{8}$ of his share for $43,000. What was the value of the ship?

1362. Two boats start on a river whose current is 5 miles per hour, and are driven by steam at the rate of 14 miles per hour in still water. If they are 475 miles apart at the commencement, in how long time will they meet?

1363. What distance will each have travelled over?

1364. If the Heidelburg wine vat is $\frac{3}{11}$ full, how many gallons will it take to make it $\frac{13}{17}$ full?

1365. If $\frac{3}{7}$ of the time past 6 A. M. is equal to $\frac{9}{10}$ of the time to midnight, what time is it? Ans. 6 o'clock $11\frac{19}{31}$ m.

1366. How much is $2\frac{3}{7}$ tons, $5\frac{3}{5}$ cwt., and 11 tons?

1367. Change $\frac{3}{51}$ of a ton to lbs.

1368. Change $\frac{3}{51}$ of a lb. to the fraction of a ton.

1369. Subtract $\frac{1}{325}$ of a ton from $9\frac{3}{7}$ lbs.

1370. Bought $19\frac{3}{7}$ cwt. of sugar at $11\frac{1}{9}$ cents per lb.; 17 bags of coffee, each $112\frac{3}{11}$ lbs., at $5\frac{3}{8}$ cents per lb.; 14 chests of tea, each $16\frac{3}{13}$ lbs., at $41\frac{3}{7}$ cents per lb. Paid for the above in wheat, at $\$1.93\frac{3}{11}$ per bushel. How many bushels did it take?

Ans. 213.48 bu.

1371. How many pint bottles will it take to bottle up 197 cases of Congress water, capable of holding $\frac{1}{9}$ of a hogshead each?

1372. A gentleman owned 5 tracts of land. The first contained $129\frac{3}{11}$ acres; $517\frac{3}{8}$ in the second; the third, $291\frac{3}{15}$ acres; fourth, $1193\frac{13}{17}$ acres; and the fifth, $284\frac{9}{13}$ acres. He offers the whole for sale at 11 cents per square rod. What will purchase it? Ans. $42526.96.

1373. A note is given June 30th, 1851, and is paid Aug. 9th, 1858. How long was it on interest?

1374. After giving $\frac{2}{3}$ of a property to my son, I gave $\frac{3}{11}$ of the remainder to the Old Ladies' Home, $\frac{5}{13}$ of what was left to the Seamen's Home, $\frac{19}{25}$ of what was left to the Orphans' Asylum, and had $78.55 left. What was given to each, and what was the whole estate? Ans. $2193.87.

1375. If 9 lbs of tea cost $\$8\frac{3}{7}$, what is the price per pound?

1376. If $\frac{7}{8}$ of an acre produce $51\frac{3}{11}$ bushels of corn, what would 1 acre produce?

1377. At the same rate what would $9\frac{3}{5}$ acres produce?

1378. If $\frac{3}{5}$ of an acre produce $27\frac{3}{9}$ bushels of wheat what would $174\frac{3}{11}$ acres produce?

1379. What could I get for it by selling it for $\$1\frac{7}{9}$ per bushel, giving every 11th bushel for cost?

1380. What will $13\frac{3}{4}$ dozen eggs be worth at $31\frac{1}{4}$ cents per dozen?

1381. How much is $\frac{13}{31}$ of $\frac{31}{13}$ divided by $\frac{31}{13}$?

1382. What is the product of $\dfrac{4\frac{1}{2}}{9\frac{1}{7}}$ by $3\frac{7}{11}$?

1383. What is the quotient of 3 6′ 7″ by $\frac{7}{9}$?

1384. What is the amount of $4\frac{5}{7}$ acres $9\frac{3}{8}$ A. and $7\frac{3}{11}$ R.?

1385. Abram Brown had $2348\frac{3}{5}$ bushels of wheat. He sold $\frac{3}{7}$ at $\$1.93\frac{1}{5}$ per bu., $\frac{2}{5}$ at $\$2.11\frac{1}{9}$ and the remainder at $\$1.77\frac{1}{2}$ per bu. What did he get for it?

1386. Bought 293 bbls of flour for \$1777, and sold it for $\$9\frac{1}{12}$ per bbl. What was my profit?

1387. What did the flour cost per bbl?

1388. Some persons in Abington agreed to send an agent to the west to buy flour for them. He went to Rochester and bought 1500 bbls of flour at $\$6.37\frac{1}{2}$ per bbl. He paid also 15 cents per bbl. to have it forwarded to Boston. At Boston he paid 3 cents per bbl. wharfage, 10 cents per bbl. truckage, and \$17.86 freight to Abington. His own travelling expenses were \$47.83 and he was allowed \$1.50 per day for his time and was gone 12 days. It also cost $12\frac{1}{2}$ cents to deliver each barrel to each man's house from the depot.

Repeat Art. 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173. Repeat Prob. and Rule 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 2, 5, 7, 9. Repeat Art. 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184. **Repeat Prob. and Rule** 13, 15, 19, 14, 16, 9, 8, 17, 18, 21, 22.

The same flour was selling in Abington at that time for \$8.25. How much did this company save per barrel?

1389. What is the sum of the answers of the last four examples?

1390. What cost $\frac{13}{17}$ of a ton at \$.08$\frac{1}{3}$ per lb.?

1391. Bought a box of sugar containing 7 cwt. 2 qrs. 27 lbs. 8 oz. After selling $\frac{7}{9}$ of it I was offered 13$\frac{3}{5}$ cents per lb. for the remainder. This I refused, and sold it for \$25.00. How much did I make or lose?'

1392. On the completion of the Capitol at Washington $\frac{1}{5}$ of $\frac{1}{4}$ of it will cost \$500,000. What will be its entire cost?

1393. James Allen bought 98 gallons of Molasses for \$58$\frac{3}{5}$. What will 39 gallons cost at the same price?

1394. If $\frac{7}{9}$ of a load of hay cost \$15$\frac{3}{6}$ what will 3$\frac{8}{11}$ loads cost?

1395. Robert exchanged 3$\frac{3}{7}$ bbls. of flour at \$11$\frac{3}{7}$ per bbl. for 2$\frac{7}{9}$ bbls of sugar at \$14$\frac{3}{4}$ per bbl. What did he gain or lose by his trade?

1396. It is estimated to cost \$17,587$\frac{9}{11}$ per mile to build a railroad. What will it cost to build one 138 m. 3 fur. 27 r.?

1397. If a pint of rum a day will kill a man in a year and a half, how many men will a cargo of 830 hogsheads kill in the same time?

1398. A certain man smokes 4 cigars every day; how many boxes would he smoke in 42 years, allowing 100 to each box.

1399. How many pounds of cotton in 37$\frac{3}{5}$ bales if each bale contains 5$\frac{1}{3}$ cwt.?

1400. What cost $\frac{9}{11}$ of an acre at \$.58$\frac{1}{3}$ per sq. foot?

1401. What was the cost of the cigars per annum, in the above example, at 3 cents apiece?

1402. What is the sum of the answers from 1390 to 1396, inclusive? Ans. 12435436.65.

Repeat Art. 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226.

Analysis of Part III. Per Centage.

DIVISIONS. CHAPTERS. SECTIONS. PAGE

- I. EQUAL TIME
  - I. Commission and Brokerage 146
  - II. Insurance 151
  - III. Custom House Business 154
  - IV. Assessment of Taxes 156
  - V. Profit and Loss 160
- II. UNEQUAL TIME
  - I. Interest
    - 1. Legal Interest U. States 163
    - 2. Bank Interest or Discount 172
    - 3. Interest at any per cent. 174
    - 4. Problems in Interest 175
    - 5. Partial Payments 176
    - 6. Accurate Interest 182
    - 7. Compound Interest 183
  - II. Discount
    - 1. Discount and present worth of a note due at a future date 185
    - 2. Notes due at different future dates 187
    - 3. Notes due at different past and future dates 188

REVIEW OF PARTS I. II. AND III. 189

PART III.

PER CENTAGE.

227. Per centage includes all those questions where there is a per cent., or so much per hundred. When parts of quantities are expressed by hundredths, as 3 hundredths of anything, it is called 3 per cent., 8 hundredths of anything is called 8 per cent., and is expressed decimally, *e. g.* .03, .08.

228. All questions occurring in per centage are included in two classes, viz:—

1. Those without time or with equal time, and
2. Those including unequal time.

229. Part III. is therefore separated into two Divisions.

Div. I.—Questions involving *two factors and their product.*

Div. II.—Questions involving *three factors and their product.*

DIVISION I.

PER CENTAGE WITH EQUAL TIME.

Prob. LX.—To compute per centage without time or with equal time, or to find the product of the two factors.

General Rule.—*Multiply the given sum by the rate per cent.*

230. This includes Commission, Brokerage, Insurance, Custom House Business, Assessment of Taxes, and Profit and Loss.

What is the subject of Part I? Part II? Part III? Give the analysis of Part I? Part II? Part III? 227. What is Percentage? What is meant by hundredths of anything? What by per cent.? What to express a quantity decimally? 228. What two classes of questions in Per centage? Why? 229. What do you mean by questions involving two factors? If you have two factors given, how do you find their product? Can you find one factor from the other, and the product? How? How find a product from three factors? How find one factor from two and the product? What does Division I. include? What is Prob. LX? Give the General Rule for per centage without time or with equal time.

CHAPTER I.

COMMISSION AND BROKERAGE.

231. *Commission* is an allowance per cent. for purchasing or selling goods.

232. *Brokerage* is an allowance per cent. for the purchase or sale of stocks.

233. *Stocks* are *certificates* of property, and are generally issued at $100.00 per share.

234. The *Par Value* of *Stocks* is when they sell for what they were issued.

235. *Above Par*, or *Premium*, is when they sell for more than they were issued.

236. *Below Par*, or *Discount*, is when they sell for less than they were issued.

237. *Dividends* are the profits of the Stocks.

238. The *Stockholders* are the owners of the *shares of stock.*

239. The *Corporation* is the entire body of stockholders, invested with peculiar privileges by a charter from a government.

240. A *Commission Merchant*, or *Factor*, is an agent who buys or sells merchandize for another.

241. A *Broker* is one who buys or sells real estate or stocks for another.

242. All questions in Commission and Brokerage are divided into two classes, viz:—

1st. When the commission or brokerage is computed on the *amount collected* or *paid out.*

2d. When a sum is received to be invested, *after the commission or brokerage is taken out.*

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231. What is commission? 232. What is brokerage? 233. What are stocks? 234. What is the par value of stocks? 235. When are stocks above par? What is premium? 236. When are stocks below par? What is discount? 237. What are dividends? 238. Who are stockholders? 239. What is a corporation? 240. What is a commission merchant? 241. What is a broker? What is a factor? 242. How are all questions in commission or brokerage divided? What is the difference between the two classes?

243. To compute Commission, Brokerage, and the Premium or Discount on Stocks, see Prob. LX., p. 145.

EXAMPLES.

1403. What is the commission on $785.00 at 5 per cent. ?

1404. What is the commission on $1371.25 at 7 per ct. ?

1405. What is the commission on $1783.37 at 3¾ per ct. ?

1406. What is the brokerage on $2300.00 at ¼ per ct. ?

1407. What is the brokerage on $7587.43 at 2⅛ per ct. ?

1408. What is 9 per cent of $47381.75 ?

1409. What is 7⅛ per cent of $2687.425 ?

1410. How much is .025 of $314.26 ?

1411. What is the commission on $785.6725 at 5⅓ per ct. ?

1412. What is the brokerage on $675.375 at 3⅙ per ct. ?

1413. How much did an auctioneer receive for selling a house and lot at $3753.25 ; 2 per ct. commission ?

1414. How much would a collector receive for collecting $37846.25 at 16⅔ per ct. ?

1415. What would a pension agent receive for collecting arrearages amounting to $70842.675 at 37½ per ct. ?

1416. What is due my agent for selling $7845.00 worth of merchandize at 3⅞ per cent. and 2¼ per cent. for insuring the payment ?

1417. A man distributed his property by will as follows: 11 per ct. to his wife ; 9 per ct. each to 4 boys ; 6 per ct. each to 5 girls ; 50 per cent of the remainder to Yale College, and the residue, which was $13786.31, to the Deaf and Dumb Asylum, Hartford, Conn.

What did each party receive ?

1418. Which is greater, 7 per ct. of $1100 or 3⅛ per ct. of $2300 ? How much ?

1419. What is ⅞ of one per ct. of $9872.63 ?

1420. What is the sum of the answers ? Ans. 83365.42.

1421. How much is ¾ per ct. of the last answer ?

PROB. LXI.—TO COMPUTE COMMISSION, OR BROKERAGE WHEN IT IS TO BE DEDUCTED FROM A SPECIFIC SUM AND THE BALANCE TO BE INVESTED.

RULE. — 1. *Divide the given amount by* $1.00 *plus the per cent. of commission.*

2. *The difference between this quotient and the given sum, is the commission.*

ILLUSTRATION.—Received $7500.00 to invest after deducting my commission of 3 per cent. What was the sum invested? What my commission?

1.00 + Com.	Sum received.	Sum to be invested.
1.03)	7500.00 (	7281.55
	721	
	290	
	206	
	840 &c.	

$7500.00	Sum received.
7281.55	Sum to be invested.
$218.45	Commission.

EXAMPLES.

1422. What can be invested out of $9000, com.; 3⅝ per ct.?

1423. Sent to a broker $6000 to be invested in bank stock; brokerage 2 per ct. What was left for investment?

1424. A commission merchant received $8750.11 to invest in flour, reserving 3 per ct. commission. How many barrels of flour could he purchase at $5.87½ per bbl.?

1425. What can be invested out of $4732.65; brokerage 2¼ per ct.?

1426. Received advice of the sale of 300 bushels of white beans at New Orleans at $2⅗ per bushel, my agent charging 1⅛ per ct. commission. He has invested the balance in sugar at 6 cents per pound, charging ½ per ct. commission. How much sugar was purchased?

1427. What is the sum of the answers! Ans. 33431.74.

What is Prob. LIX.? What is first to be done? 2d? What is the rule? How do you find a quotient? What is the difference between exact division and complete division?

PROB. LXII.—TO FIND THE PER CENT. COMMISSION OR BROKERAGE.

RULE.—*Divide the commission or brokerage by the sum invested.*

ILLUSTRATION.—Received $168.95 for investing $6758. What was my per cent. brokerage?

Sum Invested Commission.

6758) 168.95 (.025

135.16

33790

33790 Ans. 2½ per cent.

EXAMPLES.

1428. James Brown gave a broker $50.00 for purchasing $4000.00 worth of bank stock. What per cent. was it?

1429. Robert Allyn sent $11124 to New Orleans to purchase cotton.. His agent replies that he has taken out his own commission, $324.00, and invested the balance in cotton at 7⅓ cents per pound. What was his per cent. commission?

1430. How many cwt. of cotton did he purchase?

1431. Three years ago I purchased 20 shares of Erie R. R. stock at $100.00 per share, which I am now willing to sell for $1350. What is my per cent. loss?

1432. I have this day received two letters from two commission merchants at Mobile, informing me of a fine opportunity for speculation. One proposes to invest any amount for 3⅛ per cent.; the other says he will invest $6500 for $195.00. Which makes the best offer, and what is the difference?

1433. I have this day received $42.00 dividend on five shares of N. York Central R. R. stock, for which I paid $120.00 per share. What was the rate per cent.?

1434. How much will a commission of 2½ per cent. invest for $500.

1435. What is the sum of the answers? ∴ Ans. 21472.87⅝.

What is Prob. LX? Give the rule. 1430th Ex. *Will there be pounds in this answer?*

PROB. LXIII.—TO FIND THE VALUE OF STOCKS AT A PREMIUM OR DISCOUNT.

RULE.—*Multiply the par value by* $1.00, *plus the rate per cent. premium, or minus the rate per cent. discount.*

PROB. LXIV.—TO FIND THE PAR VALUE OF STOCKS AT A PREMIUM OR DISCOUNT.

RULE.—*Divide the value by* $1.00, *plus the rate per cent. premium, or minus the rate per cent. discount.*

EXAMPLES.

1436. What is the value of $5700 of stock at 15 per cent. premium?

1437. What must I pay for 41 shares of Georgetown Corporation Stock, par value $100.00, discount 42 per cent.?

1438. Sold 23 shares of the Central R. R. Stock for $115, at a discount of 95 per cent. What was the par value?

1439. What will 17 shares of the Shawmut Bank Stock sell for, par value being $100.00, at 8¾ per cent. premium?

1440. What must be paid for 127 shares of Orange and Alexandria R. R. Stock, par value being $100.00, at 95 per cent. per share?

1441. Suppose these same 127 shares be sold by the purchaser for 8½ per cent. premium. What would be the profit?

1442. Bought 19 shares of B. and O. R. R. Stock at 9 per cent. discount, and it is now worth 2 per cent. premium, par value being $100.00, and I am offered 27 shares Balt. Wil. and Phil. R. R. Stock at 21 per cent. discount in exchange. How much shall I make or lose on my original purchase, B. W. and P. Stock being worth 78 per cent.?

1443. What is the value of $7286 of stock at 17½ per cent. discount?

1444. What is the par value of $796 of stock at 3⅓ per cent. discount?

1445. What is the sum of the answers on this page?

Ans. 31818.67.

CHAPTER II.

INSURANCE.

244. *Insurance* is security given against loss by fire, water, or other accidents.

245. *Insurance Companies* are corporate bodies, and are divided into three classes, *viz.:*

1st. *Fire.* — Those which insure against loss by fire.

2d. *Marine.* — Those which insure against loss at sea.

3d. *Life.* — Those which contract to pay a certain sum on the decease of an individual.

246. The *Underwriters* are the insurers.

247. The *Policy* is the contract of the parties.

248. The *Premium* is the sum paid for insurance.

249. Insurance generally can be effected on only ⅔ the value of real estate, and on ½ the value of personal property *on land*, but the entire value, *at sea*.

250. Insurance is computed for one year or without specified time, by Prob. LX, p. 145.

EXAMPLES.

1446. What is the annual insurance of a house valued at $500, at 3 per cent.?

1447. What is the annual insurance of a store valued at $35000.00, at 7½ per cent.?

1448. What is the insurance on a cargo of sugar, from Cuba to New York, valued at $14768.57, at 3¾ per cent.?

1449. What will it cost to get my life insured one year for $7750.00 at 3⅛ per cent.?

1450. What is the sum of the answers? Ans. 3436.01.

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244. What is insurance? 245. What are insurance companies? What are corporate bodies? How many classes of insurance companies? What are the 1st class? 2d? 3d? 246. Who are Underwriters? 247. What is a Policy? 248. What is the Premium? 249. What proportion of real estate can you insure? Why get more in amount insured at sea than on land? Rule for insurance?

PROB. LXV.—TO FIND THE RATE OF INSURANCE.

RULE.—*Divide the premium by the amount insured.*

NOTE.—The premium is found by subtracting $1.00 for the cost of the policy.

EXAMPLES.

1451. Fille Brown paid $65.00 for insuring $3200 on his house for four years. Policy $1.00. What was the annual rate of insurance?

1452. Paid $163.00 for insuring my vessel from New York to Liverpool *via* New Orleans.

Vessel valued at $4050.00. Policy $1.00.

What was the rate of insurance?

1453. Jessie Riley paid $47.00 annually for a life insurance of $1175.00. What was the rate of insurance?

1454. Paid $751.00 for insuring a cargo of tea from Canton to New York, valued at $15000.00. Policy $1.00. What was the rate of insurance?

PROB. LXVI.—TO FIND THE SUM INSURED.

RULE.—*Divide the premium by the rate per cent.*

EXAMPLES.

1455. What amount can I get insured on my house for $46.00, at 4.5 per cent. Policy $1.00?

1456. An importer paid $651 for insuring a cargo from Liverpool to Boston at 1¼ per cent. Policy $1.00. The cargo being a total loss, what did he receive from the Ins. Com.?

1457. By paying $44.00 annually how much can I get insured on my life at 2⅞ per cent.?

1458. My real estate is insured at a cost of $657.00. Rate per cent. 1¾. Policy $1.00. What is its value?

1459. Paid $77.00 for insuring my library at full value. Rate 2⅓ per ct. Policy $1.00. What should I receive if it was burned?

1460. What is the sum of the answers? Ans. 95273.44.

Repeat Prob. LXV. Rule. Prob. LXVI. Rule.

PROB. LXVII.—TO FIND THE SUM TO BE INSURED, THAT THERE MAY BE NO LOSS ON THE DESTRUCTION OF THE PROPERTY.

RULE.—1. *Divide the sum to be insured by $1.00, minus the rate of insurance.*

2. *To this quotient add the value of the policy.*

ILLUSTRATION.—What sum must be insured on a cargo to London, valued at $86783.50, at 7 per cent., so that the owner shall suffer no loss if the cargo is totally destroyed?

```
 1.00
    7
 ----
 .93)86783.50(93315.60
     837          1.00 Policy.
     ---       ---------
      308      $93316.60 Ans.
      279
      ---
       293
       279
       ---
       145
        93
       ---
       520, &c.
```

EXAMPLES.

1461. What sum insured will cover the total loss of the brig Emma, valued at $73000, at 5$\frac{7}{8}$ per cent.?

1462. What sum insured will cover the total loss of cargo and ship William Jarvis—cargo valued at $215768.00, insured at 7$\frac{3}{8}$; ship valued at $115000.00, insured at 3$\frac{5}{8}$; policies, $2.00?

1463. What sum will insure a cargo from Sidney to Canton valued at $75000.00, at 15 per cent.; from Cauton to Calcutta, at 5 per cent.; Calcutta to Cape Town, at 7 per cent.; Cape Town to Cape Verde, at 3 per cent.; Cape Verde to Havana, at 11$\frac{1}{2}$ per cent.; Havana to N. York, at 13$\frac{1}{2}$ per cent.; so as to cover the total loss of the cargo in New York Harbor?

1464. What is the sum of the answers on this page?

Ans. $564895.67.

Repeat Prob. LXVII. Repeat 1st part of rule. 2d. Give the rule.

CHAPTER III.

CUSTOM HOUSE BUSINESS.

251. *Custom House Business* relates to the collection of Government revenues arising from duties, tonnage, &c. &c.

252. *Duties* are sums of money required by Government to be paid on imported goods, and are Specific or Ad Valorem.

253. *Specific Duty* is a tax by the quantity, not by the value.

254. *Ad Valorem Duty* is a tax on the value or cost of the goods in the country where they were purchased.

255. *Tonnage* is the amount paid per ton by the vessel for the privilege of entering a port.

256. *Gross Weight* is the weight of both the goods and the article that contains them.

257. *Draft* is an allowance for the waste of goods.

258. *Tare* is an allowance for the weight of the article containing the goods.

259. *Net Weight* is the weight of the goods, and is found by *first* subtracting the *draft* from the gross weight, then the *tare* from this remainder.

260. Duties are collectable only on net weight.

261. *Leakage* is an allowance for waste on liquids, and is generally two per cent.

262. Merchants generally use net weight in trade.

263. Table for Allowing Draft.

Quantities weighing less than 100 lbs., draft,							1	lb.
From	100	lbs.	to	200	lbs.,	“	2	lbs.
“	200	“		300	“	“	3	“
“	300	“		1000	“	“	4	“
“	1000	“		1800	“	“	7	“
“	1800	“		any weight,		“	9	“

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251. What is Custom House Business? 252. What are Duties? 253. What is Specific Duty? 254. What is Ad Valorem Duty? 255. Tonnage? 256. Gross Weight? 257. Draft? 258. Tare? 259. Net Weight? 260. On what are duties collectable? 261. What is Leakage?

PROB. LXVIII. — TO COMPUTE CUSTOM HOUSE CHARGES.

1st. SPECIFIC DUTY. — RULE.—*Multiply the net measure by the duty specified on a unit of the given quantity.*

2d. AD VALOREM DUTY.— RULE. — *Multiply the cost of the goods by the per cent. duty.*

3d. TONNAGE. — RULE. —*Multiply the tonnage of the vessel by the tax per ton.*

NOTE. — Draft is estimated upon each box, bale, &c., separately. In reckoning tare, over ½ lb. is called 1 lb.; less than ½ lb. is rejected.

EXAMPLES.

1465. What is the duty on 67 chests of tea, each weighing 126 lbs. Draft 2 lbs. Tare 4 per ct. Duty 13 cents per lb.?

1466. What is the duty on 6 boxes of sugar, gross weight of

No. 1. 472 lbs.
No. 2. 683 "
No. 3. 984 "
No. 4. 381 lbs.
No. 5. 1124 "
No. 6. 1281 "

Tare 9 per cent., and duty 2 cents per lb.? Ans. $89.08.

1467. What is the duty on 79 boxes of old Windsor soap, each weighing 137 lbs. Tare 21 per ct. Duty 3 cents per lb.?

1468. What is the duty on 10 hhds. tobacco, gross weight,

No. 1. 2187 lbs.
No. 2. 1934 "
No. 3. 1781 "
No. 4. 1978 "
No. 5. 2347 "
No. 6. 2037 lbs.
No. 7. 36 " = (2036.)
No. 8. 42 "
No. 9. 38 "
No. 10. 2043 "

Tare 17 per cent., duty 6¼ cents per lb.?

1469. What is the duty on 81 boxes of candles, each weighing 113 lbs. Tare 3½ per cent. Duty 1⅞ cts per lb.?

1470. What would be the port charges of the Missionary Ship "Morning Star," tonnage 747 tons at 10 cents per ton?

1471. Sum of the answers on this page? Ans. $2582.07.

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263. What is the rule for allowing draft? Tare? What is Prob. LXVIII? Rule for calculating Specific Duty? Ad Valorem Duty? Tonnage?

CHAPTER IV.

ASSESSMENT OF TAXES.

264. *A Tax* is a sum of money assessed on the person and property of "a *citizen*" for the support of the government of a Nation, State, Country, District, Parish, Town, &c.

265. *Poll Tax* is assessed on the person, and is usually a specific sum.

266. *Property Tax* is assessed on the entire valuation, *real* and *personal*, and is apportioned at a certain per cent.

267. *Real Estate* includes only fixed possessions, as lands, and buildings stationary, &c.

268. *Personal Estate* includes money, stocks, notes, bonds, mortgages, buildings in motion, ships, furniture, and all "movable property."

269. *Overlay* is an amount to be added to the net amount to be raised, to cover expense for collection and losses.

270. *Poll* is a person taxed.

PROB. LXIX.—TO ASSESS A TAX.

RULE.—1. *Multiply the number of polls by the sum assessed on each, and subtract the product from the sum to be raised, including the overlay.*

2. *Divide the remainder by the entire valuation, and the quotient will be the rate per cent. for the property tax.*

3. *Multiply individual valuations by the per cent. taxed, add the poll tax, and it gives the entire individual tax.*

4. *For convenience a table may be made showing the tax for* **1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10000, 20000, 30000, 40000,** *&c.*

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264. What is a tax? **265.** Poll tax? **266.** Property tax? **267.** Real estate? **268.** Personal estate? **269.** Overlay? **270.** Poll? Rule?

ILLUSTRATION. — The town of Holliston voted a tax of $7500.00. The number of polls was 621. Voted the poll tax to be $1.50. Entire valuation $1413700. Overlay $500.

It is required to assess the tax.

1st. 621 × $1.50 = $931.50, the entire poll tax.
2d. $7500 + 500 = 8000. Entire sum to be raised.
3d. $8000 — $931.50 = $7068.50, the entire property tax.
4th. $7068.50 ÷ $141390 = .005, the rate per cent.

The following table is now made out from this rate per cent.

Table from $1 to $10000000.

Sum.		Tax.
$ 1.00	pays	$.005
2.00	"	010
3.00	"	.015
4.00	"	.020
5.00	"	.025
6.00	"	.030
7.00	"	.035
8.00	"	.040
9.00	"	.045
10.00	"	.050

EXPLANATION.

To get the tax on $700 we take the tax against $7.00 which is .035, and remove the point two places, that is, the tax on $700 is 100 times as much as for $7.

The tax of $8763.50 will be found as follows:

Tax on $8000 = 40.00
700 = 3.50
60 = .30
3 = .015
.50 = .0025
Tax on $8763.50 = 43.8175

What is Jasper Leland's tax, whose real estate is $27900, personal $17600, and he pays for 3 polls.

Personal tax 1.50 × 3 = $ 4.50
Property tax by the above table = 227.50
Jasper Leland's tax is $232.00

What part of the U. S. silver dollar is copper? What is the composition of the U. S. gold dollar? How many cubic inches in a gallon? How many feet in a rod? In a mile?

EXAMPLES.

1472. The town of Joppa voted a tax of $12300.00. The number of polls 400; poll tax $1.50; overlay $300.00; entire valuation $3000000.00. It is required to make a table and assess Joshua Abbot's tax, who pays for 3 polls, and is taxed on $3467.82 real estate, and $4567.87 personal.

1473. What would be Richard Mayo's tax for 1 poll, $1784.56 real, and $1468.26 personal property?

1474. What would be the tax of Nathaniel Lewis for 5 polls, $17842.63 real, and $1481.00 personal property?

1475. Ebenezer Lock's tax for 3 polls, $75004.78 real, and $73476.87 personal property?

1476. What would be Ellis Holmes' tax, paying for 1 poll, $1787.56 real, and $9786.83 personal property?

1477. Jeremiah Jones' tax for 17 polls, real estate $21000.87, and personal property $127684.35?

PROB. LXVII.—TO FIND WHAT AMOUNT TO ASSESS THAT A GIVEN SUM MAY BE HAD AFTER PAYING FOR COLLECTION.

RULE.—*Divide the given sum by* $1.00, *minus the rate per cent. for collection.*

EXAMPLES.

1478. What sum must be assessed to raise the net amount of $9000.00 with 5 per cent. for collection?

1479. What sum must be assessed to raise the net amount of $433.33 with 2½ per cent. for collection?

1480. A town voted $2684 for roads, $1700 for schools, $984.58 for support of the poor, $5680 for general town expenses, and 3 per cent. for collecting taxes. What sum must be raised to cover all expenses?

1481. What is the sum of the answers on this page?

Ans. $22710.82.

What does "per cent." mean? What is the business of a broker? What is meant by stocks?

PROB. LXXI.—TO APPORTION "SCHOOL RATE BILLS."

RULE.—1. *Subtract the public money, if any, from the entire expense of the school.*

2. *Divide the remainder by the total number of days' attendance, and the quotient will be the cost for* 1 *day.*

3. *Multiply the cost of one day by the number of days of attendance of each patron's pupils, and the product will be his tax.*

EXAMPLES.

1482. The expenses of a certain school were as follows:— Teacher's salary, $197.42; board, $136.00; repairs, $23.00; fuel, $47.00; stationery, $14.21; expense for collecting, $9.28. The school had $114.98 public money. Total number of days of attendance was 4712. What was the rate per day?

Ans. 0.066199.

1483. What was Allen Brown's tax, who sent 3 children, one 75 days, one 106, and the third 91 days? Ans. $18.01.

1484. If a school is supported 6 months for $500.00, teachers' salaries; board, $150.00; stationery, $27.62; general expenses, $97.58; with a total attendance of 4976 days, what is the rate per day? Ans. 0.155787.

1485. What would Allen Smith have to pay for 8 pupils, attending as follows: 49 days, 54, 76, 83, 103, 106, and two 115 days each?

1486. If the town of Stroudsburg, Penna., pays $6000 per annum to its teachers, $250 for repairs, $100 for stationery, $187 for fuel and other current expenses, with a sum total attendance of 70400 days, what rate of tax must the town assess on each pupil per day, the State bearing 30 per cent. of the expense?

1487. What will be Robert Huston's tax, who had six pupils in attendance, as follows: two, 180 days each; one, 160; one, 158; one, 155; and one, 140 days?

What is the rule for apportioning "school rate bills"? What is meant by public money? Name six articles of stationery.

CHAPTER V.

PROFIT AND LOSS.

271. Profit and Loss is the process by which we determine the relations of the cost, the price, and the gain or loss in the purchase and sale of goods.

272. The per cent. gain or loss is determined from the *cost*, *not* from the *price*.

PROB. LXXII. IS EMBRACED IN SEVEN CASES, VIZ:

CASE 1.—To determine the *profit* or *loss* from the cost and per cent.

RULE.—*Multiply the cost by the per cent.*

CASE 2.—To determine the *price* from the cost and per cent.

RULE.—*Add the profit to, or subtract the loss from the cost.*

CASE 3.—To determine the *cost* from the profit or loss and per cent.

RULE.—*Divide the profit or loss by the per cent.*

CASE 4.—To determine the *cost* from the price and gain or loss per cent.

RULE.—*Divide the price by* $1.00, *plus the per cent. gain, or minus the per cent. loss.*

CASE 5.—To determine the *per cent.* from the cost and profit or loss.

RULE.—*Divide the profit or loss by the cost.*

CASE 6.—To find a price from which a merchant may fall a certain per cent., yet sell his goods at cost.

RULE.—*Multiply the per cent. fall by* 100, *and divide the product by the difference between* 100 *and the per cent. fall. The quotient will be the per cent. of increase.*

Case 7.—To find a price from which a merchant may fall a certain per cent., yet sell his goods above or below cost.

Rule.—*Add the profit to, or subtract the loss from the cost; then with this sum proceed as in Case 6.*

Illustration under Case 6.—What price shall I put on my horse, which cost $250, that I may fall 5 per cent., and yet lose nothing.

$\frac{100 \times 5}{100-5} = 5\frac{5}{19}$. Then $250 \times 1.05\frac{5}{19} =$ $263.1579, the price asked for the horse.

Illustration under Case 7.—James Allen paid $175 for a chaise, on which he wishes 10 per cent. profit. What price must he put on it that he may fall 15 per cent., and still realize his profit.

175 × 1.10 = $192.50, the amount he must receive to make 10 per cent.

$\frac{100 \times 15}{100-15} = 17\frac{11}{17}$. Then $192.50 \times 1.17\frac{11}{17} =$ $226.48, the price to be asked for the chaise.

EXAMPLES.

1488. Bought 750 barrels of flour, at $7.62½ per bbl. What shall I gain by selling it at 11 per cent. profit?

1489. Bought broadcloth at $4.31 per yard, which I can sell at 7 per cent. profit. What will be the price?

1490. Bought molasses at 27 cents per gallon. Am willing to lose 7½ per cent. What must be the price?

1491. Gained 39 cents per quintal by selling at 12 per cent. profit. What did it cost?

1492. Lost 45 cents per bbl. by selling at 9 per cent. loss. What did it cost?

1493. What is the sum of the answers on this page?

Ans. $639.27.

1494. Sold sugar at $8.60 per cwt., and gained 11 per cent. What did it cost?

1495. Sold a cask of rice for $27.35, and thereby lost 7 per cent. on cost. What did I pay for it?

1496. James Bright bought a farm for $3143, expended $1347 in improvements, and sold it for 47 per cent. gain on first cost. What was his profit by the trade?

1497. Bought a lot of cotton for $76000; paid $4000 for shipping to Liverpool; kept 1 year, and sold it for 31 per cent. on first cost. What was the profit, money being worth 6 per cent.? Ans. $14.760.

1498. Bought a quantity of butter for $913.00. For what must I sell it to gain $13\frac{1}{2}$ per cent., allowing $56.00 for expenses?

1499. Bought a cargo of flour at $7.63 per bbl., and sold it for $8.12 per bbl. What is the per cent. gain?

Ans. $6\frac{46}{109}$ per cent.

1500. Bought a lot of shoes at $2.65 per pair, but could sell them for only $2.41 per pair. What was the per cent. loss?

1501. Sold a horse for $175, and lost 15 per cent. What did he cost me?

1502. Sold a carriage for $210, and gained 11 per cent. What did I pay for it?

1503. Bought a farm of 85 acres for $6486. What price per acre must I ask for it so as to fall 15 per cent. and lose nothing by the trade?

1504. Sold 100 bushels of potatoes as follows: half for 85 cents per bushel, and lost 7 per cent.; the remainder I sold for 8 per cent. profit. What was gained by the whole trade?

1505. What must I charge for a house lot that cost $1147.96, so that I can in trade fall 10 per cent. from my price, and make $12\frac{1}{2}$per cent. by the trade?

1506. What is the sum of the answers of the last five examples? Ans. $1920.27.

What is the difference between the *cost* and *price* of an article? On which is the gain or loss per cent. reckoned?

DIVISION II.

PER CENTAGE WITH UNEQUAL TIME.

CHAPTER I.

INTEREST.

SECTION I.

LEGAL INTEREST U. S.

273. Interest includes all those questions relating to the payment of certain specified rates by the borrower to the lender, for the use of money.

274. The Principal is the sum of money at interest.

275. The Rate Per Cent. is the price per hundred for the use of the money for one year.

276. The Time is the time the money is drawing interest.

277. The Interest is the sum paid for the use of the money.

278. The Amount is the principal and interest added.

279. The interest may be considered as the product of three quantities, viz: the *principal*, *rate* per cent., and the *time* in years. All computations in interest, therefore, must fall under one of the four following formulas: —

280. *Prin.* = The quo. of the Int. div. by the pro. of the Time and Rate.

281. *Rate* = The quo. of the Int. div. by the pro. of the Time and Prin.

282. *Time* = The quo. of the Int. div. by the pro. of the Rate and Prin.

283. *Interest* = The pro. of the Prin. × Rate × Time in years.

The legal rate of interest for the United States generally is 6 per cent. per annum.

The legal rate of interest in N. Y., Mich., Wis., Min., Ga., and S. C., is 7 per cent. Ala. and Texas, 10. La., England, and France, 5. Canada, Nova Scotia, and Ireland, 6 per cent

284. The interest of $1.00, at 6 per cent., therefore, is 6 cents for one year, and in the same proportion for a longer or a shorter time.

Hence, for 2 months, or 60 days, it will be 1 cent.
For 1 month, or 30 " " 5 mills.
" $\frac{1}{5}$ " or 6 " " 1 mill.
" $\frac{1}{30}$ " or 1 day, " $\frac{1}{6}$ mill.

In a similar manner, the pupil can readily compute the interest of $1.00 for *any given time*, MENTALLY; thus, $\frac{1}{2}$ the number of months will express the interest in cents; $\frac{1}{6}$ the number of days will express the interest in mills.

PROB. LXXIII.—TO COMPUTE INTEREST AT 6 PER CENT.

RULE.—*Compute the interest on* $1.00 *for the given time, as above, mentally, and multiply the principal by it. Less than* 5 *mills is rejected;* 5 *or more, counted* 1 *cent.*

EXAMPLES.

1507. What is the interest on $75.00 from April 1st, 1859, to Nov. 1st, 1859? Ans. $2.63.

1508. What is the interest on $8700 from March 3d, 1860, to Sept. 3d, 1861? Ans. $2.61.

1509. What is the interest of $1100 from June 1st, 1855, to July 1st, 1855?

1510. What is the interest of $2500 from Oct. 1st, 1847, to Nov. 1st, 1853?

1511. What is the interest of $2150 from May 1st, 1842, to Jan. 1st, 1846?

1512. What is the interest of $1721 from May 1st, 1863, to Nov. 17th, 1867?

1513. What is the amount of $37.42 from Oct. 1st, 1852, to March 17th, 1853?

1514. What is the amount of the last answer from Mar. 17th, 1853, to July 1st, 1858?

1515. What is the sum of the answers of the last six examples? Ans. 1949.38.

1516. What is the interest of $499.99 from Sept. 9th, 1849, to Sept. 19th, 1850?

1517. What is the interest of $4500.00 from May 10th to May 13th? Ans. $2.25.

1518. What is the interest of $6750 from June 5th to October 17th? Ans. $148.50.

1519. What is the interest of $9750.00 from February 29th to March 3d?

1520. What is the interest of $11825.00 from February 28th to March 2d?

1521. What is the interest of $647.00 from February 28th to June 7th?

1522. Borrowed $725.00 Monday, and paid the same, with interest, Saturday. What was the amount paid? Ans. $.60.

1523. What is the amount of $5.41 from the birth of Washington to his death? Ans. $27.42.

1524. What is the amount of $684.26 from Aug. 11th, 1851, to Oct. 17th, 1857?

1525. What is the amount of $178.46 from April 11th, 1851, to Dec. 31st, 1856?

1526. What is the interest of $941.71 from March 4th, 1852, to Nov. 4th, 1856?

1527. What is the interest of 5 cents from the birth of Christ, or from the year 1, inclusive, to March 4th, 1856? Ans. $5.57.

1528. What is the amount of 5 cents from June 17th, 1775, to July 4th, 1860?

1529. What is the interest of $9436.43 from April 19th, 1843, to Dec. 11th, 1858?

1530. What is the amount of $1384.00 from Oct. 13th, 1854, to June 8th, 1859?

1531. What is the interest of $17842.71 from July 4th, 1890, to July 7th, 1895?

1532. What is the interest of $417.23 from Dec. 31st, 1861, to Feb. 29th, 1864?

PROB. LXXIV.—NEW METHOD OF COMPUTING INTEREST.

RULE.—1. *Separate the given time into such parts of 200 months, 20 months, 60 days, and 6 days, that the several interests corresponding may be expressed by the principal, or some multiple or factor of it.*

2. *Add or subtract the several interests, and the result will be the required interest.*

285. The above method of computing interest at 6 per cent. is proposed as the shortest way now known; and the especial attention of Educators and Accountants is called to the following points, upon which it is based.

1st. The principal, as it stands, expresses the interest of itself for 200 months, 20 months, 2 months or 60 days, and 6 days, by changing the decimal point.

2d. Every *multiple* or factor of the principal also expresses, *in itself*, the interest of the *principal* for several different times.

3d. In *adding* or *subtracting* these several interests, *it is not necessary to rewrite* them, but they may be added or subtracted as they stand in the principal, its factors, or multiples.

4th. By eight simple divisions of the *principal*, the interest is computed for thirty-four different periods of time; and by various combinations, by addition and subtraction, it can readily be computed for any time.

5th. The principle on which the following table is computed can be extended to various other times; *e. g.* the *interest* for 1 year may be taken and divided in the same manner as the *principal.*

What is Prob. LXXIV? Repeat the Rule. Why take 200 months as a basis? Why 20 months? Why 60 days? Why 6 days? Can the principle of this rule be extended to other times? Why are decimal factors taken? What part of the principal will express the interest for 20 months? 2 months? 6 days? 100 months? 10 months? 1 month? 30 days? 8 days? 50 months? 5 months? 150 days? 15 days? 45 days?

Remark 1.—200 months as a basis for the division of time is taken because the law makes the interest of $1.00, at 6 per cent., 6 cents for 12 months, consequently it must be 100 cents for 200 months; or, in other words, the interest becomes equal to the principal in 200 months. In 20 months it must be equal to $\frac{1}{10}$ of the principal; in 2 months, $\frac{1}{100}$ of the principal; and for 6 days, $\frac{1}{1000}$ of the principal.

Remark 2.—In the following table, commencing at the right of the page, the 1st column contains the divisors for separating the time into factors. Col. 2 contains the time a sum of money will double itself, and factors of that time. Col. 3 is $\frac{1}{10}$ of Col. 2. Col. 4 is $\frac{1}{10}$ of Col. 3, or $\frac{1}{100}$ of Col. 2. Col. 5 is $\frac{1}{10}$ of Col. 4, $\frac{1}{100}$ of Col. 3, or $\frac{1}{1000}$ of Col. 2.

Table Separating 200 Months into Factors.

$\frac{1}{1000}$ P.	$\frac{1}{100}$ Prin.	$\frac{1}{10}$ Principal.	Principal.	
6 days.	60 da. = 2 mo.	1 yr. 8 mo. = 20 mo.	16 y. 8 mo. = 200 mo.	Pri.
3 days.	30 da. = 1 mo.	10 mo = 300 days.	8 y. 4 mo. = 100 mo.	2
2 days.	20 days.	6 mo. 20 da. = 200 da.	5 y. 6 mo. 20 da. = $66\frac{2}{3}$ mo.	3
1.5 da.	15 days.	5 mo. = 150 days.	4 y. 2 mo. = 50 mo.	4
1.2 da.	12 days.	4 mo. = 120 days.	3 y. 4 mo. = 40 mo.	5
1 day.	10 days.	3 mo. 10 da. = 100 da.	2 y. 9 mo. 10 da. = $33\frac{1}{3}$ mo.	6
	7.5 days.	2 mo. 15 da. = 75 da.	2 y. 1 mo. = 25 mo.	8
.5 da.	5 days.	1 mo. 20 da. = 50 da.	1 y. 4 mo. 20 da. = $20\frac{2}{3}$ mo.	12
	4 days.	1 mo. 10 da. = 40 da.	1 y. 1 mo. 10 da. = $13\frac{1}{3}$ mo.	15

Note.—It will be found in practice that most of the times for computing interest will be easily separated into parts by the factors and their multiples of 6 and 60 days.

ILLUSTRATIONS.

1. What is the interest of $47.96 for 1 yr. 8 mo.?

As the time is $\frac{1}{10}$ of 200 months, the interest will be $\frac{1}{10}$ of the principal = $4.796 or $4.80.

2. The Int. of $651.72 for 16 yrs. 8 mo. = 200 mo. is $651.72.
" " 651.72 " 1 yr. 8 mo. = 20 mo. is 65.172.
" " 651.72 " 2 mo. = $\frac{1}{100}$ of 200 mo. is 6.5172.
" " 651.72 " 6 da. = $\frac{1}{1000}$ " " .65172.

In this, as in all similar cases, there is no necessity of rewriting the principal. These interests may be read in the principal.

3. What is the interest of $651.72 for 1 yr. 10 mo.?

Int. 1 yr. 8 mo. = $65.172
" 2 mo. = 6.517
Ans. $71.689
or
$651.72
Ans. $71.689 = 71.69

In this example the interests may be added *as they* stand in the principal; thus, 7 + 2 = 9; 1 + 7 = 8; 5 + 1 = 6; 6 + 5 = 11; 6 + 1 = 7.

NOTE.—In a similar manner the work in most examples may be materially abridged.

4. What is the interest of $651.72 for 1 yr. 10 mo. 6 days?

Int. 1 yr. 8 mo. 6 da. $65.172
" 2 mo. 6.517
" 6 days, .651
$72.340
or
$651.72
Ans. $72.34

5. What is the interest of $43.67 for 3 yr. 4 mo. 12 days?

Int. 1 yr. 8 mo. $4.367
" 1 yr. 8 mo. 4.367
" 6 days, .043
" 6 days, 43
Int. 3 yr. 4 mo. 12 da. = $8.821
or
$43.67
Ans. $8.82

6. What is the interest of $567.49 for 1 yr. 7 mo. 24 days?

Int. 1 yr. 8 mo. $56.749
" 6 days, .567
Int. 1 y. 7 mo. 24 da. = $56.182
or
$567.49
Ans. $56.18

7. What is the interest of $847.62 for 5 yr. 6 mo. 28 days?

Int. 5 yr. 6 mo. 20 da. 3)847.62
$282.54
Int. 2 days, .282
" 6 days, 847
Int. 5 yr. 6 mo. 28 da. = $283.669
or
$847.62
282.54
Ans. $283.67

8. What is the interest of $7568.43 for 1 yr. 11 mo. 9 days?

Int. 1 yr. 8 mo. = $756.843
" 2 mo. = 75.684
" 6 days = 7.568
" 1 mo. = 37.842
" 3 days = 3.784
Int. 1 yr. 11 mo. 9 da. = $881.721

or $7568.43
378.421
Ans. $881.72

9. What is the interest of $437.51 for 1 yr. 1 mo. 12 days?

$437.51
.06
Int. 1 yr. 26.2506
" 1 mo. 2.1875
" 6 da. .4375
" 6 da. .4375
$29.3131

or Int. 6 da. $ 437.51
" 360 & 36 da. 26.2506
Ans. $29.31

10. What is the interest of $547.96 for 1 yr. 9 mo. 3 days?

First Method.

$547.96 .06 = 1 yr.
1055 .045 = 9 mo.
273980 .0005 = 3 da.
273980 .1055 Int. $1.00.
54796
$57.809780 Ans.

$547.96
27.39
Ans. $57.81

11. What is the interest of $7989.97 for 2 yr. 9 mo. 11 da.?

$7989.97
$.166\frac{5}{6}$
6)3994985
665831
4793982
4793982
798997
$1332.99333

$7989.97
1331.66
Ans. $1332.99

12. What is the interest of $9486.21 for 1 yr. 7 mo. 27 days?

$9486.21
47.43
$944.88

13. What is the interest of $467.21 for 1 day?

46721
$.78 Ans.

14. What is the interest of $529.79 for 2 days?

52979
$.18 Ans.

15. What is the interest of $393.71 for 4 days?

39371
$.26 Ans.

16. What is the interest of $357.87 for 5 days?

35787
$.30 Ans.

17. What is the interest of $763.94 for 7 days?

76394
12732
$.89 Ans.

18. What is the interest of $97.68 for 13 days?

9768
1628
$.20 Ans.

19. What is the interest of $567.97 for 14 days?

56797
18932
$1.33 Ans.

20. What is the interest of $642.84 for 17 days?

64284
10714
$1.82 Ans.

21. What is the interest of $437.98 for 18 days?

437.98
$1.31 Ans.

22. What is the interest of $7682.42 for 19 days?

$7682.42
1280.40
$24.33 Ans.

23. What is the interest of $672.41 for 29 days?

672.41
112.07
$3.25 Ans.

24. What is the interest of $833.42 for 15 days?

833.42
$2.08 Ans.

EXAMPLES.

1533. What is the interest of $6781.24 for 2 months 6 days?
1534. What is the interest of $128.41 for 3 months 9 days?
1535. What is the interest of $286.23 for 3 months 12 days?
1536. What is the interest of $36.79 for 4 months 1 day?
1537. What is the interest of $167.21 for 5 months 6 days?
1538. What is the interest of $72.61 for 6 months 7 days?
1539. What is the interest of $491.29 for 7 months 3 days?
1540. What is the interest of $98.94 for 8 months 13 days?
1541. What is the interest of $17.63 for 9 months 11 days?
1542. What is the interest of $59.63 for 10 months 19 days?
1543. What is the sum of the above answers? Ans. $128.69.
1544. What is the interest of $668.65 for 3 months 4 days?
1545. What is the interest of $567.92 for 4 months 7 days?
1546. What is the interest of $68.43 for 7 months 9 days?
1547. What is the interest of $942.76 for 10 months 4 days?
1548. What is the interest of $2467.31 for 1 yr. 1 mo. 1 day?
1549. What is the interest of $1213.94 for 1 yr. 5 mo. 29 da.?
1550. What is the interest of $748.95 for 1 yr. 7 mo. 2 da.?
1551. What is the interest of $1143.96 for 2 yr. 4 mo. 5 da.?
1552. What is the interest of $137.61 for 3 yr. 7 mo. 5 da.?
1553. What is the interest of $763.84 for 4 yr. 9 mo. 11 da.?
1554. What is the interest of $6542.71 for 3 mo. 27 days?
1555. What is the interest of $97.64 for 11 months 16 days?
1556. What is the interest of $57.21 for 9 months 5 days?
1557. What is the interest of $311.79 for 10 mo. 14 days?
1558. What is the interest of $683.29 for 6 months 6 days?
1559. What is the interest of $7531.19 for 6 y. 10 mo. 14 d.?
1560. What is the interest of $329.96 for 3 yr. 4 mo. 8 da.?
1561. What is the interest of $811.11 for 1 year 2 months?
1562. What is the amount of $312.31 for 5 months 5 days?
1563. What is the amount of $93.67 for 1 month 24 days?
1564. What is the amount of $9376.83 for 6 years 3 months?
1565. What is the sum of the answers from 1543?

Ans. $17528.19.

SECTION II.

BANK INTEREST OR DISCOUNT.

286. *Bank Interest* is taken in advance, and for 3 days more than the time specified in the note. These "three days" are called "Days of Grace."

287. A sum of money "falling due" on Sunday, or any holiday, must be paid the day before. All notes *maturing Feb. 29th, 30th, or 31st, must be paid on the 28th*, unless the 28th is Sunday, or a holiday, when it must be paid on the 27th. In leap years they are payable the 28th or 29th.

288. *Bank Interest* is computed the same as interest.

EXAMPLES.

1566. $513.41. *Washington, March 5th,* 1853.

Two months after date I promise to pay Luther Green, or order, five hundred thirteen $\frac{41}{100}$ dollars, value received. Payable at the Bank of the Metropolis. SIMON GARDNER.

What is the discount on the above note?

1567. $75.21. *Tolland, Dec. 31st,* 1853.

Two months after date, for value received, I promise to pay James Ladd, or order, seventy-five $\frac{21}{100}$ dollars. Payable at Windham Bank. LYMAN GAGER.

What is the discount on the above note?

1568. $57.21. *Royalton, Nov. 15th,* 1847.

Four months after date, for value received, I promise Jesse Wright to pay him, or order, fifty-seven $\frac{21}{100}$ dollars. Payable at Royalton Bank. ALMOST GRANGER.

What is the discount on the above note?

1569. What is the sum of the answers on this page?

Ans. $17.35.

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286. What is Bank Interest? What are days of grace? 287. What of money falling due on Sunday, or on a holiday? Of those falling due Feb. 28th, 29th, 30th, or 31st. 288. How is Bank Interest computed? When was the note to James Ladd payable? When was it due?

PROB. LXXV.—TO FIND THE PRESENT WORTH OR PROCEEDS OF A NOTE.

RULE.—*Subtract the Bank Interest from the face of the note.*

EXAMPLES.

1570. $7584.96. *Green Harbor, July* 16*th*, 1856.

Four months after date, for value received, I promise Lemuel Packard to pay him, or order, the sum of seven thousand five hundred eighty-four $\frac{96}{100}$ dollars. Payable at Marshfield Bank.

JOHN BAKER.

What was obtained from the bank on the above note?

1571. $846.71. *Baltimore, June* 17*th*, 1859.

Three months after date, for value received, I promise George Hawkins to pay him, or order, the sum of eight hundred forty-six $\frac{71}{100}$ dollars. Payable at Exchange Bank.

JACOB BIGLER.

What was received for the above note at the bank?

PROB. LXXVI.—TO FIND THE FACE OF A NOTE THAT SHALL PRODUCE A CERTAIN SUM WHEN DISCOUNTED AT THE BANK.

RULE.—*Divide the sum to be received from the bank by* **$1.00**, *minus the Bank Interest of* **$1.00**, *for the rate and time.*

EXAMPLES.

1572. How large must a note be made to obtain **$474.68** from a bank for **3** months, at **6** per cent.?

1573. How large must a note be made to receive **$67.84** from a bank for **60** days, at **7** per cent.?

1574. How large must a note be made to receive **$67468.42** from a bank for **4** months, at **7** per cent.?

1575. How large must a note be made to obtain **$658.75** from a bank for **30** days, at **5** per cent.?

1576. What is the sum of the answers on this page?

Ans. 78597.25.

Repeat Prob. LXXV. Rule. Prob. LXXVI. Rule.

SECTION III.

INTEREST AT ANY PER CENT.

Prob. LXXVII.—To compute interest at any per cent.

Rule.—1. *Compute the interest at* 6 *per cent.*

2. *Divide it by* 6, *and multiply the quotient by the given rate.*

Or 2d Method.—1. *Compute the interest at* 6 *per cent.*

2. *Add to it, or subtract from it, such part of it as the given rate exceeds or falls short of* 6 *per cent.*

EXAMPLES.

1577. What is the interest of $7400 from Aug. 7th, 1851, to July 3d, 1853, at 7 per cent.?

1578. What is the interest of $854.50 from July 1st, 1853, to Feb. 17th, 1856, at 6 per cent.?

1579. What is the interest of $321.00 from April 13th, 1855, to Jan. 27th, 1857, at 8½ per cent.?

1580. What is the interest of $731.00 from March 3d, 1858, to Jan. 17th, 1859, at 1 per cent. per month?

1581. What is the interest of $2071.00 from Oct. 27th, 1859, to Oct. 3d, 1862, at ¼ per cent. per month?

1582. What is the interest of $3171.00 from Feb. 2d, 1859, to Jan. 1st, 1861, at 1⅜ per cent. per month?

1583. What is the interest of $5687.56 from April 5th, 1862, to June 15th, 1863, at 5½ per cent.?

1584. What is the interest of $2784.26 from May 17th, 1863, to March 25th, 1865, at 6¾ per cent.?

1585. What is the interest of $78421 from Jan. 29th, 1853, to Aug. 31st, 1857, at 7½ per cent.?

1586. What is the interest of $56.78 from Nov. 17th, 1864, to Sept. 30th, 1867, at 9 per cent.?

1587. What is the amount of $23.67 from March 23d, 1859, to May 17th, 1871, at 7 per cent.?

1588. What is the sum of the answers on this page?

Ans. 30201.48.

SECTION IV.

PROBLEMS OF INTEREST.

PROB. LXXVIII.—THE INTEREST, TIME, AND RATE GIVEN, TO FIND THE PRINCIPAL.

RULE.—*Divide the given interest by the interest of* $1 *for the given rate, or see Art.* 280.

PROB. LXXIX.—THE INTEREST, TIME, AND PRINCIPAL GIVEN, TO FIND THE RATE.

RULE.—*Divide the given interest by the interest of the principal at one per cent., or see Art.* 281.

PROB. LXXX.—THE INTEREST, RATE, AND PRINCIPAL GIVEN, TO FIND THE TIME.

RULE.—*Divide the given interest by the interest of the principal for one year, or see Art.* 282.

PROB. LXXXI.—THE AMOUNT, RATE AND TIME GIVEN TO FIND THE PRINCIPAL.

RULE.—*Divide the given amount by the amount of* $1 *for the given time and rate.*

EXAMPLES.

1589. The interest is $77.51, time 1 yr. 6 mo., rate 6 per ct. What is the principal? Ans. $861.22.

1590. The interest is $48.619, time 2 yrs. 3 mo. 9 days, principal $356.184. What is the rate? Ans. 6 per cent.

1591. The interest is $1.685, principal $17.49, rate 6 per ct. What is the time? Ans. 1 7 8.

1592. The amount is $166.67, time 1 yr. 6 mo. 3 days, rate 6 per ct. What is the principal?

1593. The interest is $412.29, the principal $9162, time 3 months. What is the rate?

SECTION V.

PARTIAL PAYMENTS

289. *Partial Payments* includes all questions relating to the interest on notes, bonds, &c., when several partial payments have been made.

PROB. LXXXII.—RULE FREQUENTLY USED WHEN THE FINAL PAYMENT TAKES PLACE WITHIN ONE YEAR.

RULE.—*Find the amount of the principal for the whole time, also the amount of each payment from the time it was paid to the time of settlement. From the amount of the principal subtract the sum of the amounts of the payments.*

290. RULE OF THE SUPREME COURT OF THE UNITED STATES, AND OF THE STATES GENERALLY.

I. "The rule for casting interest, when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due."

II. "If the payment exceeds the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of the principal remaining due."

III. "If the payment be less than the interest, the surplus of the interest must not be taken to augment the principal, but interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied towards discharging the principal, and the interest is to be computed on the balance as aforesaid." —*Chancellor Kent.*

291. *This may be put in the following form:—*

RULE.—*Compute the interest on the principal to the time of the first payment. If the payment exceeds the interest, subtract the difference from the principal. If the interest exceeds the payment, add the difference to the next interest. Thus proceed computing interest on the unpaid principal, from one payment to the next, till the time of settlement.*

ILLUSTRATION OF RULE UNDER ART. 289.

$\$1275.\frac{00}{100}$. *Northampton, Feb. 12th,* 1853.

On demand, for value received, I promise to pay to the order of R. E. Nickerson, twelve hundred and seventy-five dollars, with interest from date. REUBEN YOUNG.

Endorsements.

1853, March 7th.	Received on the within note,			$67.00.
1853, June 12th.	"	"	"	127.75.
1853, Aug. 17th.	"	"	"	721.00.

What was due Dec. 3d, 1853?

The interest of $1275 from Feb. 12th, 1853, to Dec. 3d, 1853, is $61.8375; added to the principal, amounts to $1336.8375.

The amount of the first payment, $67, from March 7th, 1853, to Dec. 3d, 1853, is $70.4925.

The amount of the second payment, $127.75, from June 12th, 1853, to Dec. 3d, 1853, is $131.390875.

The amount of the third payment, $721, from Aug. 17th, 1853, to Dec. 3d, 1853, is $733.737.

Amount of payments, $935.62.

Young owes Nickerson	$1336.84
Nickerson owes Young	935.62
Balance in favor of Nickerson	$401.22 Ans.

ILLUSTRATION OF RULE UNDER ART. 290.

$718.23. *Trenton, May 8th,* 1856.

For value received, I promise to pay Lemuel Packard, or order, seven hundred eighteen dollars and twenty-three cents, on demand, with interest. M. J. PACKARD.

Endorsements.

1856, Aug. 12th.	Received on the within note,			$\$263.\frac{00}{100}$.
1857, Nov. 13th.	"	"	"	$26.\frac{00}{100}$.
1858, Apr. 17th.	"	"	"	$57.\frac{00}{100}$.

What was due June 7th, 1858?

Interest on $718.23 to 1st payment, is $11.25227.

Principal, $718.23 added, is	$729.48227
Subtract the 1st payment	263.00
Aug. 12, 1856. New Principal	$466.48227
Interest to April 17th, 1858	47.0369
	$513.51917
Subtract 2d and 3d payments	83.00
Apr. 17, 1858. New Principal	$430.51917
Interest to June 7th, 1858	3.58765
Due June 7th, 1858	$434.10 Ans.

EXAMPLES.

1594. $1341.72. *Concord, Dec. 5th,* 1854.

On demand, for value received, I promise Henry Owen to pay him, or order, thirteen hundred forty-one $\frac{72}{100}$ dollars, with interest from date. FRANKLIN PIERCE.

Endorsements.

Received, Jan. 5th, 1855, $347.31.
" Mar. 17th, " 84.34.
" June 17th, " 412.31.

What was due Nov. 7th, 1855? Ans. $541.67.

1595. $3476.55. *Bangor, Feb. 23d,* 1850.

For value received, I promise to pay to the order of John Howard, three thousand four hundred seventy-six $\frac{55}{100}$ dollars, with interest, on demand. JAMES GREW.

Endorsements.

April 23d, 1850,	$71.85.	Nov. 12th, 1853,	$918.75.
May 25th, 1850,	12.67.	Dec. 6th, 1854,	63.82.
Aug. 11th, 1851,	78.45.	Feb. 7th, 1855,	19.87.
Sept. 25th, 1852,	468.75.	Mar. 17th, 1856,	71.85.
Oct. 9th, 1853,	976.83.	May 19th, 1856,	1786.41.

What was due Sept. 23d, 1856? Ans. $25.81.

ILLUSTRATION OF RULE 3D UNDER ART. 291.

$1781.$\frac{39}{100}$. *Roxbury, Feb. 17th,* 1847.

On demand, for value received, I promise to pay L. Allen Kingsbury, or order, seventeen hundred eighty-one $\frac{39}{100}$ dollars, with interest.
GEORGE E. CLARK.

Endorsements.

Received on the within,	Mar. 27th, 1847,	$71.53.
" "	July 12th, 1847,	11.71.
" "	Aug. 30th, 1847,	85.00.
" "	Dec. 3d, 1847,	11.81.
" "	Feb. 17th, 1848,	17.13.
" "	July 31st, 1848,	21.73.
" "	Sept. 6th, 1849,	603.25.
" "	Mar. 17th, 1850,	763.21.

What was due May 30th, 1850?

Interest to Mar. 27th, 1847,	$11.88	Principal	$1781.39
1st payment	71.53		
Difference	$59.65	Applied to note,	59.65
Amount due March 27th, 1847			$1721.74
Interest to July 12th, 1847	$30.13		
2d payment	11.71		
Interest not paid	$18.42		
Interest to Aug. 30th, 1847,	13.77		
Interest now due	$32.19		
3d payment	85.00		
Difference applied to Prin.	$52.81		52.81
Amount due Aug. 30th, 1847			$1668.93
Interest to Dec. 3d, 1847	$25.87		
4th payment	11.81		
Interest not paid	$14.06		
Interest to Feb. 17th, 1848,	20.50		
Interest now due	$34.64		
5th payment	17.13		
Interest not paid	$17.51		

Interest to July 31st, 1848	$45.62	
Interest now due	$63.13	
6th payment	21.73	
Interest not paid	$41.40	
Interest to Sept. 6th, 1849	110.15	
Interest now due	$151.55	
7th payment	603.25	
Difference applied to Principal	$451.70	$451.70
Amount due Sept. 6th, 1849		$1217.23
Interest to Mar. 17th, 1850	$38.85	
8th payment	763.21	
Difference applied to Principal	$724.36	724.36
		$492.87
Interest to May 30th, 1850		6.00
Amount due May 30th, 1850		$98.87

EXAMPLES.

1596. $18521.73. *Washington, Sept. 5th,* 1850.

For value received, I promise to pay Suter, Lea & Co., or order, eighteen thousand five hundred and twenty-one $\frac{73}{100}$ dollars, with interest, on demand. JACOB BROWN.

Endorsements.

Dec. 4th, 1850,	$814.72.	Aug. 17th, 1853,	$927.31.
March 5th, 1851,	76.00.	Sept. 3d, 1854,	4721.17.
June 5th, 1851,	314.37.	Dec. 7th, 1855,	13721.00.

What was due Jan. 1st, 1856? Ans. $3487.71.

1597. $387.50. *Albany, Sept. 17th,* 1849.

For value received, I promise Thomas Dodge to pay him, or order, on demand, three hundred eighty-seven $\frac{50}{100}$ dollars, with interest. LINCOLN HAMLET.

Endorsements.

Mar. 21st, 1851,	$38.59.	Sept. 7th, 1852,	$89.97.

What was due May 17th, 1856? Ans. 400.37.

1598. $6000.00. *Berlin, June 5th,* 1851.

Six months after date I promise Joseph Brown to pay him, or order, six thousand dollars, value received.

RICHARD CHADBAND.

Endorsements.

Received,	Jan. 3d, 1852	$357.62
"	May 17th, 1853	78.36
"	Aug. 19th, 1854	91.48
"	Sept. 12th, 1855	31.87
"	Oct. 23d, 1856	93.58
"	Dec. 31st, 1856	486.12
"	April 3d, 1857	3846.31
"	May 21st, 1857	2879.65

What was due June 3d, 1857? Ans. $210.82.

1599. $5473.25. *Lisbon, May 17th,* 1851.

One year after date I promise Calvin Bronson to pay him, or order, five thousand four hundred seventy-three $\frac{25}{100}$ dollars, value received.

SILAS WRIGHT.

Endorsements.

Jan. 17th, 1852,	$187.00.	May 3d, 1854,	$89.37.
Feb. 19th, 1852,	17.00.	June 17th, 1854,	683.41.
April 27th, 1853,	146.00.	Oct. 25th, 1854,	3486.37.

What was due Jan. 7th, 1856?

1600. $5000.$\frac{00}{100}$. *Bolton, Dec. 8th,* 1855.

On demand, for value received, I promise Zelotes Long to pay him, or order, five thousand dollars.

ELEAZAR LOOMIS.

Endorsements.

Jan. 25th, 1856	$571.85
Feb. 27th, 1857	2321.83
Mar. 17th, 1857	1682.31
June 5th, 1857	1800.00

What was due Oct. 29th, 1857?

SECTION VI.

ACCURATE INTEREST.

292. By the common methods of computing interest for days, we obtain one year's interest in 360 days, and as there are 5 days more in the year, the interest is $\frac{5}{365}$ too much. Hence,

Prob. LXXXIII.—To compute accurate interest at any per cent.

Rule.—*Compute the interest by the common method, and diminish it by* $\frac{5}{365}$ *or* $\frac{1}{73}$ *part of itself.*

Note.—The *actual number of days* in the time must be taken, and thirty allowed to a month.

EXAMPLES.

1601. What is the interest of $786.42 from April 9th, 1856, to May 29th, 1856? (50 days = 1 m. — 20 d.) Ans. $6.46.

1602. What is the interest of $682.41 from May 5th, 1856, to Oct. 31st, 1856? (179 = 5 m. — 29 d.) Ans. $20.08.

1603. What is the interest of $76.84 from April 17th, 1853, to Dec. 29th, 1854, at 7 per cent.?

1604. What is the interest of $36842.68 from June 17th, 1854, to March 3d, 1856, at 5 per cent.?

1605. What is the interest of $7684.21 from May 19th, 1853, to July 21st, 1854, at 7 per cent.?

1606. What is the interest of $56.27 from April 19th, 1851, to Sept. 9th, 1857, at 7 per cent.?

1607. What is the interest of $348.75 for 320 days?

1608. What is the interest of $56.87 for 119 days?

1609. Interest of $35.42 from Feb. 29th, to April 17th?

1610. Interest of $684.26 from March 6th, to Nov. 30th?

1611. What is the interest of $38.91 from Feb. 17th, 1856, to Dec. 29th, 1859?

1612. What is the sum of the four last answers?

Ans. 40.68.

What is accurate interest? Why does the common method give too much? Repeat Prob. LXXXIII. Rule.

SECTION VII.

COMPOUND INTEREST.

293. When the interest is added to the principal at regular periods, forming new principals, it is called compound interest, or interest on interest.

Prob. LXXXIV.—To compute compound interest.

Rule.—1. *Find the amount to the time when it is to be compounded.*

2. *Take this amount as a new principal and compute as before; and in like manner upon each new principal to the time of settlement.*

3. *The difference between the Final Amount and First Principal will be the compound interest.*

Ill.—What is the compound interest on $375 for two years.

Principal	$375.00
Interest one year	22.50
New Principal	$397.50
Interest 2d year	23.85
Amount	$421.35
1st Principal	375.00
Compound interest	$46.35 Ans.

EXAMPLES.

1613. What is the compound interest of $721.42 for 3 years?

1614. What is the compound interest of $537.27 for 6 years?

1615. What is the compound interest of $53.72 for 3 years, compounded quarterly?

1616. What is the compound interest of $5712.73 for 2 years, compounded bi-monthly?

1617. What is the sum of the above answers? Ans. 1097.59.

1618. What is the compound interest of $7841.47 for 2 years 8 months, compounded monthly? Ans. $1356.9.

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293. What is Compound Interest? Repeat Prob. LXXXIV. Rule, 1st part. 2d. 3d. Rule.

Table for computing Compound Interest for any length of time.

Yrs.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	Yrs.
1	1.040000	1.050000	1.060000	1.070000	1.080000	1
2	1.081600	1.102500	1.123600	1.144900	1.166400	2
3	1.124864	1.157625	1.191016	1.225043	1.259712	3
4	1.169859—	1.215556+	1.262477—	1.310796+	1.360489—	4
5	1.216653—	1.276282—	1.338226—	1.402552—	1.469328+	5
6	1.265319+	1.340096—	1.418519+	1.500730+	1.586874+	6
7	1.315932—	1.407100+	1.503630+	1.605781+	1.713824+	7
8	1.368569+	1.477455+	1.593848+	1.718186+	1.850930+	8
9	1.423312—	1.551328+	1.689479—	1.838459+	1.999005—	9
10	1.480244+	1.628895—	1.790848—	1.967151+	2.158925—	10
11	1.539454+	1.710339+	1.898299—	2.104852—	2.331639—	11
12	1.601032+	1.795856+	2.012196+	2.252192—	2.518170+	12
13	1.665074—	1.885649+	2.132928+	2.409845+	2.719624—	13
14	1.731676+	1.979932—	2.260904—	2.578534+	2.937194—	14
15	1.800944—	2.078928+	2.396558+	2.759032—	3.172169+	15
16	1.872981+	2.182875—	2.540352—	2.952164—	3.425943—	16
17	1.947900+	2.292018+	2.692773—	3.158815+	3.700018+	17
18	2.025817—	2.406619+	2.854339+	3.379932+	3.996019+	18
19	2.106849+	2.526950+	3.025600—	3.616528—	4.315701+	19
20	2.191123+	2.653298—	3.207135+	3.869684+	4.660957+	20

Illustration.—What is the interest of $600 for 6 yrs. 4 mo. at 6 per cent.

$1.418519 Amt. of $1.00 for 6 yrs. from table.
600
$851.1104
1.02 Amt. of $1.00 for 4 mo.
17022208
8511104
$868.132608 — $600 = $268.13, Com. Int.

To compute for any number of years, separate the given time into parts. Find the amount for one part of time, then write this as a new principal; find the amount for another part, and so on till all the time is taken.

EXAMPLES.

1619. What is the compound interest of $500 for 40 years, at 6 per cent.? (20 + 20 = 40.)

1620. What is the compound interest of $76823 for 37 years, at 5 per cent.?

CHAPTER II.

DISCOUNT.

SECTION I.

DISCOUNT AND PRESENT WORTH OF A NOTE DUE AT A FUTURE DATE.

294. When an allowance of interest is made for the payment of a sum of money before it is due, it is called *Discount.*

295. The *Present Worth* is the amount paid, or the actual value of the note after the discount has been deducted.

PROB. LXXXV.—TO FIND THE PRESENT WORTH.

RULE.—1. *Divide the given sum by the amount of one dollar for the given rate and time.*

2. *The quotient will be the present worth.*

3. *The difference between this present worth and the given sum, is the discount.*

296. The above rule gives the true discount, but merchants use the following rule as more convenient.

PROB. LXXXVI.—TO COMPUTE MERCHANTS' DISCOUNT.

RULE.—*Multiply by the rate of discount.*

EXAMPLES.

1621. What is the discount of $715.00 from March 3d, 1851, to Jan. 5th, 1853?

1622. What is the discount of $821.00 from June 25th, 1855, to March 7th, 1856?

1623. What is the discount of $3500.00 at $\frac{7}{8}$ per cent.?

1624. What is the discount of $1718.00 at $\frac{1}{4}$ off?

1625. What is the discount of $7000.00 at 97 per cent.

1626. What is the sum of the answers on this page?

Ans. $7354.27.

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294. What is Discount? 295. What is Present Worth? Repeat Prob. LXXXV. Rule 1st. 2d. 3d. Prob. LXXXVI. Rule. What is the difference between merchants' discount and discount?

1627. What is the discount of $718.25 from Apr. 17th, 1851, to June 17th, 1856?

1628. What is the discount of $684.56 from May 21st, 1853, to Oct. 3d, 1854, at 7 per cent.?

1629. What is the discount of $35.76 from Aug. 18th, 1854, to Jan. 9th, 1855, at 9 per cent.?

1630. What is the discount of $568.26 from Nov. 29th, 1853, to Dec. 3d, 1856, at $6\frac{1}{2}$ per cent.?

1631. What is the discount of $78.23 from Oct. 17th, 1852, to Jan. 5th, 1856, at 5 per cent.?

1632. What is the sum of the above answers?

Ans. $334.82.

1633. What is the discount of $368.27 from Jan. 31st, 1851, to Feb. 13th, 1853, at $7\frac{1}{2}$ per cent.?

1634. What is the discount of $5687.87 from Feb. 29th, 1856, to March 17th, 1857?

1635. What is the discount of $56.87 from May 9th, 1857, to May 25th, 1858?

1636. What is the merchants' discount of $36.87 at $4\frac{1}{2}$ per ct.?

1637. What is the merchants' discount of $21.78 at 3 per ct.?

1638. What is the merchants' discount of $368.75 at $\frac{3}{8}$ per ct.?

1639. What is the merchants' discount of $4000.00 at 9 pr. ct.?

1640. What is the merchants' discount of $845.68 at $4\frac{1}{2}$ pr. ct.?

1641. What is the merchants' discount of $3787.68 at 17 pr. ct.?

1642. What is the merchants' discount of $42.60 at 19 per ct.?

1643. What is the merchants' discount of $82.56 at $\frac{3}{8}$ per ct.?

1644. What is the merchants' discount of $78.46 at $\frac{7}{8}$ per ct.?

1645. What is the merchants' discount of $30.46 at 23 per ct.?

1646. What is the merchants' discount of $187.63 at $25\frac{1}{2}$ pr. ct.?

1647. What is the merchants' discount of $426.82 at $87\frac{1}{2}$ pr. ct.?

1648. What is the merchants' discount of $568.76 at $\frac{1}{16}$ pr. ct.?

1649. What is the merchants' discount of $568.78 at $\frac{3}{16}$ pr. ct.?

1650. What is the merchants' discount of $979.68 at $3\frac{1}{16}$ pr. ct.?

1651. What is the sum of the answers from the 1636th, inclusive?

Ans. $1514.51.

SECTION II.

NOTES AT DIFFERENT FUTURE DATES.

PROB. LXXXVII.—TO COMPUTE THE DISCOUNT AND PRESENT WORTH OF A NOTE DUE AT DIFFERENT FUTURE DATES.

RULE.—1. *Compute the present worth of the several sums separately by Prob. LXXXI, p.* 167.

2. *The sum of these will be the present worth.*

3. *Subtract the present worth from the face of the note, and it will give the discount.*

EXAMPLES.

1652. Bought a house for $1600.00, for which I gave the following note:—

$1600. *Pelham, Feb. 4th,* 1854.

For value received, I promise James Grew to pay him, or order, sixteen hundred dollars, as follows: May 1st, 1854, $500; Sept. 6th, 1854, $500; May 1st, 1855, $500; Sept. 6th, 1855, $100, without interest.

SIMON PETTIGREW.

What was the value of the note April 7th, 1854?

Ans. $1549.62.

1653. For what should I sell the following note April 19th, 1854?

$1500. *Marlboro', Oct. 17th,* 1850.

For value received, I promise Solomon Weeks to pay him, or order, fifteen hundred dollars, as follows: $500, Jan. 1st, 1855; $500, Jan. 1st, 1856; $500, Jan. 1st, 1857.

Ans. $1363.86. E. S. HUNT.

What is discount? What is present worth? What is the difference between interest and present worth? Interest and discount? How does Prob. LXXXIII. differ from Prob. LXXXI? What is done first under Prob. LXXXIII? Second? Third? Give the Rule entire. Give an illustration.

SECTION III.

NOTES AT DIFFERENT PAST AND FUTURE DATES.

PROB. LXXXVIII.—TO COMPUTE THE DISCOUNT AND PRESENT WORTH OF A NOTE DUE AT DIFFERENT PAST AND FUTURE DATES.

RULE.—1. *Find the amount of the sums past due, and the present worth of the sums future due.*

2. *The sum of these will be the present value of the note.*

EXAMPLES.

1654. What sum is necessary to liquidate the following note June 6th, 1855?

$2300. *Providence, Jan. 17th,* 1849.

For value received, I promise James Brown to pay him, or order, twenty-three hundred dollars, as follows: May 12th, 1849, $267.50; Dec. 15th, 1851, $357.25; April 10th, 1854, $473.00; June 6th, 1857 $573.12; Dec. 12th, 1858, $629.13; all without interest.

Ans. $2333.62. EDWIN WHIPPLE.

1655. What sum of money will pay the following note July 12th, 1858?

$2781. *Pawtucket, May 12th,* 1849.

For value received, I promise to pay John Hickman, or order, twenty-seven hundred eighty-one dollars, without interest, as follows: $500, May 12th, 1851; $500, May 12th, 1856; $500, May 12th, 1857; $500, May 12th, 1858; $781, May 12th, 1860. CHARLES MASON.

Ans. $3023.60.

Repeat Prob. LXXXVIII. Repeat first part of Rule. Second. Third. Give the Rule entire.

NOTE.—*Let the pupil make a thorough review of Part III.*

REVIEW OF PARTS I., II. AND III.

MISCELLANEOUS QUESTIONS.

1656. What is the product of seventeen, thousandths; eight hundred and seventy-two, ten thousandths; and seventeen hundredths? Ans. .000252008.

1657. What is the product of nine hundred and twenty-one, ten thousandths, and eighty-nine hundredths. Ans. .081969.

1658. What is the difference between seven, ten thousandths, multiplied by nine hundred and eleven, ten millionths, and eleven hundred thirty-one, ten thousandths? Ans. .11309993623.

1659. What is the quotient of five, millionths, divided by seven tenths? Ans. .000007142+.

1660. Divide three hundred eighty-one and seventy-five, hundredths, by seven hundred ninety-three and two, tenths.

1661. Divide seven hundred and ninety-one by thirty hundred and fifty-seven, hundred thousandths.

1662. What is the shortest method of finding the cost of 17 plows at $7.50, 17 carts at $23.75, and 17 horses at $62.75? What did they cost? Ans. to the last 3, 27473.522167.

1663. Change $\frac{144}{168}$ to its lowest form of expression.

1664. Change $\frac{567}{630}$ to its lowest form of expression.

1665. Change $\frac{663}{1885}$ to its lowest form of expression.

1666. Change $\frac{2093}{2717}$ to its lowest form of expression.

1667. Add $6\ 7''$, $5'\ 9''$, $10'\ 11''$ to $7'\ 3''$.

1668. From $3\ 6'\ 7''\ 8'''\ 9^{iv}$ take $1\ 5'\ 3''\ 9'''\ 11^{iv}$.

1669. Multiply $6'\ 3''\ 4'''\ 5^{iv}$ by $6\ 2'$.

1670. The sum of two numbers is $14\frac{17}{19}$, and one number is $6\frac{3}{8}$. What is the other?

1671. The difference of two numbers is $8\frac{13}{19}$, and one number is $23\frac{7}{8}$. What two numbers will answer this condition?

1672. The sum of two numbers is 45, and their difference 7. What are the numbers?

Repeat Art. 227, 228, 229, 230, 231, 232, 233.

1673. A ship sails 13½ knots per hour for 17⅔ hours. What is the distance sailed?

1674. In reckoning 365 days to a year we actually lose 5 hr. 48 min. 49.6 sec. Every leap year we reckon 366 days. What is the actual gain in four years? Ans. 44 min. 41.6 sec.

1675. How long time will it take to gain 1 day?

1676. What is the difference in the salary of the President of the United States per hour, for the years 1856 and 1857?

1677. What is the time from June 5th, 1775, to July 10th, 1871? Ans. 35098 days.

1678. When is the solar year of 1856 exactly ½ expired? It commences Jan. 1st, 12 o'clock, noon.

Ans. July 2d, at 2 h. 54 min. 24.8 sec. A. M.

1679. A family uses about 7 tons of coal in 1 year. How many cubic feet of space would it occupy?

1680. James Ellis bought 4½ tons, and concluded to put it in old flour barrels. How many barrels will his coal fill, if each barrel will contain 34 liquid gallons?

1681. A ship needs 13½ tons ballast, and is under the necessity of taking sand. How many loads will it take, carrying 25½ feet to the load? Sum of the last 3 Ans., 360.223864.

1682. Edwin Field offers me 10¼ cents per lb. for 7 cwt. 2 qrs. 19 lbs. of sugar, cash. Charles Brown offers 10½ cents per lb. for the same, on a note of 4 months. What is the difference of their offers?

1683. Suppose I trade with Brown, and get his note discounted at the Metropolitan Bank. What should I receive in cash?

1684. Borrowed of Lucius Smith $793, Aug. 17th, 1858. What would be the face of a note dated Nov. 3d, 1858, to run 4 months, that would enable Smith to obtain from bank the amount due?

1685. What is the sum of the answers of the last three examples? Ans. $897.73

Repeat Art. 234, 235, 236, 237, 238, 239, 240, 241, 242. Repeat Prob. and Rule. Prob. 57, 58, 59, 60, 61. Rule 57, 58, 59, 60, 61.

1686. What is the difference between simple and compound interest of $216.00 for 2 years and 6 months?

1687. Boston Common contains 48 acres of land. Suppose it had been sold in 1830 for 50 cents per foot, and the sum put at compound interest at 6 per cent., what would it amount to in 1867?

1688. At that rate, what would it be worth per foot in 1867?

1689. One-fifth of the space burnt over by the great fire of 1666, London, was 87 A. 0 R. 32 rds., and covered 218 squares. How many square rods in a square?

1690. In 1642 fifty towns and villages were settled in Massachusetts Bay Colony, at an expense of $1000000. What would that sum amount to in 1857, at simple interest?

1691. John Harvard, a minister of Charlestown, Massachusetts, founded Harvard College by a bequest of $35000, in 1638. What would that sum amount to in 1857, simple interest, and at 6 per cent.?

1692. The loss at the "Great Fire in Boston," Aug. 8th, 1679, was $200000. Suppose that had been invested in a school fund at 6 per cent. compound interest, and allowed to accumulate till 1850, then the annual income to be distributed among 1000000 children for schooling, what would each receive?

1693. New Hampshire, in 1682, contained 4000 inhabitants; in 1850, 317864 persons. What was the per cent. of increase? (See Case 5, Prob. LXXII.)

1694. The population of the American Colonies, according to the census for the "Board of Trade, 1715," was — whites, 375750; negroes, 58850. Population in 1850 was — whites, 19630133; negroes, 3626839. What is the difference in the rate of increase?

1695. What is the commission for purchasing 7000 lbs. of tea at 65 cents per lb., at $2\frac{3}{4}$ per cent.?

1696. What is the sum of the answers? Ans. 23432645.99.

Repeat Art. 243, 244, 245, 246, 247, 248, 249, 250. Repeat Prob. and Rule 62, 63, 64, 65, 66, 6, 7, 8, 9, 10, and 11.

1697. March 20th, 1660. A fire raged in Cornhill, Boston, and destroyed 174 dwelling houses and 175 warehouses, together with property of every description, to the amount of $500000. What would have been the poll tax on the people of Massachusetts, 241000, to have paid this loss, with an overlay of 5 per cent. for collection?

1698. The population of Massachusetts, in 1775, ascertained by the Continental Congress, was 352000. By the law of Massachusetts, 14 per cent. nearly are enrolled as able-bodied militia. How many might have been enrolled at that time, according to this law? Ans. to last two examples, 49282.18.

1699. The warlike stores of that State, by returns of April 14th, 1776, were as follows:—

Firearms...............	21549	Number of Flints...	144699
Pounds of Powder...	17441	" Bayonets	10108
Ball.......	22191	" Pouches.	11979

How much was this to each man? Ans. About 1 to 2; 1 to 3; 1 to 2; 3 to 1; 1 to 5; and 1 to 5.

1700. It was a tradition of the Indians that lived in the vicinity of Niagara Falls, that the Water God required the sacrifice of a human being annually. It is supposed that the actual number of fatalities is about 35 per cent. more than that estimate. How many have occurred, according to this estimate, between the years 1650 and 1859?

1701. It is said that 670000 tons of water pass over the Falls in a minute. How many cubic feet?

1702. The Philadelphia Water Works furnished the city with 6317903116 gallons in 1857. About 127 per cent. of $\frac{1}{12}$ was pumped in the month of August. How many gallons?

1703. Four and ninety, hundredths per cent. was solid matter. How many cubic feet in the entire quantity?

1704. What is the sum of the last four answers?

Ans. 731530335.12.

Repeat Art. 251, 252, 253, 254, 255. Prob. 67, 68, 69, 70. Rule 67, 68, 69, 70.

1705. Reduce $\frac{1889}{2083}$ to its lowest terms.

1706. Reduce $\frac{4394}{4862}$ to its lowest terms.

1707. Change $\frac{14195}{17729}$ to its lowest terms.

1708. Change $\frac{11123}{11131}$ to its lowest terms. (?)

1709. I have a room 13 ft. 6′ wide, and 16 7′ long. How many square feet in it?

1710. How many square yards of Turkey carpeting will cover it?

1711. How many yards in length of carpeting will cover it, if the carpet is 30 inches wide?

1712. Multiply 6 4′, 5 3′, and 4 7′.

1713. What is the solid contents of a box 6 ft. 5′ high, 5 ft. 4′ wide, and 7 ft. 4′ long?

1714. How many gallons will it contain, suppose the above are inside dimensions?

1715. The product of 3 numbers is 729, and two of them are 3 and 27. What is the third?

1716. The product of 3 numbers is 729.856, and two of the numbers are $4\frac{7}{8}$ and $4\frac{1\frac{3}{4}}{5}$. What is the third?

1717. Resolve 76, 92, 112, 726, and 117 into their prime factors.

1718. At a certain election there were three parties, Democrats, Whigs, and Americans. Whole number of votes cast, 1571, of which the Democrats had 785, Whigs 580, and Americans 206. Before the second trial, the Americans had secured the vote of one Democrat, and the Whigs had brought one more voter to the polls. How many votes was it necessary for the Democrats to obtain to secure the election? Ans. 5 votes.

1719. How many did the Democrats want of an election at the first trial? Ans. 2 votes.

1720. This day paid the Bank of Charleston $12.50 discount on a note to run four months. What was the face of the note?

Repeat Art. 256, 257, 258, 259, 260, 261, 262, 263. Repeat Prob. and Rule 71, 72, 73, 74, 75, 76, 77, 78, 79.

1721. What is the time from May 5th, 1855, to June 6th, 1856?

1722. What is the time from Aug. 1st, 1854, to Feb. 28th, 1857?

1723. What is the time from Nov. 12th, 1857, to Jan. 1st, 1858?

1724. From Oct. 30th to April 17th, 1856?

1725. From Dec. 31st to Jan. 13th, 1857?

1726. What is the interest of one dollar for 30 days?

1727. For 31 days? 42 days? 36? 27? 19? 1? 7? 11? 17? 27?

1728. 18 days? 11 days? 9? 8? 7? 19? 13?

1729. 21 days? 30? 27? 25? 43? 48? 8? 12? 19? 21? 31? 42? 43?

1730. 72 days? 69? 53? 37? 3? 2? 5?

1731. 33 days? 9? 11? 13? 18? 17? 9? 8? 17? 11? 9? 3? 1? 2? 5? 2?

1732. 71 days? 59? 49? 39? 27? 14? 13? 9? 10? 14? 13? 18? 26? 28? 1?

1733. 37 days? 12? 15? 25? 35? 14? 17?

1734. 91 days? 82? 41? 38? 28? 8? 7? 6? 7? 4? 8? 17? 12? 13? 29? 28?

The interest in the above sums is to be computed mentally, and answers to be given instantly.

1735. What is the interest on $587.93 from Aug. 23d, 1856, to Jan. 15th, 1858?

1736. What is the commission of the sale of 377 bu. white beans, @ $$3\frac{3}{7}$ per bu., at $2\frac{7}{9}$ per cent.?

1737. Sent to my agent in Liverpool 700 bales of cotton on commission of $3\frac{3}{7}$ per cent. He informs me that he has sold it for $$27\frac{3}{9}$ per bale, and after deducting his commission, he had invested the residue for me in United States Stocks, after deducting a further commission of $1\frac{3}{8}$ per cent. for investment. How much have I invested in stocks?

1738. What is the sum of the last three answers?

Ans. 18311.81.

1739. I sold my hat to James Brown for $5.00, and lost 10 per cent.; he sold it to John Burdwin, and lost 10 per cent.; Burdwin sold it, and gained 10 per cent.; I then bought the hat for what the last man paid for it. Did I make or lose by the trade? How much? Ans. $50\frac{5}{9}$ cents.

1740. Allen Smith bought of Sampson White—

10 firkins of butter, 56 lbs., @ .$21\frac{1}{2}$ =
11 cheese, 46 lbs., @ $7\frac{1}{2}$ cts.
46 bu. potatoes, @ .$83\frac{1}{3}$.
278 bu. wheat, @ $1.$98\frac{1}{2}$.
178 bu. corn, @ .$58\frac{1}{3}$.

In payment, gave his note for 4 months, without interest. On the back of the note was the following: "*Due Feb. 6th,* 1857."

Feb. 12th, 1857, received $86.95.
June 13th, 1858, received $17.81.
Dec. 15th, 1859, received $208.47.
Mar. 12th, 1860, received $197.81.

What was due April 12th, 1860?

1741. This day received Allen Smith's note for 2 months for balance due, and had the same discounted at the Bank of Metropolis. What was the amount received?

1742. Two years ago this day I purchased 106 tons of old iron at 7 mills per lb., and hired it stored for $150 a year, payable quarterly. I borrowed the money to purchase with, and also to pay rent, at 9 per cent. I have this day sold my iron without moving it, for two and three eighths cents per lb. How much shall I have left after paying my borrowed money?

1743. G. Hall holds the following notes, viz: No. 1, $786.29, due Jan. 13th, 1864, without interest; No. 2, $537.21, due Mar. 3d, 1863, without interest. What will be the face of a note, to run four months, that will cancel them Aug. 12th, 1861?

Repeat Art. 264. 265, 266, 267, 268, 269, 270, 271, 272, 273. Repeat Prob. and Rule 80, 81, 82, 83, 84. Article 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285. Prob. and Rule 7, 9, 11, 12, 13, 17, 19, 23, 26, 29. Article 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296.

Analysis of Part IV. Proportion.

CHAPTERS.	SECTIONS.	PAGE
I. RATIO		197
II. PROPORTION	1. Questions dependent on a single ratio	199
	2. Questions dependent on several ratios	203
III. PARTITIVE PROPORTION	1. Equal Time	207
	2. Unequal Time	208
	3. General Average	210
IV. MEDIAL PROPORTION		213
V. EQUATED PROPORTION		217

REVIEW OF PARTS I. II. III. AND IV. 222

PART IV.

RATIO AND PROPORTION.

CHAPTER I.

RATIO.

297. Ratio is the number of times one number is contained in another.

298. The *Divisor*, or *Denominator*, is the *Antecedent.*

299. The *Dividend*, or *Numerator*, is the *Consequent.*

300. The *Quotient*, or *Value of the Fraction*, is *Ratio.*

301. It is not essential that the division be performed, as the ratio may be fractionally expressed: thus, the ratio of 3 to 6 is 2, or $\frac{6}{3}$; of 4 to 7, is $\frac{7}{4}$, &c.

302. Therefore, all operations that pertain to *Division*, or *Fractions*, are *applicable to Ratios.* The theory of *Ratios*, *Fractions*, and *Division*, are *identical.*

303. When *Ratio* is applied to magnitudes, they *must be of the same kind*, otherwise *one could not be considered a part of the other.*

304. All *Ratios* should be reduced to their lowest terms, when used in proportions. This may be done by cancelling.

305. A Ratio is expressed thus, A : C, and is read as A is to C, or $\frac{C}{A}$, and read C divided by A, or the ratio of A to C.

306. The 1st term, A, is the Antecedent; the 2d term, C, is the Consequent.

307. Both terms together are called a couplet.

308. When the Antecedent is largest, it is called a ratio of *lesser inequality.*

What is a Ratio? Dividend? Numerator? Antecedent? Divisor? Denominator? Consequent? Quotient? Ratio? Is the Division essential? Repeat 302, 303, 304, 305, 306, 307, 308.

309. When the Consequent is largest, it is called a ratio of *greater inequality.*

310. The Antecedent and Consequent are called the *terms* of the *ratio.*

311. A *Simple* Ratio has one Antecedent, and one Consequent, or one couplet: thus, 7 : 5, &c.

312. A Compound Ratio has several couplets, or is the product of several couplets: thus,

7 : 12 is a Simple Ratio.
6 : 2 " "
5 : 3 " "
210 : 72 is a Compound Ratio.

313. NOTE.—Let the pupil bear in mind that *all rules* previously given for *fractions*, ARE REQUIRED in the *successful management of* RATIOS.

EXAMPLES.

1744. What is the ratio of 21 to 7?
1745. Of 46 to 23? Ans. $\frac{1}{2}$.
1746. Of 74 to 37? Ans. $\frac{1}{2}$.
1747. Of 36 to 18? Ans. $\frac{1}{2}$.
1748. Of $8\frac{1}{2}$ to $4\frac{1}{4}$?
1749. Of $5\frac{3}{4}$ to 2?
1750. Of $7\frac{5}{8}$ to $2\frac{1}{2}$?
1751. Of 31 to 17?
1752. Of $46\frac{1}{2}$ to $31\frac{3}{4}$?
1753. Of 78 to $25\frac{1}{2}$?
1754. Of 327 to 721? Ans. $2\frac{67}{327}$.
1755. Of 463 to 831?
1756. Of 8741 to $46\frac{3}{5}$?
1757. Of $32\frac{7}{8}$ to $16\frac{5}{8}$?
1758. Of $51\frac{2}{8\frac{6}{12}}$ to $21\frac{3}{9}$?
1759. Of 56 to $7\frac{4}{18}$?
1760. Of 19 to 209?
1761. Of 5 miles to $2\frac{1}{2}$ miles?
1762. Of 7 miles to 80 rods?
1763. Of 4 miles to 2 rods?
1764. Of 6 miles to 18 yds.?
1765. Of $3\frac{1}{2}$ miles to $27\frac{1}{2}$ ft.?
1766. Of 3 lbs. to 27 lbs.?
1767. Of $6\frac{3}{4}$ lbs. to 41 ounces?
1768. Of 89 days to 7 hours?
1769. Of 7m. 3f. 6in. to 19rds.?
1770. Of $7\frac{3}{5}$ mi. to 4 ft. $\frac{3}{5}$ in.?
1771. Of $82\frac{3}{17}$ mi. to $41\frac{1}{17}$ mi.?
1772. Of 93 cwt. to $16\frac{3}{8}$ drs.?
1773. Of 42 yds. to $\frac{7}{8}$ in.?
1774. Of $21\frac{4}{7}$ mi. to $57\frac{3}{11}$ rds.?
1775. Of 19 cwt. to $11\frac{3}{31}$ lbs.?

What is a couplet? Ratio of greater inequality? Of less inequality? What are the terms of a Ratio? Simple Ratio? Compound Ratio? Repeat *Art. 313.* Repeat it again. What *does* it *mean?*

CHAPTER II.

PROPORTION.

SECTION I.

QUESTIONS DEPENDENT ON A SINGLE RATIO.

314. A *Single Proportion* is expressed by an equality of two *Ratios.*

315. Every Proportion, therefore, must have, at least four terms, two Antecedents and two Consequents, which are arranged in the following manner, viz.: A : C : : A : C. They are read as A is to C so is A to C; or by figures, as 4 is to 8 so is 11 to 22.

316. The four terms, taken together, are called *Proportionals.*

317. The Ratios, taken together, are called *Couplets.*

318. The 1st and 4th are called *Extremes.*

319. The 2d and 3d are called *Means.*

320. In a proportion, from the definition, the ratios are equal; *hence,* the product of the *Extremes* are equal to the product of the *Means.*

321. From this principle, *if any one term be wanting* in a proportion, it may easily be found, as we shall have the product and one factor to find the other.

322. If denominate numbers occur, they must be reduced to the same denomination.

323. Change all ratios in a proportion to their lowest terms by cancelling.

324. Make the decimal places, if any, equal in the antecedents and consequents.

325. Arrange all fractional terms the same as whole numbers. Then remove all the denominators in the antecedents to the consequents, and all denominators in the consequents to the antecedents, and proceed as in whole numbers.

---

314. What is a Single Proportion? 315. Consists of how many terms? How written? How read? What are Proportionals? Couplets? Extremes? Means? Product of the Extremes equal to what? Why? How supply a wanting term? On what principle? How manage denominate numbers? Decimal? Fractional? Why?

326. Can you assign any reason for that peculiar arrangement of the denominators in the last article?

327. Every example in Proportion is divided into two parts, the *statement* and the *question.*

328. The position of every ratio in a proportion must be determined by this *question.* For, by the definition of proportion, if the ratio connected with the answer is a greater inequality, the other equal ratio must be the same, or if a lesser inequality, the other equal ratio must be of a lesser inequality.

PROB. LXXXIX.—TO FIND THE 4TH TERM OF A SINGLE PROPORTION.

RULE.—1. *Two of the three terms given will be of the same kind and form a ratio.*

2. *The remaining term will be of the same kind as the answer, and is called the term of demand.*

3. *This term of demand is the third term of the proportion.*

4. *Ascertain from the question whether the answer is to be greater or less than the term of demand.*

5. *If the answer is to be greater, make the ratio of a greater inequality.*

6. *If the answer is to be less, make the ratio of a lesser inequality.*

7. *Multiply the 2d and 3d terms together, and divide the product by the 1st term, or take such a part of the term of demand as the ratio represents.*

ILL.—If 75 bu. of corn cost $112.50, what will 270 bu. cost?

The "Statement" is, that "75 bushels cost $112.50."

The "Question" is, "What will 270 cost?"

The "Term of Demand" is $112.50.

The "Ratio" either $\frac{75}{270}$ or $\frac{270}{75}$.

---

327. Into how many parts is every example in Proportion divided? What is the statement? What is the question? 328. How is the position of every ratio determined? Explain fully? What is a ratio of greater inequality? Of lesser inequality? Repeat Prob. LXXXIX. Repeat 1st part of rule. 2d. 3d. 4th. 5th. 6th. 7th. Repeat the entire rule.

In this question it is readily seen that 270 bushels will cost more than 75, hence the proportion is as 75 : 270 : : 112.50 : Ans.; or the following form is preferable :—

$$\begin{array}{c|ll} & \$\cancel{1}\cancel{1}\cancel{2}.\cancel{5}\cancel{0} & 450 \\ \cancel{2}\cancel{5} \quad \cancel{7}\cancel{5} & \cancel{2}\cancel{7}\cancel{0} & 90 \\ \hline & \$405.00 & \text{Ans.} \end{array} \qquad \text{Cancel 3 and 25.}$$

Ill. 2.—If $\frac{7}{8}$ of a ton of hay cost \$$14\frac{3}{4}$, what will $2\frac{5}{8}$ tons cost?

The term of demand is \$$14\frac{3}{4}$. Ratio $\frac{2\frac{3}{8}}{\frac{7}{8}}$.

OPERATION.

$$\begin{array}{c|c} & 14\frac{3}{4} \\ \frac{7}{8} & 2\frac{5}{8} \end{array} = \begin{array}{c|c} & \frac{59}{4} \\ \frac{7}{8} & \frac{21}{8} \end{array} = \begin{array}{c|cc} 4 & 59 & \\ \cancel{8} & \cancel{2}\cancel{1} & 3 \\ \cancel{7} & \cancel{8} & \\ \hline 4 & 177 & \end{array}$$

Ans. \$44.25

Ill. 3.—If 4 tons 6 cwt. 10 lbs. of hay cost \$175.00, what will 6 tons 3 cwt. 3 lbs. of hay cost?

Term of demand, \$175.

Ratio, $\frac{123.03}{86.10}$.

OPERATION.

$$\begin{array}{r|ll} & \$\cancel{1}\cancel{7}\cancel{5}.\cancel{0}\cancel{0} & 2.50 \\ 41—\cancel{2}\cancel{8}\cancel{7}\cancel{0}—\cancel{8}\cancel{6}.\cancel{1}\cancel{0} & \cancel{1}\cancel{2}\cancel{3}.\cancel{0}\cancel{3} & 41.01 \\ \hline 41 & 1025.25 & \\ \end{array}$$

$25.00\frac{25}{41}$ Ans.

EXAMPLES.

1776. If 17 bbls. of flour cost \$187.00, what will 13 bbls. cost?

1777. If 681 sheep cost \$2724, what will 793 cost?

1778. If 943 lbs. of coffee cost \$66.01, what will 1787 lbs. cost?

1779. If 179 acres of grain cost \$8690, what will $17\frac{1}{2}$ A. cost?

1780. If a man travels 917 miles in 23 days, how far will he go in 7 days?

1781. If 67 oxen plow 87 acres, what will 13 oxen plow in the same time?

1782. If a ship sail 7189 miles in 37 days, how far will she sail in 11 days?

1783. What is the sum of the answers? Ans. 6722.91.

1784. If sugar cost \$11 per cwt., what will 27 lbs. cost?

1785. If 16 lbs. of rice cost \$.78, what will $19\frac{3}{4}$ lbs. cost?

1786. If $\frac{7}{8}$ of a ton of lead cost \$$7.58\frac{1}{3}$, what will $23\frac{3}{5}$ lbs. cost?

1787. If $\frac{5}{9}$ of a cord of wood cost \$$3\frac{4}{17}$, what will $9\frac{3}{11}$ cost?

1788. Paid \$$58\frac{3}{7}$ for $9\frac{3}{8}$ tons of coal. What will $11\frac{3}{7}$ cost?

Sum of the above answers, 127.69.

1789. Spent \$$39\frac{5}{17}$ in travelling $758\frac{9}{17}$ miles. What will it cost to travel $3428\frac{6}{11}$ miles, at the same rate? Ans. $177\frac{86387}{141845}$.

1790. If a hind wheel is $9\frac{8}{17}$ feet in circumference, and turns 11000 times in going a certain distance, how many times will the fore wheel, $7\frac{7}{22}$ ft. in circumference, turn in going the same distance?

1791. A scaffold of hay will keep a horse 23 weeks, or a cow twice as long. How long will it last both?

1792. How many yards of paper, 30 inches wide, will cover the walls of a room that is 64 feet around and $8\frac{1}{2}$ feet high?

1793. A man failed. Property was valued at \$11784.26; total debts, \$17842.67. What will James Smith get, who has \$3784.50 due him?

1794. If the discount for 1 year on \$106.00 is \$6.00, what will it be on \$784.75 for the same time?

1795. If 57 gallons of currant wine be worth \$$211\frac{1}{16}$, what will $717\frac{1}{8}$ gallons be worth?

1796. If 21 is 72, what is 71? Ans. $243\frac{3}{7}$.

1797. If $17\frac{3}{5}$ hhds. of sugar are worth $987\frac{3}{8}$ bbls. of flour, what would purchase 2000 bbls. of flour? Ans. $35\frac{5135}{7899}$.

1798. Borrowed \$750 of a friend for $\frac{7}{8}$ of a year. How long shall I lend him \$478.62 to requite the favor?

1799. What will 78 bags of salt-petre cost, if $19\frac{1}{2}$ cost \$158?

1800. If $3\frac{1}{8}$ tons of hay cost \$75.00, what will $17\frac{3}{8}$ cost?

1801. If $17\frac{1}{4}$ cwt. of sugar cost $\frac{1}{4}$ of \$$86\frac{1}{4}$, what will $\frac{3}{5}$ of 35 cwt. cost?

1802. If $\frac{1}{2}$ of $17\frac{1}{4}$ cwt. of sugar cost \$$86\frac{1}{4}$, what will 35 cost?

1803. If $\frac{14\frac{1}{28}}{28}$ lbs. of rice cost $3\frac{17}{28}$ cents, what will 3 cwt. 2 qr. cost?

1804. Sum of last five answers? Ans. \$3943.83.

SECTION II.

QUESTIONS DEPENDENT ON SEVERAL RATIOS.

Part 1. Compound Proportion.

329. Compound Proportion is a proportion in which the term of demand is effected by several ratios instead of one.

330. Hence, the principles laid down under Ratio and Single Proportion, are applicable here.

PROB. XC.—TO SOLVE QUESTIONS DEPENDENT ON SEVERAL RATIOS.

RULE.—1. *Arrange the several ratios separately with the term of demand, as in single proportion.*

2. *Then proceed to cancel, multiply, and divide.*

ILLUSTRATION.—If 15 compositors, in 17 days of 11 hours each, compose 3 volumes, each of 17 sheets of 24 pages of 44 lines of 44 ems, in how many days of 10 hours each will 18 compositors compose 9 volumes, each of 24 sheets of 36 pages of 66 lines of 32 ems?

OPERATION.	RATIOS.		
	A's.	C's.	
Term of demand..................................		$\cancel{1}\cancel{7}$	
More compositors, less days. Compositors	$\cancel{3}$ $\cancel{1}\cancel{8}$	$\cancel{1}\cancel{5}$	
Less hours, more days. Hours..............	$\cancel{1}\cancel{0}$	$\cancel{1}\cancel{1}$	
More volumes, more days. Volumes.......	$\cancel{3}$	$\cancel{9}$	3
More sheets, more days. Sheets............	$\cancel{1}\cancel{7}$	$\cancel{2}\cancel{4}$	
More pages, more days. Pages..............	$\cancel{2}\cancel{4}$	36	
More lines, more days. Lines................	$\cancel{4}\cancel{4}$	$\cancel{6}\cancel{6}$	
Less ems, less days. Ems	$\cancel{4}\cancel{4}$	$\cancel{3}\cancel{2}$	

Cancel 17, 24, 11, 11, 32, 5, 6, 3, and the Ans. is 108 days.

---

329. What is a compound proportion? 330. Are the principles different from single proportion? Repeat Prob. XC. Rule. In the illustration, how can you know which is the term of demand? How, which is the question? Of how many parts does the statement consist? How do you arrange a compound proportion? How many parts to the illustration? What *is the difference between single and compound proportion?*

EXAMPLES.

1805. If a family of 9 persons in 4 weeks spend $184.00, how long ought a family of 13 persons to be in spending $143.00? Ans. $2\frac{7}{46}$.

1806. If 4444 lbs. of bread will supply a garrison of 8800 men for 11 days, how much bread will supply 9450 men for $77\frac{1}{8}$ days?

1807. If 12 men can build a wall 30 ft. long, 6 ft. high, and 4 ft. thick, in 18 days of 10 hours each, how long will it take 36 men to build a wall 360 ft. long, 8 ft. high, and 6 ft. thick, working 6 hours per day?

1808. A ship's crew of 427 men had provision for 16 months at the rate of 27 oz. per day each. 7 months at sea, and in a storm 138 men were washed overboard and $\frac{1}{3}$ of the provision then on hand spoiled. What must be their daily allowance to continue the voyage 13 months?

1809. If a bar of iron $9\frac{1}{2}$ ft. long, 6 in. wide, and 4 in. deep, weighs $749\frac{3}{5}$ lbs., what will be the weight of an anvil $2\frac{1}{2}$ ft. long, $1\frac{3}{4}$ ft. wide, and 2 ft. deep?

Ans. to the last four, 37860.92.

1810. If 430 men agree to dig a canal 367 ft. long, 57 ft. wide, and 10 ft. deep, in 38 days, and find after digging 21 days that they have dug a canal 301 ft. long, 48 ft. wide, and 6 ft. deep, what number of men must they employ to finish it in the requisite time? Ans. 751.

1811. If a man can walk 378 miles in 9 days when the days are 11 hours long, how many days will it take him at the same rate to travel 478 miles when the days are 9 hours long?

1812. If 16 men build a wall 78 ft. long, 12 ft. high, 7 ft. thick, in 27 days, how many will it take to build a wall 378 ft. long, 19 ft. high, and 8 ft. thick, in 73 days?

1813.

Part 2. Conjoined Proportion or Chain Rule.

331. When the *antecedent* in one ratio *is equal* in value to the *consequent* in the next, it is called "Conjoined Proportion" or "Chain Rule."

332. The *correct method* of solving questions under this rule, is given under compound proportion, but the following rule is sometimes used.

PROB. XCI.—TO SOLVE QUESTIONS IN CONJOINED PROPORTION OR CHAIN RULE.

RULE.—*Taking the terms in pairs, place the 1st term as an antecedent and against it its equal, and in like manner arrange all the pairs. If the answer is to be of the same kind as the first term place the odd term under the antecedents, if not, place it under the consequents, then proceed to cancel, multiply, and divide.*

ILLUSTRATION.—If 6 lbs. of tea are worth 8 lbs. of coffee, and 10 lbs. of coffee are worth 32 lbs. of sugar, how many lbs. of sugar can be had for 18 lbs of tea?

The term of demand is 32 lbs. of sugar.

If 10 lbs. of coffee are worth 32 lbs. of sugar, "how much for 8 of coffee."

If 6 lbs. of tea are worth 32 lbs. of sugar, "how much sugar for 18 of tea."

By Compound Proportion.

$$\begin{array}{r|l} & 32 \\ 10 & 8 \\ \cancel{6} & \cancel{18}\ 3 \\ \hline \cancel{10} & 76.8 \text{ lbs.} \end{array}$$

By Conjoined Rule.

$$\begin{array}{lrl} \text{Tea, lbs.} & \cancel{6} = & 8 \text{ lbs. coffee.} \\ \text{Coffee, lbs.} & 10 = & 32 \text{ lbs. sugar.} \\ \text{Ans.} & = & \cancel{18} \text{ lbs. tea.} \\ \hline & \cancel{10}) & 76.8 \text{ lbs. Ans.} \end{array}$$

By the Chain Rule.

$$\begin{array}{rcl} \text{Ans.} & : & 32 \\ 10 & : & 8 \\ 6 & : & 18 \end{array}$$

It will be perceived that the solution is precisely the same in all cases, there being but a slight difference in the arrangement of the terms.

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331. When the antecedent in one ratio is equal to the consequent in the next, what is it called? On what principles are questions in this rule solved? Repeat Prob. and Rule LXXXIX. What is the difference between the solutions?

Illustration 2. — Suppose 100 lbs. of Venice equal to 70 lbs. of Lyons, 60 lbs. of Lyons to 50 lbs. of Rouen, 20 lbs. of Rouen to 25 lbs. of Toulouse, and 50 lbs. of Toulouse to 37 Geneva, then how many lbs. of Geneva are equal to 25 Venice?

Term of demand.

Geneva			37
Toulouse		~~50~~	~~25~~ 5
Rouen	2	~~20~~	~~50~~
Lyons	12	~~60~~	~~70~~ 7
Venice	4	~~100~~	~~25~~
		96	1295
		Ans. $13\frac{47}{96}$	

The Chain.

Ans. : 37
50 : 25
20 : 50
60 : 70
100 : 25
Ans. $13\frac{47}{96}$: 37
50 : &c.

It will be perceived that by dropping the column of consequents we have the numbers corresponding to the rule for Conjoined Proportion; thus:

The ratios are expressed by the antecedent in *one* line, and the consequent in the next, as 50 : 25.

Toulouse 50 = 37 Geneva.
Rouen 20 = 25 Toulouse.
Lyons 60 = 50 Rouen.
Venice 100 = 70 Lyons.
Ans. $13\frac{47}{96}$ = 25 Venice.

EXAMPLES.

1814. If 13 lbs. of figs are worth 34 lbs. of almonds, and 17 lbs. of almonds are worth 28 lbs. of raisins, and 35 lbs. of raisins are equal to 45 lbs. of dates, how many lbs. of dates are equal to 39 lbs. of figs?

1815. If 48 bbls. of flour are equal to 240 bushels of wheat, and 47 bu. of wheat equal 141 bu. of corn, and 7 bu. of corn are equal to 5 bu. of rye, and 11 bu. of rye to 34 bu. of oats, how many bbls. of flour are equal to 4827 bu. of oats?

1816. If 84 lbs. of fish are equal to 98 lbs. of beef, and 17 lbs. of beef to 21 veal, and 38 veal to 34 mutton, and 73 mutton to 87 pork, how many fish are equal to 6846 pork?

1817. What is the sum of the answers? Ans. 7161.19.

CHAPTER III.

PARTITIVE PROPORTION.

SECTION I.

PARTITIVE PROPORTION. EQUAL TIME.

333. Partitive Proportion consists in dividing a quantity into several parts, which shall have a given relation to each other.

NOTE.—This is commonly called *Fellowship* or *Partnership*.

PROB. XCII.—TO DETERMINE QUESTIONS IN PARTITIVE PROPORTION, EQUAL TIME.

RULE.—*Add together the several parts. Then, as the sum of these parts is to each individually, so is the quantity to be divided to its corresponding part.*

Or it may be expressed thus:—The whole stock : Each man's stock : : The entire profit : Each man's profit.

ILLUSTRATION.—A, B, and C purchased a lot of flour. A put in $700, B $800, and C $300. They sold it for $2000. What was each man's share of the proceeds?

OPERATION.

A's,	$700		$\cancel{2000}$ 10		$\cancel{2000}$ 10		$\cancel{2000}$ 10
B's,	$800	$\cancel{1800}$	700	$\cancel{1800}$	800	$\cancel{1800}$	300
C's,	$300	9	7000	9	8000	9	3000
	$1800	A's, $\$777.77\frac{7}{9}$		B's, $\$888.88\frac{8}{9}$		C's, $\$333.33\frac{3}{9}$	

$2000 Entire Profit.

EXAMPLES.

1818. A, B, and C went into partnership. A furnished $2350.00, B $3200.00, and C $1820. They lost $860. On dissolving partnership, what was each man's share?

1819. G, H, I, and L engaged in partnership. G put in $160, H $140, I $170, and L $370. They gained $3000. L was allowed $300 for keeping the books. What was each man's share?

1820. How much did A, C, G, I, and L have?

Ans. $3036.59.

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333. What is Partitive Proportion? Repeat Prob. XCII. Rule.

1821. D, E, and F engaged in trade. D put in 600 bbls. of flour, valued at $8.00 per bbl., but which *cost him* $7.00 per bbl.; E put 9000 bu. of wheat, valued at $1.60 per bu., but which cost him $1.75 per bu.; and F put in 10000 bu. of corn, valued at $.67 per bushel, which cost 63 cents. They sold out to a new firm for $30000. Did the company make or lose by the trade?

1822. What was D's gain or loss?

1823. What was E's gain or loss?

1824. What was F's gain or loss per cent. on the cash invested in buying his corn?

1825. Did D make as much *by the sale*, as he did *by forming the partnership*?

SECTION II.

PARTITIVE PROPORTION. UNEQUAL TIME.

334. It frequently happens, that a member of a firm wishes to withdraw his capital, in whole or in part, before the company wish to close the concern, or that the company wish to introduce new capital, either by admitting new members, or by allowing some one to increase his stock in trade.

335. This act brings in a new element into consideration, in questions of Partnership. For it is evident, that if A has in $400 for 1 month, and B $200 for 2 months, that the profits should be equal.

NOTE.—This is sometimes called Fellowship on time, Double Fellowship, or Compound Fellowship.

PROB. XCIII.—TO DETERMINE QUESTIONS IN PARTITIVE PROPORTION, UNEQUAL TIME.

RULE.—*Multiply each part by the time it is in trade, or the interest on each part may be taken. Then proceed with these products as in Prob. XCII.*

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334. What frequently happens in partnership? 335. What new element does this bring in? Repeat Prob. XCIII. Rule.

Note. — The method by interest, it will be perceived, differs from the first method only in multiplying by half the number of months, instead of the whole.

EXAMPLES.

1826. A, B, and C formed a partnership for one year. May 1st, A put in $600, B $400, and C $700. July 1st, $400 more was put in by A. Aug. 1st, B drew out $200. Oct. 1st, C drew out $500. Jan. 1st, B put in $700. At the end of the year the profits were $3000. What was each man's share of the profits?

Ans. A's, $1534.25.
B's, 794.52.
C's, 671.23.

1827. A and B formed a partnership for 2 years, with a capital of $6000.00. At the end of 15 months, B withdrew ½ of his capital. The profits were $4000, and B's share was $1550.00. How much did each put in?

Ans. A, $3373.34.
B, 2626.66.

1828. M, N, and P formed a company. M employed 6 hands at 90 cents a day, and labored with them 56 days. N employed 7 hands at $1.87½ per day, and labored with them 43 days. P employed 4 hands at $2.00 per day, and labored 17 days with them, but kept his hands 17 days longer. They agreed to allow themselves $2.50 per day. They gained $1800. What was each man's part of the proceeds, after paying his help?

Ans. M's, $254.94.
N's, 282.06.
P's, 124.23.

1829. G, H, and P hire a pasture for 9 months, and agree to pay $57.00 rent. G puts in 6 oxen for 4 months, then puts in 10 more for the remaining time; H puts in 22 oxen for 5 months; and P has in 11 oxen for the whole time. What ought each to pay?

SECTION III.

GENERAL AVERAGE.

336. General Average relates to the proper adjustment of the loss of property at sea, when a portion has been voluntarily destroyed to save the remainder.

337. *Jettison* is the property destroyed.

338. *Contributory Interests* are the *entire property* afloat, viz: *Ship and its furniture, cargo,* and *freight, after* having been freed from all charges.

339. In most ports, $\frac{1}{3}$ the freight money is deducted from freight to pay for seamen's wages, pilotage, and other small charges, before it goes into General Average. In New York, $\frac{1}{2}$ is deducted.

340. The cargo is valued at the *destined port* price. The property sacrificed *is included.* Storage and other necessary charges are deducted.

341. One third must be deducted from the value of new masts, rigging, &c., as they are estimated to be so much better.

PROB. XCIV.—TO APPORTION GENERAL AVERAGE.

RULE.—1. *Find the contributory interests by ascertaining the value of all property, after the various charges have been deducted. Then say,*

2. *Contributory Interests : To $1.00 : : Jettison : To the per cent. loss.*

NOTE.—It will be perceived that this rule is the same as for Partitive Proportion, equal time, Prob. XCII, p. 207.

ILLUSTRATION.—The ship Emma sailed for Boston May 6th, 1857, from Havre. In a storm she lost 2 masts, and all the furniture and cargo on deck. Off Provincetown fell in with steamer James Adger, and was towed to the wharf. Paid James Adger $378. Surveyors report her damage to be $1718. Pre-

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336. What is General Average? 337. Jettison? 338. Contributory Interests? 339. What is deducted? 340. How is the cargo valued? 341. What is deducted for new work? Repeat Prob. XCIV. Rule. Note.

mium for insurance, $408.37. Loss of furniture, $173.81. Cargo thrown overboard—

Hincks & Jenkins	$1784.51
Brown & Knight	943.71

Value of ship		$9000.00
Freight, gross amount	$2341.00	
Cargo shipped by Hincks & Jenkins		2983.51
Brown & Knight		7843.27
Atlas & Fugitt		978.42

What is the loss per cent., and what does each part pay?

OPERATION.

Statement of Loss.

Ship's damage, per surveyors' report	$1718.00
Cash paid James Adger	378.00
Hincks & Jenkins	1784.51
Brown & Knight	943.71
Ship furniture	173.81
Total Loss	$4998.03

Contributory Interests.

Value of ship	$9000.00	
Less for premium	480.37—	$8591.63
Freight, $2341, less $\frac{1}{3}$, 780.33\frac{1}{3}$		1560.66$\frac{2}{3}$
Hincks & Jenkins		2983.51
Brown & Knight		7843.27
Atlas & Fugitt		987.42
Contributory Interests		$21966.50

21966 50 : 1.00 : : 4998.03 : 22$\frac{3}{11}$+ per cent. loss.

The ship will then contribute $8591.63 × 22$\frac{3}{11}$ = Ans.

Freight contributes .... 1560.67 × 22$\frac{3}{11}$ = Ans.

Hincks & Jenkins contribute 2983.51 × 22$\frac{3}{11}$ = Ans.

Brown & Knight contribute 7843.27 × 22$\frac{3}{11}$ = Ans.

Atlas & Fugitt contribute... 987.42 × 22$\frac{3}{11}$ = Ans.

$4998.03

EXAMPLES.

1830. Steamer Acorn, on her passage from Havana to New York, grounded near the Narrows, lost an anchor, broke a windlass, and finally, by aid of the steam-tug Ajax, got off and came into port.

Statement of Loss.

Expense of steamer Ajax	$117.81
Freight lost by jettison	9.87
John Barstow's goods thrown overboard	1187.93
Anchor	178.41
Other damages, per rep. surveyor	218.96
Protest and other charges	68.43
Agents' commission	85.25
Total Loss	$1866.66

Contributory Interests.

Value of steamer in New York	$17843.50
Freight, $3387.68, less ½ =	1693.84
Cargo by Briggs & Allen	7684.16
Cargo by John Barstow	1187.93
Cargo by Jacob Benson	21486.79

What was the loss per cent.? How much does the ship, freight, and shippers pay of total loss, individually?

Ans.
Loss per cent. =
Ship pays =
Freight pays =
Briggs & Allen =
John Barstow =
Jacob Benson =

CHAPTER IV.

MEDIAL PROPORTION.

342. *Medial Proportion* treats of the relations of a *compound* and *its value*, to the *ingredients* of which it is composed and *their values*.

NOTE.—This is generally called Alligation Medial, and Alternate.

PROB. XCV.—CASE 1.—TO FIND THE PRICE OF A COMPOUND FROM THE SIMPLES OF WHICH IT IS COMPOSED, AND THEIR PRICES.

RULE.—*Divide the entire cost by the sum of the ingredients or articles.*

ILLUSTRATION.—I have 16 lbs. of sugar at 6 cents, 18 lbs. at 8 cents, and 26 lbs. at 10 cents. What is a compound of these worth per pound?

OPERATION.

16 lbs. at 6 cents = $ .96
18 lbs. at 8 cents = 1.44
26 lbs. at 10 cents = 2.60
60 lbs. are worth $5.00, and 500 ÷ 60 gives 1 lb. worth 8⅓ cents, Ans.

PROB. XCVI.—CASE 2.—THE PRICE OF THE INGREDIENTS GIVEN, TO FIND HOW MUCH OF EACH MUST BE TAKEN TO PRODUCE A COMPOUND OF A GIVEN VALUE.

RULE.—1. *Take one of each ingredient, and find the gain or loss.*

2. *Then take such a quantity of any ingredient as will make the gain and loss equal.*

ILLUSTRATION.—I have sugar at 5, 6, 7, and 11 cents per lb. I wish to mix these in such a manner that I may have it at 9 cts. per lb. What quantities of each kind must be taken?

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342. What is Medial Proportion? Note? Repeat Prob. XCV. Rule. Prob. XCVI. Rule. Give an illustration of each.

If I take 1 lb. at 5, I gain	4 cents.	
" 1 lb. at 6, "	3 "	
" 1 lb. at 7, "	2 "	
" 1 lb. at 11, I lose		2 cents.
Total	Gain, 9 cents.	Loss, 2 cents.
If I take 3½ lbs. at 11, I lose		7 cents.

Which makes the gain 9 cents, and the loss 9 cents.

Ans. 1 lb. at 5, 1 lb. at 6, 1 lb. at 7, and 4½ at 11 cts.

NOTE.—It will be perceived that an innumerable number of answers may be obtained.

PROB. XCVII.—CASE 3.—THE PRICES OF ALL, AND THE QUANTITY OF PART OF THE INGREDIENTS, GIVEN, TO FIND THE QUANTITY OF THE REMAINING INGREDIENTS, TO PRODUCE A COMPOUND OF A GIVEN VALUE.

RULE.—1. *Find the gain or loss on the given quantities, and on one pound or part of each of the remaining ingredients.*

2. *Then take a sufficient quantity of one or more of these ingredients to make the total gain and loss equal.*

ILLUSTRATION.—I have coffee worth 6, 8, 9, 11, and 13 cents per lb. I wish to make a mixture worth 10 cts. per lb., taking 3 at 6 cts., and 5 at 13 cts. How much of the others must I take?

OPERATION.

	Gain.	Loss.
If I take 3 lbs. at 6 cents, I gain	12 cents.	
" 5 lbs. at 13 " I lose		15 cts.
" 1 lb. at 8 " I gain	2 cents.	
" 1 lb. at 9 " I gain	1 cent.	
" 1 lb. at 11 " I lose		1 ct.
Total	Gain, 15 cents.	Loss, 16 cts.

Now take 1 lb. at 9 cents, and gain 1 cent, and it makes the total gain 16 = 16, the loss.

Ans. 1 lb. at 8 cts., 2 lbs. at 9, and 1 at 11 cts.

Repeat Prob. XCVII. What is 1st part of Rule? 2d? Repeat entire Rule. Give an illustration?

PROB. XCVIII. — CASE 4. — THE PRICES OF THE COMPOUND AND THE INGREDIENTS, AND THE QUANTITY OF THE COMPOUND, GIVEN, TO FIND THE QUANTITY OF THE INGREDIENTS.

RULE.—1. *Find the proportion of each ingredient by Case 2.*

2. *Then divide the quantity of the compound by the sum of the ingredients found, and multiply this quotient by the quantity of each.*

ILLUSTRATION. — I have an order for a bbl. of sugar, to contain 210 lbs., at 10 cts. per lb. My sugars are worth 6, 7, 9, 12, and 13 cts. per lb. How shall I mix them to fill the order?

OPERATION.

	Gain.	Loss.
If I take 1 lb. at 6 cents, I gain	4 cents.	
" 1 lb. at 7 " "	3 "	
" 1 lb. at 9 " "	1 cent.	
" 1 lb. at 12 " I lose		2 cents.
Now take 2 lbs. at 12 " "		6 "
This makes the total gain	8 cents =	8 cts. loss.

Then 210 ÷ 6 = 35.

35 × 1 = 35 lbs. at 6, worth $2.10
35 × 1 = 35 lbs. at 7, " 2.45
35 × 1 = 35 lbs. at 9, " 3.15
35 × 1 = 35 lbs. at 12, " 4.20
35 × 2 = 70 lbs. at 13, " 9.10
Number of lbs. 210, and at 10 cts. = $21.00 Ans.

EXAMPLES.

1831. A drover bought 870 lambs at 75 cents apiece, and 290 sheep at $1.25 apiece. At what price per head must he sell the lot to gain $50.00?

1832. If 20 bushels of wheat, at $1.35 per bushel, be mixed with 15 bushels of rye, at $.85, what is a bushel of the mixture worth?

1833. How much wine at 5 shillings and 3 shillings per gallon must be mixed with wine at 8 shillings per gallon, to make the mixture worth 6 shillings per gallon?

Ans. { 1 gallon at 5 shillings.
1 " at 3 "
2 " at 8 "

1834. How much brandy at 60, 70, 75, and 80 cents per gallon must be mixed with water, to make the compound worth 50 cents per gallon?

1835. A person wishes 15 bushels of wheat, at $1.70, mixed with rye at $.80, 30 bushels of corn, at 95 cents, with barley at 42, and oats at 60, so that a bushel of the mixture may be worth 87 cents. How much of each must be taken?

Ans. { 9 bu. of rye.
31 bu. of barley.
1 bu. of oats.

1836. How much gold at 15, 17, 18, and 22 carats fine, must be mixed together to form a composition of 4 ounces 20 carats fine?

1837. How much water must be mixed with wine, at 9 shillings per gallon, to make 126 gallons at 7 shillings per gallon?

1838. I have a bin containing 46 bushels, and have apples worth 80, 90, 105, and 125 cents per bushel. How many bushels of each kind must I take to fill the bin for $16.00?

1839. A merchant wishes to fill a cask with 215 pounds of raisins, at 13 cents per pound. His raisins are worth 7, 9, 10, 11, 15, 17, and 22 cents per pound. How much of each sort would he take?

1840. How much wine at 15, 17, 18, 36, and 42 shillings per gallon, will fill a pipe of 256 gallons, at 26 shillings per gallon?

1841. Luther Fitch has 40 gallons of wine at $.75, and 26 gallons at $1.37½, that he wishes to mix with wines at $1.75, $2.25, $2.50, and $3.00, to make the mixture worth $2.00 per gallon. How much of each kind must he take?

CHAPTER V.

EQUATED PROPORTION.

343. Equation of Payments is the method of finding the average date for the payment of several sums, due at different periods of time.

Prob. XCIX.—TO FIND THE AVERAGE DATE FOR THE PAYMENT OF SEVERAL SUMS, DUE AT DIFFERENT PERIODS.

RULE.—1. *Compute the interest on each sum, from the 1st day of any month, previous to the time of the 1st payment, to the time it is due.*

2. *Multiply the sum of these interests, in mills, by 6, and divide the product by the amount due. (See Art. 282.)*

3. *The quotient will be the number of days from the assumed date to the date for the average time for payment.*

ILLUSTRATION 1.—What is the *average date* of the following account?

All Cash Bills, or on Equal Time.

Jacob Emmerson. *Dr.*

1860.	September 1st.	Amt. of Bill.	Int. 6 per cent.
0 mo. Sept. 5th.	To Merchandize	$450.00	$ .375
1 mo. Oct. 12th.	" "	687.30	4.811
2 mos. Nov. 15th.	" "	37.85	.479
2 mos. Nov. 20th.	" "	276.21	3.729
3 mos. Dec. 12th.	" "	308.00	5.275
		$1759.36	$14.669

14669. × 6 ÷ 1759.36 = 49.99 days = 50 days from the assumed date, Sept. 1st, to average date, Oct. 20th.

Ans. Oct. 20th, 1860.

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343. What is Equation of Payments? What is Proportion? Equated Proportion? How do you compute interest? What is Problem XCIX? Rule? Why multiply the interest, in mills, by 6? What is meant by average date?

Remark.—It will be seen in this illustration, that, by assuming the 1st of the month as a date from which to reckon, and by writing the number of months from the assumed date in the margin against each month, the time is approximately given by the date. Then correct by adding 1 day for any intervening month of 31 days, or subtracting for February, and the true time in days is found: thus, the time for Nov. 15th is 2 mos. 16 days.

Illustration 2.—What is the *average due* of the following account?

Cash and Unequal Time Bills.

Henry Case. *Dr.*

1862.	October 1st.	Amt. of Bills.	Int. 6 per cent
0 mo. Oct. 20th.	To Merchandize, 6 mos....	$468.91	$15.708
1 mo. Nov. 6th.	“ “ 4 mos....	87.68	2.294
2 mos. Dec. 8th.	“ “ cash......	487.31	5.604
3 mos. Jan. 12th.	“ “ 7 mos....	583.94	30.748
4 mos. Feb. 9th.	“ “ 2 mos....	435.87	13.871
5 mos. Mar. 13th.	“ “ cash......	267.14	7.301
6 mos. April 21st.	“ “ 4 mos....	873.46	47.312
		$3204.31	$122.838

$122838. \times 6 \div 3204.31 = 230.011$ days $= 230$ days from the assumed date, Oct. 1st, to the average due.

Ans. May 18th, 1863.

Remark.—In examples of this class, interest must be computed to maturities.

Illustration 3.—What is the average date and due of the following bill?

In this third class of accounts, by dropping the time on which they were purchased, and calculating as in Ill. 1, it gives the cash bills equal time with the others, whereas they had no time. Hence, the interest for the time on which the goods were purchased, on cash bills, must be subtracted from the total interest. Then proceed as before.

Part Cash and part on Equal Time.

James Allen. *Dr.*

1857.		April 1st.		Amt. of Bills.	Int. 6 per cent.
0 mo.	April 15th.	To Mdze., 6 mos.		$418.34	$1.045
2 mos.	June 6th.	" cash...	$9.369	312.31	3.488
3 mos.	July 12th.	" 6 mos.		891.42	15.302
3 mos.	July 15th.	" cash...	30.372	1012.41	17.886
4 mos.	Aug. 5th.	" cash...	14.305	476.83	10.093
5 mos.	Sept. 4th.	" 6 mos.		842.31	22.040
			$54.046	$3953.62	$69.854
					54.046
					$15.808

15808. × 6 ÷ 3953.62 = 23.9 days = 24 days from the assumed date. Ans. Apr. 25th, 1857, and

The average due will be 6 months after, or Oct. 25th, 1857.

REMARK. — When the cash interests *exceed* the credit interests, proceed the same with their difference, *but count the time backward instead of forward.*

344. To find the average date or due of an account where there has been an exchange of goods, proceed in the same manner as above with each side, separately. Then, if the amount and interest of one side exceed the other, find their differences, and proceed with them. But if the interest of the smaller side be larger than the interest of the larger side, find the difference, and proceed as before, but count the time backward from your assumed date.

The average of storage is found by the same method, by computing interest on the several amounts stored. The computer is again referred to the method of getting the time. See Remark, p. 218.

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344 How find the average date or due where there has been an exchange of goods? How find the average of storage? What is meant by counting the time backward?

EXAMPLES.

1842. What is the equated time for the payment of **$4000**, due as follows: **$200**, Mar. 5th; **$700**, Apr. 3d; **$750**, May 17th; **$1250**, June 23d; **$980**, Aug. 17th; and the balance Sept. 9th, 1856? Ans. 104 days from Mar. 1st.

1843. What is the average time for the payment of **$578.64**, due as follows: **$210**, Aug. 5th, 1856; **$230**, Sept. 4th, 1856; and the balance Dec. 1st, 1857?

1844. What is the average time for the payment of a sum of money due as follows, viz: $\frac{1}{2}$ on May 3d; $\frac{1}{6}$, July 3d; $\frac{5}{24}$, Aug. 17th; and $\frac{1}{8}$, Sept. 5th?

1845. Bought a farm for **$7000**, and agreed to pay for it in semi-annual instalments of **$500** each. After paying 5 instalments, I wish to pay the balance at once. What is the average time for payment?

1846. I owe Batchelder **$500**, due to-day, May 1st; **$700**, June 15th; **$430**, Aug. 11th; and **$280**, Sept. 23d. When is the average due?

1847. A owes B **$1500**, to be paid in 9 months, but A offers to pay him **$1000** in 3 months, provided the remainder may remain till the average due. In how many months should it be paid?

1848. A debt of **$4875** is to be paid in 11 months, but the debtor agrees to pay **$3500** down. When should the balance be paid?

1849. What is the average date of the following account?

	James Asper.	*Dr.*
1858, Mar. 3.	To Merchandise, 6 mos.	$4267.91
April 7.	" " 3 mos.	2567.23
June 12.	" " cash	1856.71
Aug. 15.	" " cash	5782.31

1850. When is the average due of the last example?

1851. When is the average due of the following bill?

1859. *Jacob Emmerson in account with David Clark.*

Jan. 11.	To Merchandise	$476.00	Mar. 5.	Cash..	$850.00
Jan. 24.	“ “	347.00	July 17.	“	900.00
Mar. 17.	“ “	851.00	July 20.	“	1100.00
July 1.	“ “	729.00			
Aug. 3.	“ “	937.00			

1852. What is the average date of the above example?

1853. When is the average due of R. P. Cooper's account with C. W. Upham, 1860, as follows?

April 7.	To Mds., 6 mos.	$648.71	Feb. 1.	By Mds., 6 mos.	$1841.23
May 3.	“ 3 mos.	537.82	June 7.	“ cash..	1271.33
June 5.	“ cash...	793.41	Oct. 3.	“ 8 mos.	1941.62
July 17.	“ 7 mos.	957.31			
Aug. 29.	“ cash...	1321.17			
Sept. 1.	“ 2 mos.	335.17			

1854. When is the average due of Lyman Cobb's account with Daniel French, 1864, as follows?

Mar. 17.	To Mds., 6 mos.	$458.62	Apr. 17.	By Cash.........	$300.00
May 19.	“ “	398.41	Aug. 25.	“	700.00
June 25.	“ “	682.31	Nov. 30.	“	900.00
Aug. 27.	“ “	846.51	Dec. 1.	“	100.00
Nov. 30.	“ “	436.87	Dec. 15.	“	700.00
Dec. 31.	“ “	984.26			

1855. What is the storage of flour at 5 cents per barrel per month, received and delivered as follows?

Received.		*Delivered.*	
1858, May 17.	6000 bbls.	1858, Dec. 1.	2687
June 12.	4286 “	1859, Jan. 3.	6289
Aug. 30.	3287 “	Mar. 12.	Balance.

REVIEW OF PARTS I, II, III, IV.

MISCELLANEOUS QUESTIONS.

1856. A certain book contains 356 pages, and has 33 lines on a page. How many letters in the book, if there are 64 in each line?

1857. James Allen expended $\frac{1}{5}$ of his salary in travelling, $\frac{1}{8}$ of the remainder in amusement, and $\frac{1}{7}$ of the residue he placed at interest, when he found he had $658.37 left. What did he place at interest?

1858. How much spend for amusement?

1859. How much for travelling?

1860. What was his salary?

1861. Samuel Brown is now taking Harper's Monthly at .25; National Intelligencer, daily, at .50 per month; The States, daily, at $6\frac{1}{4}$ cents per week; also the Evangelist, a weekly paper, at $2.00 per year. He also adds about $27.00 worth of books yearly to his library. How much money at interest will pay for the above annual expenses?

1862. Thomas Bowie smokes 4 cigars every day at 5 cents apiece. How much money at interest will pay his smoking bills? Sum of the above answers? Ans. 755422.10.

1863. A man wishes to make a coal bin 6 feet long, 5 feet deep, inside measure. How wide must it be to contain 12 tons of hard coal?

1864. A cloak contains $5\frac{5}{8}$ yards of broad cloth $2\frac{1}{4}$ yards wide. How much flannel that is $1\frac{1}{8}$ yards wide will it take to line it?

Ans. $11\frac{1}{4}$ yards.

1865. What is the value of $5\frac{7\frac{1}{2}}{3\frac{7}{8}} - 4\frac{\frac{3}{5}}{7\frac{1}{2}}$? Ans. $2\frac{663}{275}$.

1866. How many bushels of wheat must a person take to mill, to obtain 6000 lbs. of flour; the miller taking $\frac{1}{16}$ for toll, and $\frac{1}{10}$ being bran? Ans. $118\frac{14}{27}$ bu.

1867. Multiply 768256789 by 256128643284.

1868. James Sprague bought 6 bbls. of sugar, containing $1357\frac{1}{2}$ lbs., at $9\frac{1}{2}$ cents per pound. He paid .$37\frac{1}{2}$ cartage, .$56\frac{1}{4}$ freight, and \$$1.87\frac{1}{2}$ for hauling to his store. On the way one barrel burst, and he estimated he lost 25 lbs. He is to sell this sugar at retail, and calculates a waste of 12 lbs. in weighing. He has spent 1 day in purchasing, and it will take $\frac{2}{3}$ of a day to weigh the sugar out. His time is worth \$$1\frac{3}{4}$ per day. What must he charge per pound to make $14\frac{1}{2}$ per cent. on his money?

Ans. 12 cents, nearly.

1869. James Allen has a farm containing 347 A. 3 R. He gave $\frac{3}{5}$ to his son John, and $\frac{3}{7}$ of the remainder to his daughter Lucy. What was the share, each, of the father, son, and daughter?

Ans. { Father's, 79 A. 1 R. 37.71392 P.
Daughter's, 59 " 2 " 18.28544 "
Son's, 208 " 2 " 23.99904 "

1870. The daughter's part being near the city, she sold $\frac{3}{5}$ of it for $15\frac{1}{2}$ cents per square foot, $\frac{2}{7}$ at $23\frac{1}{2}$ cents, and the remainder, which was worth $53\frac{1}{4}$ cents per square foot, she kept and built a house on it for her own use for \$40000.00. The balance of her money she put at interest, at 6 per cent. What was her annual income?

1871. What would her tax be at 5 mills on a dollar?

1872. The son sold $\frac{1}{2}$ of his land for $17\frac{1}{2}$ cents per square foot, and built him a house that cost $\frac{1}{2}$ as much as he got for his land. Reserving \$760.00 he invested the remainder at 6 per cent. He spent 30 cents per day for cigars, and 27 for drink, took three daily political papers, at \$8.00 per year each, and paid a weekly tax to his political club of \$$1.12\frac{1}{2}$. What would be his annual income?

1873. Sold James Budgin $827\frac{1}{3}$ bu. of wheat at \$$1.37\frac{1}{2}$ per bushel, taking his note for 6 months, payable at Bank of Old Dominion, Alexandria, without interest. What was the cash price per bushel?

1874. Suppose a man had deposited $7786 in Suffolk Bank, Boston, at 6 per cent. interest, the day before the battle of Bunker Hill, at Charlestown, and withdrew it the day after the surrender of Cornwallis, at Yorktown, how long was it on deposit?

1875. What did it amount to?

1876. If $17\frac{1}{5}$ barrels of flour cost 171.66\frac{2}{3}$, how much will $9\frac{3}{4}$ barrels cost?

1877. What is the interest of $768.42 from August 27th, 1856, to March 1st, 1857, at 7 per cent.?

1878. A man hired $769.30 at 6 per cent., *at the Bank*, and purchased flour at 9\frac{1}{8}$ per barrel. After paying $.50 per bbl. expenses, he sold the flour for 11\frac{1}{2}$ per bbl., and paid his note at the Bank at the end of 3 months. How much did he make by the transaction? Sum of last 4 answers, 3232.37.

"*New York, July 27th, P. M.*

1879. "It is stated that the stock of sugar on hand at this port is 115000 hogsheads" (average weight of each 1342 lbs.), "and the decline in price on this stock is fully $1\frac{1}{2}$ cents per pound." What is the total loss?

1880. What is the per cent. loss, the cost being $11\frac{1}{2}$ cents?

1881. "The import of sugar," says the same paper (New York Evening Post of July 27), "for 1857, first 6 months, exceeds the import for the same period in 1856 by 16500000 lbs." What is that worth at 231\frac{3}{4}$ per ton?

Sum of last 3 answers, 4226787.63.

"*New York, July 27th,* 1857.

1882. "Recent importations from the West Indies, costing $11\frac{1}{2}$ cents, have sold in lots at $9\frac{1}{2}$ cents." What was the per cent. loss? Ans. $17\frac{9}{23}$ per cent.

1883. June 20th, corn was quoted at 91 cents per bushel. July 20th, it was quoted at 86 cents. What was the per cent. loss, *allowing interest?* Ans. 6 per cent. nearly.

1884. Received $36 dividend for 6 months on 12 shares of Balt. & Ohio R. R. stock, which cost me $60. What was the per cent. income? Ans. 10 per cent.

1885. Rufus Chamberlain owns a house that cost him $6700. He rents this house to a good tenant at 10 per cent., payable quarterly. The wear of the house is fully $3\frac{1}{4}$ per cent. Insurance costs him $2\frac{1}{8}$ per cent. Taxes, $7\frac{1}{2}$ mills on a dollar. If he had $6700 in cash he could loan it at 7 per cent. on real estate, interest payable quarterly, without taxes. What would the difference between these two modes of investment amount to in 5 years, supposing the income to be without interest?

1886. John Wright lent $450 to Reuben Whitney for 6 years. On the note he finds he has received $75 at the end of each year for the first 5 years. What is due at the end of the time?

1887. Israel Packard gave his note for $500, payable in 4 years, without interest. What sum would pay it in 1 year and 6 months from the date of the note?

Sum of the above answers, 1671.84.

1888. Richard Blackford owes 4 notes to Abel Kingham, all dated June 12th, 1856. One of $376.00, is payable Oct. 31st, 1856; the second, of $537.82, is payable Jan. 13th, 1857; the third, of $943.31, is payable Mar. 21st, 1857; and the last is for $946.83, payable Nov. 6th, 1857. All without interest till due. What sum of money would pay the whole July 4th, 1856?

Ans. $2671.19.

1889. A. can do a certain job of work in 3 days, and B. can do it in 4 days. How long would it take both to do it, A. working one half of the time? Ans. $2\frac{2}{5}$ days.

1890. A cistern holding 1250 gallons has two pipes. One fills 28 gallons in an hour, and the other empties 100 gallons in $1\frac{1}{2}$ hours. How long would it take to empty the full cistern, if both pipes are discharging? Ans. $32\frac{19}{58}$ hours.

1891. A vessel has 4 receiving pipes, which will fill it as follows: A. B. C. in 6 hours, B. C. D. in 8, C. D. A. in 10, and D. A. B. in 12 hours. It has also 4 discharging pipes, which will empty it as follows: E. F. G. in 7 hours, F. G. H. in 6, G. H. E. in 5, and E. F. H. in 4 hours. Suppose all the pipes are opened at the same time, how long will it take to discharge the full vessel? Ans. $10\frac{130}{239}$ hours.

1892. At what time will the hour and minute hands be together between the hours of 7 and 8 o'clock?

Ans. 7 h. 38 m. $10\frac{10}{11}$ sec.

1893. *What two numbers are there* of which $\frac{3}{5}$ of one shall exceed $\frac{5}{7}$ of another by $96\frac{1}{2}$? *Why will this question admit of many answers?*

1894. Isaac Brown plowed 34 A. in 4 days with 26 oxen. How many days will be required to plow 51 A. with 18 oxen?

Ans. $8\frac{2}{3}$ days.

1895. If 56 men eat 82 pounds of bread in 3 days, how many pounds of bread will be sufficient for a garrison of 1376 for 42 days?

1896. If $\frac{7}{9}$ of a ton of hay costs $5.21, what will $7\frac{8}{19}$ tons cost? Sum of last 2 answers, 28258.76.

1897. A man can mow $\frac{3}{7}$ of a meadow in $5\frac{1}{2}$ days, and a boy can mow $\frac{3}{13}$ of the same lot in $6\frac{1}{3}$ days. How long will it take both to mow it? Ans. $8\frac{1619}{2175}$.

1898. I have sugar at $3\frac{1}{2}$ cents, 6, 7, 9, $10\frac{1}{2}$, 11, and 13 cents. How shall I mix them that I may sell the compound at 9 cents per pound, and make a profit of $12\frac{1}{2}$ per cent.?

1899. A grocer has 41 pounds of unsaleable tea, which cost 42 cents per pound. *He determines to sell it*, and to accomplish this he mixes it with other teas at 55, 65, and 67 cents per pound. How much of each quantity must he take to sell the mixture at 58 cents?

1900. A wine merchant has wine that cost him $3.84 per gallon. How much water must he put in so that he can sell it for $3.00 per gallon, and make a profit of 15 per cent.?

Ans. $\frac{17}{221}$.

1901. Asa White has wine that cost him $3.94 per gallon. What must be his asking price that he may fall 10 per cent., yet make a profit of 15 per cent., he having added $12\frac{1}{2}$ per cent. of water? Ans. $4.47 per gallon.

Analysis of Part V. Involution, Evolution, and Series.

DIVISIONS. CHAPTERS. SECTIONS.

- I. INVOLUTION AND EVOLUTION.........
  - I. INVOLUTION 228
  - II. EVOLUTION ....
    - 1. Square Root.................. 230
    - 2. Mensuration of Surfaces... 237
    - 3. Cube Root...................... 242
    - 4. Mensuration of Solids 248
- II. SERIES ....
  - I. ARITHMETICAL PROGRESSION.............................. 252
  - II. GEOMETRICAL PROGRESSION 255
  - III. ANNUITIES.. 257
  - IV. LOAN AND FUND ASSOCIATIONS, OR BUILDING ASSOCIATIONS, 259

REVIEW OF PARTS I. II. III. IV. AND V. 263

PART V.

INVOLUTION, EVOLUTION, AND SERIES.

DIVISION I.

INVOLUTION AND EVOLUTION.

CHAPTER I.

INVOLUTION.

345. *Involution* teaches the method of raising a number to any required power.

346. The *Root* is the number multiplied by itself.

347. The *Power* is the product of a number multiplied by itself any number of times.

348. The different powers take their names from the number of times *the root* is used as a factor in producing them.

349. The *index* or *exponent* is a little figure at the right of the root, denoting its power; thus,

$3^1 = 3$, the 1st Power or Root.
$3^2 = 9$, the 2d Power or square of 3.
$3^3 = 27$, the 3d Power or cube of 3.
$3^4 = 81$, the 4th Power or squared square of 3.
$3^5 = 243$, the 5th Power or 1st sursolid of 3.
$3^6 = 729$, the 6th Power or cubed square of 3.
$3^7 = 2187$, the 7th Power or 2d sursolid of 3, &c. &c.

PROB. XCIX.—TO FIND ANY POWER OF A NUMBER, WHOLE OR FRACTIONAL.

RULE.—*Take the root as a factor, as many times as the index or power denotes, and find the product.*

Give an analysis of Part V? 345. What is Involution? 346. What is the Root of a number? 347. The Power of a number? 348. What names are given to different powers? 349. What is an exponent? Repeat Prob. XCIX. Rule.

EXAMPLES

1902. What is the cube of 11?
1903. What is the square of 17?
1904. What is the 4th power of 9?
1905. What is the 5th power of 5?
1906. What is the 2d power of $2\frac{1}{2}$?
1907. What is the 3d power of 31?
1908. What is the 4th power of $3\frac{1}{8}$?
1909. What is the sum of the last 7 examples?

Ans. 41198.61743

1910. What is the 5th power of $7\frac{1}{4}$?
1911. What is the 3d power of 281?
1912. What is the 5th power of 17?
1913. What is the 4th power of $4\frac{1}{2}$?
1914. What is the 12th power of 3?
1915. What is the 7th power of $\frac{7}{8}$?
1916. What is the 5th power of 5 6′?
1917. What is the 3d power of .57?
1918. What is the 8th power of .7?
1919. What is the 6th power of .91?
1920. What is the 5th power of $5\frac{1}{8}$?
1921. What is the 3d power of $11\frac{1}{6}$?
1922. What is the 5th power of $3\frac{1}{2}$?
1923. What is the 9th power of $15\frac{1}{8}$?
1924. What is the 7th power of $3\frac{2}{5}$?
1925. What is the difference between the square of 31, and 4 times the square of $15\frac{1}{2}$? Ans. Equal.

350. Table of Powers.

1st Powers	1	2	3	4	5	6	7	8	9	10
2d Powers	1	4	9	16	25	36	49	64	81	100
3d Powers	1	8	27	64	125	216	343	512	729	1000
4th Powers	1	16	81	256	625	1296	2401	4096	6561	10000

Let this table be committed to memory.

CHAPTER II.

EVOLUTION.

351. Evolution is the reverse of Involution, or the process of finding the root from any given power.

352. Two roots only are used in the common transactions of life, viz., the square and cube roots. Other roots are treated in higher mathematics.

SECTION I.

SQUARE ROOT.

353. Square Root is the process of determining the root from the second power, and is frequently expressed by a sign, thus: $\sqrt{}$, or the fractional index $\frac{1}{2}$.

354. *The following principles are very important, and the application of them should be thoroughly understood before any attempt to extract square root.*

355. PRIN. 1.—THE SQUARE OF THE GREATEST NUMBER EXPRESSED BY A SINGLE FIGURE, CANNOT CONTAIN MORE THAN TWO FIGURES, AND GENERALLY, THE SQUARE OF ANY NUMBER, CANNOT OCCUPY MORE THAN DOUBLE THE NUMBER OF PLACES OCCUPIED BY ITS ROOT. IT MAY OCCUPY ONE LESS.

ILLUSTRATION.—Root 9. Square 81.
" 99. " 9801.

INF.—The root of 7853 will be more than 10 and less than 100. The root of 785634 must contain three figures. Hence, if we mark off a number into periods of two figures each, every period will contain one figure of the root.

---

351. What is Evolution? 352. What roots are in common use? 353. Square Root? Repeat Art. 354. Repeat it again. Do you understand its language? Repeat Prin. 1. Illustration. Inf

356. PRIN. 2.—**THE SQUARE OF THE SUM OF TWO NUMBERS, IS EQUAL TO THE SQUARE OF THE FIRST, PLUS TWICE THE PRODUCT OF THE FIRST BY THE SECOND, PLUS THE SQUARE OF THE SECOND.**

NOTE.—This principle demands close attention.

ILLUSTRATION.—What is the square of 27?

27 is equal to...........................	20 + 7
Multiply this by itself..............	20 + 7
Product by 7	140 + 49
Product by 20	400 + 140
Square, or 2d Power.......	400 + 280 + 49

Thus far is *involution*, and as evolution is the reverse, we *must undo* what has been done, and find the root from the square 400 + 280 + 49.

We find that the square root of 400 is 20, and the square root of 49 is 7, which gives 20 + 7 = 27, Ans.

Had the numbers been added together before extracting the root, the process, although nearly the same, would not be so readily perceived.

Thus 400 + 280 + 49 = 729.

By referring to Prin. 1, we point off this number into periods of two figures each, and at once see that the root of the tens period must be 2 tens, or 20. By squaring 20, and subtracting the result, 400, we have a remainder, 329 = 280 + 49. By referring to the process of involution we find the second number of the square, 280, to be composed of twice the product of the first number, 20, by the second, 7. Therefore, as we have the first number of the root, 20, twice that number must be one factor of 280. Hence, we use 40 as part of the divisor of the remainder, 329.

OPERATION.

```
     729(20
     400  7
  47)329 27
     329
Ans.  27
```

What is Involution? Evolution? Why is involution used in illustrating square root? What is Prin. 1? Why refer to it? Why take twice 20 for a partial divisor? How do you complete the divisor?

But as 329 is equal to 288 + 49, we must add to this divisor, 40, one factor of 49. This factor is found by *trial* with that part of the divisor already found, viz., 40. We then say 40 is contained in 329 7 times (allowing 1, as our number, 40, is only part of the divisor), and add this 7 to the 40, making a full divisor 47. With this divide as in the operation above.

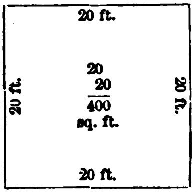
This principle admits of a Geometrical illustration, as well as the Algebraical above.

GEOMETRICAL ILLUSTRATION OF SQUARE ROOT.

ILLUSTRATION.—Suppose we have 729 blocks, each one foot square, and wish to arrange them into a square. How many will there be on each side?

From Prin. 1, as before, we perceive that the root will consist of two figures, as there are two periods; also that the root of the second period is 2 tens, or 20.

FIG. 1



Then draw a diagram representing a square, with 20 blocks on each side.

This has taken 400 blocks. (See Fig. 1.) The question now is, how we can add on the remaining number of blocks, and keep the figure in a square form.

This may be done in two ways. 1st, by adding on to two sides, as in Fig. 2, and, 2d, by adding on to 4 sides, as in Fig. 3. The 1st method is generally preferred. The 2d may be used.

FIG. 2.

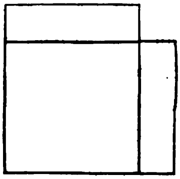
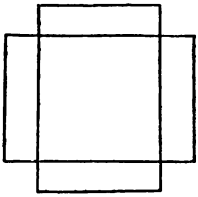


FIG. 3.



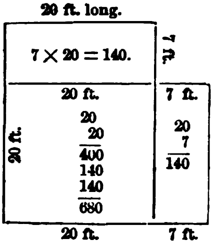
As the root already found is the length of one side of the large square, it will take as many blocks as double the root to make one addition, by the 1st method.

Therefore double the root already found will be the divisor to determine how many additions may be made. By dividing the 329 remaining blocks by 40 (twice the root, 20), we find that 7 additions may be made, and 49 blocks still left.

OPERATION.

40)329(7
280
49

Fig. 4.



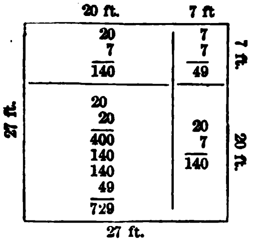
The following represents the present condition of the blocks. (See Fig. 4.)

The corner not yet filled must be determined by the width of the addition.

In this instance it is 7 feet each way, making 7 × 7 = 49 blocks.

The entire square may be represented by Fig. 5. The numerical calculation by the following

Fig. 5.



OPERATION.

729 (20
400 \ 7
40)329 27 Ans.
280
49
49

This work may be abridged in two particulars: 1st, by rejecting the ciphers; and 2d, by having but one subtrahend. See process in margin.

Abridged Process.

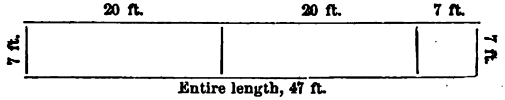
729(27 Ans.
4
47)329
329

The composition of the divisor may be more distinctly seen by reference to Fig. 6, and remembering that the divisor, in all cases, is the *entire length* of the addition, and that the number found by the partial divisor is the width of the addition.

In this case we had 2 blocks, each 20 feet long, and one 7 feet long, making the entire addition 47 feet long, and 7 feet wide.

The *surface of this addition is the subtrahend.*

Fig. 6.



NOTE.—*These two methods of illustrating square root should be carefully compared, in every particular, before proceeding further.*

357. PRIN. 3.—THE SQUARE OF THE DIFFERENCE OF TWO NUMBERS, IS EQUAL TO THE SQUARE OF THE FIRST, MINUS TWICE THE PRODUCT OF THE FIRST BY THE SECOND, PLUS THE SQUARE OF THE SECOND.

ILLUSTRATION 1.—Required the square of 27

$27 = 30 - 3$. The square of 27 then is $900 - 2 \times 90 + 9 = 729$ Ans.

ILLUSTRATION 2.—Required the square of 43.

$43 = 50 - 7$. $43^2 = 2500 - 2 \times 350 + 49 = 1849$. Or by Prin. 2: $43 = 40 + 3$. $43^2 = 1600 + 2 \times 120 + 9 = 1849$.

358. PRIN. 4.—THE PRODUCT OF THE SUM AND DIFFERENCE OF TWO QUANTITIES, IS EQUAL TO THE DIFFERENCE OF THEIR SQUARES.

ILLUSTRATION.—

$$\begin{aligned} 11 + 7 &= 10. \\ \underline{11 - 7} &= \underline{\ 4\ } \\ 121 - 49 &= 72. \text{ Ans.} \end{aligned}$$

359. PRIN. 5.—EVERY NUMBER MAY BE CONSIDERED AS BEING THE SUM, OR THE DIFFERENCE OF TWO OTHER NUMBERS.

Repeat Principle 1, 2, 3, 4, 5. Give an illustration of each. Why does the length of the addition form the divisor? Why does the surface form *the subtrahend?*

PROB. C.—TO EXTRACT THE SQUARE ROOT OF ANY NUMBER.

RULE.—1. *Point off the number into periods of two figures each, commencing at the decimal point.*

2. *Subtract the greatest possible square from the left hand period, writing its root in the quotient.*

3. *Annex the second period to the remainder for the 1st dividend.*

4. *Take twice the root already found for a partial divisor.*

5. *Find how many times this divisor is contained in the dividend, and annex it to the quotient and the divisor.*

6. *Multiply the divisor thus completed by the number last placed in the quotient, and subtract the product from the dividend.*

7. *Annex the third period to the remainder.*

8. *Take twice the quotient for a second partial divisor, and proceed as before till the desired root is found.*

9. *Annex periods of ciphers, and a decimal root will be obtained.*

10. *Change Common or Duodecimal Fractions to decimals to obtain their roots.*

11. *Change Denominate Numbers to the same denomination to obtain their square root.*

Square Root is applicable to the Mensuration ot Surfaces.

EXAMPLES.

1926. What is the square root of 1849

1927. What is the square root of 625?

1928. What is the square root of 7225?

1929. What is the square root of 11025?

1930. What is the square root of 24025?

1931. What is the square root of 65025?

1932. What is the square root of 81929?

1933. Sum of the answers? Ans. 954.2324.

Repeat Prob. C. What is the 1st part of the Rule? 2d? 3d? 4th? 5th? 6th? 7th? 8th? 9th? 10th? 11th? Repeat the Rule entire. Repeat Prin. 1, 2, 3, 4, 5. Give an illustration of each.

1934. What is the square root of 56.25? Of 17 7′?
1935. What is the square root of 41.25? Of 187.93862?
1936. What is the square root of 800.? Of 602.6026?
1937. What is the square root of 70.? Of $784\frac{3}{7}$?
1938. What is the square root of $\frac{9}{12}$? Of $19\frac{3}{5}$?
1939. What is the square root of $\frac{13}{17}$? Of 83.75?
1940. What is the square root of $\frac{6}{7}$ of $\frac{11}{14}$? Of $81\frac{3}{21}$?
1941. What is the square root of 6 6′? Of $777\frac{7}{17}$?
1942. Sum of the answers?
1943. What is the square root of 800 to six decimal places?

OPERATION BY TWO METHODS.

Common Method.

```
         8˙00(28.284271
         4
         ---
       48)400
          384
          ----
      562)1600
          1124
          -----
     5648)47600
          45184
          ------
    56564)241600
          226256
          -------
   565682)1534400
          1131364
          --------
  5656847)40303600
          39597929
          ---------
 56568541)70567100
          56568541
          --------
          13998559
```

By this method of extracting square root we have used 137 figures.

This process may be materially shortened by rejecting the right hand figures of the divisor. See operation below.

Short Method.

```
        8˙00(28.284271
        4
        ---
      48)400
         884
         ----
     562)1600
         1124
         -----
    5648)47600
         45184
         ------
56|5|6|4)241600
         226256
         ------
          15344
          11312
          -----
           4032
           8955
           ----
             77
             56
             --
             21
```

By this method of extracting square root, only 86 figures were used.

Common Method...... 137
Short Method.......... 86

SECTION II.

MENSURATION OF SURFACES.

360. For definitions and demonstration of principles, the student is referred to Geometry.

361. Areas of all similar surfaces are to each other as the squares of their like dimensions.

1. Triangles.

PROB. CI.—TO FIND THE HYPOTHENUSE.

RULE.—*The square root of the sum of the squares of the base and perpendicular.*

PROB. CII.—TO FIND THE PERPENDICULAR.

RULE.—*The square root of the difference of the squares of the hypothenuse and base.*

PROB. CIII.—TO FIND THE BASE.

RULE.—*The square root of the difference of the squares of the hypothenuse and perpendicular.*

PROB. CIV.—TO FIND THE AREA OF A TRIANGLE, THE BASE AND ALTITUDE BEING GIVEN.

RULE.—*Half the product of the base and perpendicular, or the product of half the base and perpendicular, or the product of half the perpendicular and base.*

PROB. CV.—TO FIND THE AREA OF A TRIANGLE, THE SIDES BEING GIVEN.

RULE.—*From half the sum of its sides, subtract each side separately. Then the square root of the product of the half sum, and these differences, will be the area.*

2. Quadrangles.

PROB. CVI.—TO FIND THE SIDE OF A SQUARE.

RULE.—*Take the square root of its area.*

PROB. CVII.—TO FIND A SIDE OF A RECTANGLE, PARALLELOGRAM, OR RHOMBUS.

RULE.—*Divide its area by its altitude.*

PROB. CVIII.—TO FIND THE AREA OF A SQUARE, RECTANGLE, PARALLELOGRAM, RHOMBUS, OR RHOMBOID.

RULE.—*Multiply the length by the perpendicular breadth.*

PROB. CIX.—TO FIND THE AREA OF A TRAPEZOID.

RULE.—*Half the product of the perpendicular breadth by the sum of the parallel sides.*

3. Polygons.

PROB. CX.—TO FIND THE SIDE OF ANY REGULAR POLYGON.

RULE.—*Divide the area by the product of the number of sides and half the perpendicular, from one of its sides to the centre.*

PROB. CXI.—TO FIND THE AREA OF ANY IRREGULAR POLYGON.

RULE.—*Divide the figure into triangles, and find the sum of their areas.*

PROB. CXII.—TO FIND THE AREA OF ANY REGULAR POLYGON.

RULE.—*The product of the perimeter, and half the perpendicular, from one of its sides to the centre.*

4. Circles.

PROB. CXIII.—TO FIND THE CIRCUMFERENCE OF A CIRCLE.

RULE.—*Multiply the diameter by* π.

NOTE.—π is equal to 3.1415926535897932984626+, or 3.1416, nearly.

PROB. CXIV.—TO FIND THE DIAMETER OF A CIRCLE.

RULE.—*Divide the circumference by* π.

PROB. CXV.—TO FIND THE AREA OF A CIRCLE.

RULE.—*Multiply the square of the diameter by* $\frac{3}{4}\pi$; *or* $\frac{1}{4}$ *the product of the circumference and diameter; or the product of half the diameter, by half the circumference.*

PROB. CXVI.—TO FIND THE SIDE OF A SQUARE EQUAL TO A GIVEN CIRCLE.

RULE.—*Multiply the diameter by* .886227.

NOTE.—.886227 is equal to $\frac{1}{2}\sqrt{\pi}$.

PROB. CXVII.—TO FIND THE SIDE OF AN INSCRIBED SQUARE.

RULE.—*Multiply the diameter by* .707106.

NOTE.—.707106 is equal to $\sqrt{.5}$.

PROB. CXVIII.—TO FIND THE CIRCUMFERENCE FROM AN INSCRIBED SQUARE.

RULE.—*Divide the side of the square by* .225079.

PROB. CXIX.—TO FIND THE SIDE OF AN INSCRIBED SQUARE FROM THE CIRCUMFERENCE.

RULE.—*Multiply the circumference by* .225079.

5. Spheres.

PROB. CXX.—TO FIND THE SURFACE OF A SPHERE.

RULE.—*Multiply the diameter by the circumference.*

6. Cones and Pyramids.

PROB. CXXI.—TO FIND THE SURFACE OF A CONE OR PYRAMID.

RULE.—*Half the product of the circumference or perimeter of the base, and slant height, added to the area of the base.*

PROB. CXXII.—TO FIND THE SLANT HEIGHT.

RULE.—*Subtract the area of the base from its entire surface. Divide the remainder by* $\frac{1}{2}$ *the circumference of base.*

7. Ellipses.

PROB. CXXIII.—TO FIND THE CIRCUMFERENCE OF AN ELLIPSE.

RULE.—*Multiply* π *by the square root of half the sum of the squares of the two axes.*

PROB. CXXIV.—TO FIND THE AREA OF AN ELLIPSE.

RULE.—*Multiply* $\frac{1}{4}\pi$ *by the product of the axes.*

8. Cylinders.

PROB. CXXV.—TO FIND THE CONVEX SURFACE OF A CYLINDER.

RULE.—*Multiply the length by the circumference.*

PROB. CXXVI.—TO FIND THE LENGTH OF A CYLINDER.

RULE.—*Divide the convex surface by the circumference.*

9. Polyhedrons.

PROB. CXXVII.—TO FIND THE AXIS OF A CUBE.

RULE.—*Take the square root of three times the square of one side.*

PROB. CXXVIII.—TO FIND THE SURFACE OF A REGULAR POLYHEDRON.

RULE.—*Multiply the area of one side by the number of sides.*

EXAMPLES.

1944. The base of a triangle is 21 feet, the perpendicular 32 feet. What is the length of the hypothenuse?

1945. A ladder 50 feet long, with its foot 17 feet from a building. To what height will it reach on the building?

1946. A rope is tied to the top of a pole 100 feet high. Suppose the rope to be 400 feet long, how far may it be extended on a level from the foot of the pole?

1947. What is the contents of a triangular field, whose base is 121 rods and altitude 57 rods?

1948. What is the contents of a field, whose sides are 57 rods, 81 rods, and 36 rods?

1949. There is a square field containing 675 square rods. What is the length of one side?

1950. What is the distance of the opposite corners in the abovementioned field?

1951. What is the length of a roof 20 feet wide, containing 6000 square feet?

1952. What is the area of a rhombus, length 7 inches, perpendicular breadth 6 inches?

1953. What is the difference between a square field, 207 rods on a side, and a rectangular field, 200 rods by 214 rods?

1954. What is the difference between 4 fields 21 rods square, and one field 42 rods square?

1955. I have a six-sided field, containing 5 acres. What is the length of one side, they being equal? (See Art. 361.)

1956. I have a field bounded by six sides, each 37 rods. How many acres does the field contain?

1957. How far distant from each other are the opposite sides of the abovementioned field?

1958. Sum of the answers? **Ans. 5645.601.**

1959. I have an irregular field, the length of whose sides are as follows: 1st, 21 rods; 2d, 31; 3d, 41; 4th, 46; 5th, 67; 6th, 71; and 7th, 25. The distance from the 1st corner to the 3d is 27 rods; from the 1st to the 4th, 51 rods; from the 1st to the 5th, 8 rods; from the 1st to the 6th, 69 rods. How many acres does the field contain?

1960. What is the circumference whose diameter is 6 feet?

1961. What is the diameter of a circle, whose circumference is 314159.265. miles?

1962. How many acres in a field 43 rods in diameter?

1963. How many acres in a field 75 rods in circumference?

1964. What is the difference in rods between a field 44 rods in diameter, and a square field 42 rods on each side?

1965. How large a square can be cut out of a circle 33 inches in circumference?

1966. How large a square is equal to a circle 33 inches in circumference?

1967. What is the diameter of a circle that shall be equal in area to a field 24 rods square?

1968. What is the diameter of a circle that will contain a field 27 rods square?

1969. What is the surface of a sphere 9 feet in diameter?

1970. What is the convex surface of a cone, slant height 9 feet, diameter of base 3 feet?

1971. What is the entire surface of a cone, slant height 27 feet, diameter of base 4 feet.

1972. What is the circumference of an ellipse whose axes are 7 and 11 feet?

1973. What is the area of an ellipse whose axes are 15 and 21 feet?

1974. What is the convex surface of a cylinder, diameter 4 feet, length 2 feet?

1975. What is the length of a cylinder whose convex surface is 12000 square feet, and circumference 400?

1976. Sum of the last 16 answers? Ans. 101166.7824.

SECTION III.

CUBE ROOT.

362. Cube Root is the process of determining the root from the third power, or cube, and is expressed thus, $\sqrt[3]{\ }$, or by the fractional index $\frac{1}{3}$.

363. *The following principles are very important, and the application of them should be thoroughly understood before an attempt to extract cube root.*

364. PRIN. 1.—THE CUBE OF THE GREATEST NUMBER EXPRESSED BY A SINGLE FIGURE CANNOT CONTAIN MORE THAN THREE PLACES; AND, GENERALLY, THE CUBE OF ANY NUMBER CANNOT OCCUPY MORE THAN THREE TIMES THE NUMBER OF PLACES OCCUPIED BY ITS ROOT.

ILLUSTRATION 1.—The cube of 9 is 729.

The cube of 99 is 970299.

INF.—The root will contain as many figures as the cube has periods of 3 figures each.

ILL. 2.—76·321·431 has a root of 3 figures, as there are 3 periods.

365. PRIN. 2.—THE CUBE OF THE SUM OF TWO NUMBERS IS EQUAL TO THE CUBE OF THE FIRST, PLUS 3 TIMES THE SQUARE OF THE FIRST MULTIPLIED BY THE SECOND, PLUS 3 TIMES THE SECOND MULTIPLIED BY THE SQUARE OF THE FIRST, PLUS THE CUBE OF THE SECOND.

ILLUSTRATION.—27 is equal 20 + 7, the root.

$$
\begin{array}{lrrrr}
 & & & 20\,+ & 7 \\
 & & & 20\,+ & 7 \\
\cline{4-5}
\text{Product by } 7 \ldots\ldots & & & 140\,+ & 49 \\
\text{Product by } 20 \ldots\ldots & & 400\,+ & 140 & \\
\cline{3-5}
 & & 400\,+ & 280\,+ & 49 \\
 & & & 20\,+ & 7 \\
\cline{3-5}
\text{Product by } 7 \ldots\ldots & & 2800\,+ & 1960\,+ & 343 \\
\text{Product by } 20 \ldots\ldots & 8000\,+ & 5600\,+ & 980 & \\
\cline{2-5}
\text{Cube of } 27 = & 8000\,+ & 8400\,+ & 2940\,+ & 343
\end{array}
$$

$$
\text{or } 8000 \left\{ \begin{array}{l} +\,2800 + 980 \\ +\,2800 + 980 \\ +\,2800 + 980 \end{array} \right\} + 343 = 19683
$$

We will now reverse the operation, and first point off the number into periods of 3 figures each, according to Prin. 1.

We then find the greatest cube of the left hand period, 19,000, to be 8000, and its root 20. Subtracting this cube, 8000, from the given number, 19683, and it leaves a remainder, 11683.

1200	19·683	20
420	8 000	7
49	11 683	27
1669	11 683	Ans.

We now wish to obtain the second figure of the root, and, by inspecting the work by involution, we find the remaining numbers to be composed of three separate terms. The 1st term, 8400, is composed of 2800, multiplied by 3. This 2800 was composed of the square of the first figure of the root multiplied by the second figure, or the one we now wish to find. Hence, if we divide 2800 by the square of 20 (the figure already found), we shall obtain the second figure of the root; but as we have not the 2800, we set aside 3 times the square of 20 as a divisor to 8400, or 3 times 2800, which is 20 × 20 × 3 = 1200.

We now consider the second term, 2940, or 3 times 980. This 980 was composed of the first figure of the root, 20, multiplied by the square of the last figure (the one to be found). We must now use the 1200 above as a partial divisor to obtain the second figure of the root. And as this partial divisor is much smaller than the real divisor, allowance must be made in the quotient figure; thus, 1200 is contained in 11683 9 times, but the real divisor will be contained only 7 times. Hence, 3 × 7 × 20 will be the divisor to 3 times 980, or 2940. We then set aside, under the 1200, 3 × 7 × 20 = 420.

Examining the third term, 343, we find it is composed of the cube of the second figure in the root. Hence, 7 × 7 = 49, is the divisor of the third term. Set this aside under 420, and add these three partial divisors, which gives 1669 as the true divisor, and 27 as the root.

---

362. What is cube root? How may it be expressed? 364. What is Prin. 1? Give an illustration. What inference from Prin. 1? How can you tell how many figures the root of any number will contain? 365. Repeat Prin. 2.

```
        19·683 | 27
  12     8     |
       --------
  42  | 11683
   49 | 11683
 -----  -----
 1669 |     0
```

This method may be shortened by rejecting the ciphers, and the work will then take the following form.

This principle will also admit of a geometrical illustration, as well as the algebraical above.

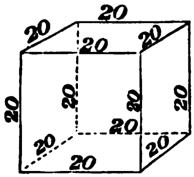
GEOMETRICAL ILLUSTRATION OF CUBE ROOT.

Suppose we have 19683 blocks, each 1 foot every way, and we wish to form them into a pile of equal length, height, and breadth. What will be the dimensions of the pile?

By pointing off into periods of 3 figures each, according to Prin. 1, we perceive the root of the first period to be 20, and its cube 8000; that is, if we make a pile 20 blocks each way, it will contain 8000 blocks, and we have left 11683 blocks, thus:

```
19·683(20
 8000
-----
11683
```

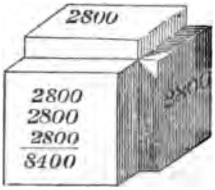
Fig. 1.



This is represented by Fig. 1.

It will be perceived that we must add on three sides at once to preserve the form of the figure, and that each side is 20 feet in length. Hence, it must take 20 × 20 = 400 blocks to cover one side, and 3 × 400 = 1200 to cover 3 sides, one foot in thickness. Therefore divide 11683 by 1200 to obtain the number of additions. Making allowance, as in square root, for a partial divisor, we find that 7 additions may be made. As one contains 1200 blocks 7 will contain 7 × 1200 = 8400.

Fig. 2.



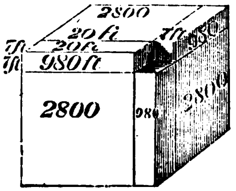
This addition is represented by Fig. 2.

These three additions contain 2800 blocks each, making the entire addition 8400 blocks.

There will now be left 3 spaces, each 20 feet long and 7 feet square. See Fig. 2.

It will then take 7 × 7 × 20 = 980 to fill up one space, and 3 times 980 = 2940 blocks to fill the entire spaces. The pile will then take the form of Fig. 3. There is now wanting a corner cube, 7 feet on a side, to complete the additions which will contain 7 × 7 × 7 = 343 cubic feet.

FIG. 3.



The entire cube (see Fig. 4) is composed of 8 blocks, as follows :—

1 cube	containing	8000	cu. ft.
3 square slabs,	"	8400	"
3 long blocks,	"	2940	"
1 corner cube,	"	343	"
Total contents		19683	

The divisor in cube root is composed of the entire *surface* of the additions, and its composition is best seen by reference to Fig. 5.

FIG. 4.

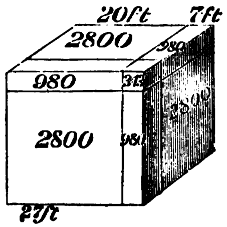


FIG. 5.

20	20	20	20	20	20	7
20	20	20	7	7	7	7
400	400	400	140	140	140	49

In the above example the 3 square blocks contain 1200 sq. ft., the 3 long blocks contain 420 sq. ft., and the corner square contains 49 sq. ft.; added gives a divisor, 1669, and the numerical operation is thus given :—

```
         19683 ( 20        Or, rejecting ciphers, thus:
 1200     8000   7              12     19·683(27
 420  |  11683   27 ft.         42      8
   49 |  11683                  49  |  11683
 1669        0                1669  |  11683
```

Prob. CXXIX.—To extract cube root.

Rule.—1. *Point off the number into periods of three figures each, commencing at the decimal point, and making full periods by annexing ciphers, if necessary, to the decimal part.*

2. *Subtract the greatest possible cube from the left hand period, and write its root in the quotient.*

3. *Annex the second period to the remainder for the first dividend.*

4. *Take three times the square of the root already found for a partial divisor.*

5. *Find how many times this partial divisor is contained in the dividend, rejecting its two right hand figures, and write the number in the root.*

6. *Complete the divisor by adding, 1st, three times the product of the last figure in the root multiplied by the other figures in the root, and, 2d, the square of the last figure in the root, placing each one step to the right. (See divisor at foot of page 245.)*

7. *Multiply the divisor thus completed by the last figure in the root, and subtract the product from the dividend.*

8. *Annex the third period to the remainder for a second dividend.*

9. *Proceed in the same manner till the desired root is obtained.*

10. *A decimal root may be obtained by annexing periods of ciphers.*

11. *Change Common or Duodecimal Fractions to Decimals to obtain their roots.*

12. *Change Denominate Numbers to the same denomination to obtain their root.*

Repeat Prob. CXXIX. Repeat 1st part of the rule. 2d. 3d. 4th. 5th. 6th. 7th. 8th. 9th. 10th. 11th. Repeat the entire rule. Repeat Prin. 1, 2.

EXAMPLES.

1977. Required the cube root of 12167.
1978. What is the cube root of 140608?
1979. What is the cube root of 551368?
1980. What is the cube root of 6666?
1981. What is the cube root of .008?
1982. What is the cube root of .81?
1983. Required the cube root of .729.
1984. Required the cube root of 7.29.
1985. Required the cube root of 72.9.
1986. Required the cube root of 729.
1987. Required the cube root of 2.
1988. What is the cube root of 3?
1989. What is the cube root of $\frac{7}{8}$?
1990. What is the cube root of $4\frac{1}{5}$?
1991. What is the cube root of $\frac{4}{38}$ of $4\frac{1}{2}$ of 7.6?
1992. What is the cube root of 24 9′ 8″ 11‴?
1993. What is the sum of the answers? Ans. 202.6887.

1994. Required the cube root of 800. by common method to the 5th decimal place.

This method may be shortened as follows:—

800.(9.2831	9	9	2
729	9	2	2
24844)71000	81	18	4
49688	3	3	
2561344)21312000	243	54	
20490752	54		
821248	4		
768402	24844 1st Divisor.		
52846	92	92	8
25613	92	8	8
27233	184	736	64
	828	3	
	8464	2208	
	3		
	25392		
	2208		
	64		
	2561344 2d Divisor.		

It requires 327 figures, by the common method, to obtain the above result. By the short method, 125.

SECTION IV.

MENSURATION OF SOLIDS.

All similar solids are to each other as the cubes of their like dimensions.

PROB. CXXX.—TO FIND A SIDE OF A CUBE.

RULE.—*Extract the cube root of its contents.*

PROB. CXXXI.—TO FIND THE CONTENTS OF A CUBE, OR ANY BODY WHOSE SIDES ARE PERPENDICULAR TO EACH OTHER.

RULE.—*Find the product of the length, breadth, and height.*

PROB. CXXXII.—TO FIND A SIDE OF A PARALLELOPIPEDON.

RULE.—*Divide the contents by the product of two sides.*

PROB. CXXXIII.—TO FIND THE CONTENTS OF A PRISM, CYLINDER, OR UNGULA.

RULE.—*The product of the height and area of the base.*

PROB. CXXXIV.—TO FIND THE CONTENTS OF A CONE OR PYRAMID.

RULE.—*The product of $\frac{1}{3}$ of the perpendicular height and area of the base.*

PROB. CXXXV.—TO FIND THE CONTENTS OF A FRUSTRUM OF A CONE.

RULE.—*The product of $\frac{1}{4}\pi$, $\frac{1}{3}$ the perpendicular height, and the quotient of the difference of the cubes of the diameters of the two ends, divided by the difference of the diameters.*

PROB. CXXXVI.—TO FIND THE CONTENTS OF A FRUSTRUM OF A PYRAMID OR CONE.

RULE.—*Add to the sum of the two areas, the square root of their product. Multiply this sum by $\frac{1}{3}$ the perpendicular height.*

PROB. CXXXVII.—TO FIND THE CONTENTS OF A WEDGE.

RULE.—*Add twice the length of the back to the length of the edge. Then take $\frac{1}{6}$ of the product of this sum, the height of the wedge, and the breadth of back.*

PROB. CXXXVIII.—TO FIND THE CONTENTS OF A SPHERE.

RULE.—*The product of the cube of the diameter and $\frac{1}{6}\pi$.*

NOTE.—$\frac{1}{6}\pi = .5236$.

Prob. CXXXIX.—To find the contents of a spheroid.

Rule.—*The product of the square of the shorter axis by the longer axis, and* $\frac{1}{6}\pi$.

Prob. CXL.—To find an inscribed cube in a sphere.

Rule.—*The square root of* $\frac{1}{3}$ *the square of the diameter.*

Prob. CXLI.—To find the contents of a cask.

Rule.—*The product of the length, the sum of the squares of the head, twice the mean, and bung diameters and* .1309.

Prob. CXLII.—To find the ullage of a cask.

Rule.—*When the cask is standing. The product of the net inches, and* $\frac{1}{3}$ *the sum of the head, mean, and bung diameters.*

Prob. CXLIII.—To find the government tonnage of vessels.

Rule.—*For a double-decked vessel. Subtract from the length* $\frac{3}{5}$ *of the breadth. Multiply this difference by twice the square of* $\frac{1}{2}$ *the breadth, and divide by* 95.

Prob. CXLIV.—For a single-decked vessel.

Rule.—*Subtract from the length* $\frac{3}{5}$ *of the breadth. Multiply this difference by the breadth and depth, and divide by* 95.

Prob. CXLV.—To find the carpenters' tonnage.

Rule.—*For a double-decked vessel. The product of the length, the breadth, and* $\frac{1}{2}$ *breadth, divided by* 95.

Prob. CXLVI.—For a single-decked vessel.

Rule.—*The product of the length, breadth, and depth, divided by* 95.

Prob. CXLVII.—To find the tonnage of steamships.

Rule.—*Deduct the tonnage due to the engine-room from the whole tonnage.*

Prob. CXLVIII.—To find the contents of any irregular body.

Rule.—*Fill a vessel full of water, immerse the body, and measure the quantity of water it displaces.*

EXAMPLES.

1995. What is the length of one side of a cubical block, containing 166375 cubic inches?

1996. What is the distance between the opposite corners of a cube, 17 inches on a side?

1997. What is the distance between the opposite corners of a school-room 21 feet long, 20 feet wide, and 9 feet high?

1998. What are the contents of a cubical block 13½ feet on a side?

1999. How many gallons of atmosphere in a room 21 feet long, 20 wide, and 9 feet high?

2000. What is the difference in capacity, between a cart 6 feet long, 4 feet wide, 1½ deep, and one 5½ feet long, 4½ wide, and 2½ deep? Sum of the above answers, 30877.4224.

2001. I have a bin containing 100 bushels. It is 4 feet wide and 6 feet deep. How many feet long is it?

2002. How many cubic feet of earth must be thrown out in digging a well 6 feet in diameter and 80 feet deep?

2003. What are the contents of a pyramid 13 feet high, with a triangular base, 7, 10, and 12 feet on a side?

2004. What are the contents of a frustrum of a cone 6 feet in diameter at top, 20 feet at the base, and 40 feet high?

2005. What are the contents of Bunker Hill Monument, 20 feet square at the base, and 15 feet at the top, and 220 feet high? Sum of the last 5 answers, 76074.4721.

2006. What are the contents of a wedge, length of the back 2 inches, length of the edge 1½ inches, height 11 inches, and breadth 3 inches?

2007. What are the contents of a sphere 31 inches in diameter?

2008. How large a cube will hold 8 times as much as the sphere in the last example?

2009. What are the contents of a spheroid, shorter axis 13 inches, longer axis 21 inches?

Sum of the last 4 answers, 17537.0181.

2010. How large a cube can be cut from a sphere 66.6 feet in diameter? Ans. 38.451.

2011. What are the contents of a barrel 41 inches long, head diameter 28 inches, mean diameter 29 inches, and bung diameter 30 inches? Ans. 27092.11 cu. in. or 117.28 gal.

2012. What is the ullage of a cask, wet inches 21, head diameter 17 inches, mean diameter 19 inches, and bung diameter 22 inches? Ans. 406. cu. in.

2013. What is the government tonnage of a vessel, length 112 feet, breadth of beam 23 feet, depth 12 feet?

2014. What is the government tonnage of a double-decked ship 250 feet long, 45 feet breadth, and 35 feet deep?

2015. What is the carpenters' tonnage of a vessel, with the same dimensions as in the 2013th example?

2016. What is the carpenters' tonnage of a ship, dimensions the same as in the 2014th example?

2017. What is the government tonnage of a steamship 230 feet long, 40 feet wide, depth of hold 32 feet, tonnage of the engine-room 150 tons?

2018. Required the contents of 4 crabs and 2 lobsters, which, placed in a cylinder 18 inches in diameter, partly full of water, caused it to rise 11 inches?

2019. What is the sum of the last six answers?

Ans. 10035.74.

How do you find the side of a cube? Give the rule for extracting cube root. How find the contents of a cube? How of any body whose sides are perpendicular to each other? How find the side of a parallelogram? The contents of a cone? Frustrum of a cone? What is π? How much is it? How find the frustrum of a pyramid? The contents of a wedge? Sphere? Spheroid? Inscribed cube? Cask? Ullage of a cask? How find government tonnage? For a single-decked vessel? Carpenters' tonnage? Tonnage of steamships? How find the contents of an irregular body?

DIVISION II.

SERIES.

366. A Series is a progression of numbers or quantities, which succeed each other according to some determined law.

367. Of the various kinds of Series, Arithmetical Progression, or Equi-different Series, is that which progresses by a constant difference.

368. Geometrical Progression is that which progresses by a constant factor or multiplier.

CHAPTER I.

ARITHMETICAL PROGRESSION.

369. The series in Arithmetical Progression, may increase or decrease; forming an ascending or descending series.

370. The numbers which form the series, are called terms. The *first* and *last* are called *extremes*. The remaining ones are called *means*.

371. Connected with every series, are five parts, so intimately related, that any three being given, the other two may be found.

These are—1st. The First Term.
2d. The Last Term.
3d. The Number of Terms.
4th. The Common Difference.
5th. The Sum of the Series.

Prob. CXLIX.—Given, the first term and common difference, to construct the series.

Rule.—*Add the common difference to any term, and it will give the following term.*

---

366. What is a Series? 367. What is an Equi-different Series? 368. What is Geometrical Progression? 369. What two kinds of Arithmetical Series? 370. What are the several parts of a series called? What is the first term? 371. How many parts must be given to find the remaining ones? Last term? Number of terms? Sum of series?

PROB. CL.—GIVEN, THE FIRST TERM, COMMON DIFFERENCE, AND NUMBER OF TERMS, TO FIND THE LAST TERM.

RULE.—*Add the first term to the product of the common difference, and the number of terms, less one*

PROB. CLI.—GIVEN, THE COMMON DIFFERENCE, NUMBER OF TERMS, AND LAST TERM, TO FIND THE FIRST TERM.

RULE.—*Subtract the product of the common difference, and the number of terms, less one, from the last term.*

PROB. CLII.—GIVEN, THE EXTREMES AND COMMON DIFFERENCE, TO FIND THE NUMBER OF TERMS.

RULE.—*Add one to the quotient of the difference of the extremes, divided by the common difference.*

PROB. CLIII.—GIVEN, THE EXTREMES AND NUMBER OF TERMS, TO FIND THE COMMON DIFFERENCE AND MEANS.

RULE.—*Divide the difference of the extremes by the number of terms, less one, for the common difference. Then construct the series by Prob. CL.*

PROB. CLIV.—GIVEN, THE EXTREMES AND NUMBER OF TERMS, TO FIND THE SUM OF THE SERIES.

RULE.—*Multiply ½ the sum of the extremes by the number of terms, or the sum of the extremes by ½ the number of terms.*

PROB. CLV.—GIVEN, THE EXTREMES AND COMMON DIFFERENCE, TO FIND THE SUM OF THE SERIES.

RULE.—*Find the number of terms by Prob. CLII, and the sum of the series by Prob. CLIV.*

PROB. CLVI.—GIVEN, THE EXTREMES AND SUM OF THE SERIES, TO FIND THE COMMON DIFFERENCE.

RULE.—*Divide the sum of the series by ½ the sum of the extremes, and the difference of the extremes by this quotient, less one.*

PROB. CLVII.—GIVEN, THE FIRST TERM, NUMBER OF TERMS, AND SUM OF THE SERIES, TO FIND THE LAST TERM.

RULE.—*Subtract the first term from the quotient of the sum of the series, divided by ½ the number of terms.*

EXAMPLES.

2020. What is the last term of an arithmetical series, whose first term is seventeen, number of terms 21, and common difference 11?

2021. What is the common difference of a series, whose extremes are 3 and 10, and number of terms 12?

2022. What is the number of terms in an equi-different series, whose first term is seven, last term 205, and common difference 11?

2023. What is the sum of a series, whose first term is 21, last term 2232, and number of terms 202?

2024. What is the sum of a series, whose first term is 23, last term 4434, and common difference 11?

2025. What is the common difference of an arithmetical series, whose first term is five, last term 55, and the sum of the series 630?

2026. What is the last term of an equi-different series, whose first term is seven, number of terms 31, and sum of the series twelve hundred and forty?

2027. What is the 12th term of a series, whose first term is $\frac{1}{2}$, and common difference $\frac{7}{8}$?

2028. What is the sum of a series, whose first term is $\frac{9}{13}$, last term $98\frac{4}{13}$, and number of terms 78?

2029. Which is the most, an arithmetical series whose first term is 7, last term 71, and number of terms 41, or an equi-different series whose first term is 41, last term 71, and number of terms 21? How much?

2030. Find 7 means between 183 and 215.

2031. What is the sum of the answers on this page?

Ans. $1129429\frac{23}{88}$.

Give Prob. CXLIX. Rule. Prob. CL. Rule. Prob. CLI. Rule. Prob. CLII. Rule. Prob. CLIII. Rule. Prob. CLIV. Rule. Prob. CLV. Rule. Prob. CLVI. Rule. Prob. CLVII. Rule.

CHAPTER II.

GEOMETRICAL PROGRESSION.

372. The series in a Geometrical Progression, may increase or decrease, forming an ascending or descending series.

373. There are also 5 parts, as in Equi-different Series, any three of which being given, the remaining two may be found.

These are — 1st. The First Term.
2d. The Last Term.
3d. The Number of Terms.
4th. The Common Ratio.
5th. The Sum of the Series.

374. The first and last are called *extremes.*

375. The remaining terms are called *means.*

PROB. CLVIII. — GIVEN, THE FIRST TERM AND COMMON RATIO, TO FIND THE MEANS, OR TO CONSTRUCT A SERIES.

RULE.—*Multiply any term by the ratio, and it will give the following term.*

PROB. CLIX.—GIVEN, THE FIRST TERM, RATIO, AND NUMBER OF TERMS, TO FIND THE LAST TERM.

RULE.—*Multiply the ratio, raised to a power, one less than the number of terms, by the first term.*

PROB. CLX.—GIVEN, THE LAST TERM, RATIO, AND NUMBER OF TERMS, TO FIND THE FIRST TERM.

RULE.—*Divide the last term by the ratio, raised to a power one less than the number of terms.*

PROB. CLXI.—GIVEN, THE EXTREMES AND RATIO, TO FIND THE SUM OF THE SERIES.

RULE.—*Divide the difference of the extremes, by the ratio, less one, and add the greater extreme to the quotient.*

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372. How many kinds of Geometrical Series? 373. How many parts has a series, and what are they? Which are extremes? Which means? Give Prob. CLVIII. Rule.

PROB. CLXII.—GIVEN, THE EXTREMES AND RATIO, TO FIND THE NUMBER OF TERMS.

RULE.—*Divide the last term, by such a power of the ratio as will give a quotient equal to the first term, and, one, plus the index of that power of the ratio, will be the number of terms.*

PROB. CLXIII.—GIVEN, THE EXTREMES AND NUMBER OF TERMS, TO FIND THE RATIO.

RULE.—*Divide the greater extreme by the less, then extract that root of the quotient whose index is one less than the number of terms.*

NOTE.—By this rule any number of means may be inserted between any two extremes.

EXAMPLES.

2032. The first term, 6, and the ratio, 2, being given, construct a geometrical series of 15 terms.

2033. What is the last term of a geometrical series, whose first term is 3, ratio 7, and number of terms 9?

2034. What is the first term of a series, whose last term is 1048576, ratio 2, and number of terms 24?

2035. What is the number of terms in a series, whose first term is $\frac{1}{2}$, last term 108, and ratio 6?

2036. What is the sum of the series, whose first term is 3, last term 384, and ratio 2?

2037. What is the ratio of a series, whose first term is 12, last term 108, and number of terms 3?

2038. Insert two means between 12 and 324.

2039. What is the last term of a series, whose first term is 2, ratio 10, and the number of terms 12?

2040. What is the first term in a series, whose last term is 885735, the ratio 3, and number of terms 12?

2041. What is the sum of the answers on this page?

Ans. 2000174919926$\frac{1}{8}$.

Give Prob. CLIX. Rule. Prob. CLX. Rule. Prob. CLXI. Rule. Prob. CLXII. Rule. Prob. CLXIII. Rule. How may any number of means be inserted?

CHAPTER III.

ANNUITIES.

376. This subject includes questions relating to salaries, taxes, rents, funded debts, and payments of any kind made at regular periods. It is a most important rule in Arithmetic.

377. An *Annuity* is a certain sum payable annually, or at regular intervals, either for a limited or an unlimited time.

378. *Annuity Certain,* is limited to a certain number of years.

379. *Annuity Contingent,* is limited by one or more lives.

380. *Annuity in Possession,* is one commenced.

381. *Annuity in Reversion,* is one to commence in future.

382. *Annuity in Arrears,* is one due, but not paid.

383. *Perpetuity,* is an unlimited annuity.

PROB. CLXIV.—TO FIND THE AMOUNT OF AN ANNUITY, AT SIMPLE INTEREST, FOR A GIVEN TIME.

RULE.—*Consider the annuity the first term of an arithmetical series; the interest for one year, or from one payment to another, the common difference; and the number of payments the number of terms. Then find the sum of the series by Prob. CL. and CLIV.*

PROB. CLXV.—TO FIND THE AMOUNT OF AN ANNUITY, AT COMPOUND INTEREST, FOR A GIVEN TIME.

RULE.—*Make* **$1.00** *the first term of a geometrical series; the amount of* **$1.00**, *from one payment to the next, the ratio; and the number of payments the number of terms. Then find the sum of this series by Prob. CLIX. and CLXI, and multiply it by the annuity.*

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376. What does Annuities include? What common business transactions are performed by this rule? 377. What is an annuity? 378. Annuity Certain? 379. Annuity Contingent? 380. Annuity in Possession? 381. Annuity in Reversion? 382. Annuity in Arrears? 383. Perpetuity? Repeat Prob. CLXIV. Rule. Prob. CLXV. Rule. Prob. CLXVI. Rule.

Prob. CLXVI.—To find the present worth of an annuity.

Rule.—1. *Find the sum of the series as in the last problem.*

2. *Divide this sum by the amount of* $1.00, *at compound interest, for the same intervals of time and rate.*

3. *Multiply this quotient by the annuity.*

Prob. CLXVII.—To find the value of a perpetuity.

Rule.—*Divide the perpetuity by the rate per cent.*

Prob. CLXVIII.—To find the present worth of an annuity in reversion.

Rule.—1. *Find the present value of the annuity from the present time until it terminates.*

2. *Subtract from this, the present value of the annuity, from the present time to the time the annuity commences.*

Prob. CLXIX.—To find an annuity from its present value.

Rule.—*Divide its value by the present value of an annuity of* $1.00, *for its time and rate.*

Prob. CLXX.—To find an annuity from its amount.

Rule.—*Divide the amount of the annuity by the amount of an annuity of* $1.00, *for the same rate and time.*

EXAMPLES.

2042. What is due for the rent of a house for 4 years, at $200 per year, payable quarterly, remaining unpaid till the end of the lease, at simple interest?

2043. What would it amount to at compound interest?

2044. What will purchase an annuity of $275 for 15 years?

2045. What is a perpetuity of $1400 worth?

2046. A child has an annuity of $300, to commence when he arrives at the age of 21, and continue 30 years. What is it worth when the child is 16 years old?

2047. What annuity will $16000 purchase, to continue 21 years?

2048. What tax must be assessed each year to pay a debt of $200,000, at 6 per cent., in 15 years?

2049. What is the sum of the answers? Ans. $40401.52.

CHAPTER IV.

LOAN AND FUND ASSOCIATIONS, OR BUILDING ASSOCIATIONS.

384. These institutions are founded for the promotion of two objects: 1st, to accumulate funds; and, 2d, to furnish loans for building and other purposes.

385. They may be temporary or permanent, and the number of shares may be limited or unlimited.

386. The final value of a share is determined by each association, and is usually from $100 to $500.

387. The first object, the accumulation of funds, is accomplished by the periodical payment of certain sums, till the institution is able to redeem its own issues at a certain pre-determined value, when the accumulated fund is returned to those holding shares.

388. Each share thus becomes an annuity to the association.

389. The second object, furnishing loans, is accomplished by lending money to shareholders upon their surrendering their title to the fund when accumulated, and paying a specified annuity or interest; or, in other words, by permitting a shareholder to draw out his fund, at a discount, whenever he desires.

390. The funds are put up at auction at every meeting, and bid off to the shareholder paying the highest discount.

391. The cost of a share, at simple interest, is found by Prob. CLXIV, Annuities.

392. The cost of a share, at compound interest, is found by Prob. CLXV, Annuities.

NOTE.—Computations under this chapter and the preceding, may be facilitated by using the following tables.

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384. What two objects do these institutions embrace? 385. Are they temporary or permanent? Limited or unlimited shares? 386. How is the final value of a share determined? 387. How is the first object accomplished? 388. What does each share thus become? 389. How is the second object accomplished? 390. How are the funds disposed of? 391. How is the cost of a share, at simple interest, found? 392. The cost at compound interest?

TABLE,

Showing the amount of an Annuity of one dollar per annum, compound interest, from 1 *year to* 40, *inclusive.*

Yrs.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000
2	2.030 000	2.035 000	2.040 000	2.050 000	2.060 000	2.070 000
3	3.090 900	3.106 225	3.121 600	3.152 500	3.183 600	3.214 900
4	4.183 627	4.214 943	4.246 464	4.310 125	4.374 616	4.439 943
5	5.309 136	5.362 466	5.416 323	5.525 631	5.637 093	5.750 739
6	6.468 410	6.550 152	6.632 975	6.801 913	6.975 319	7.153 291
7	7.662 462	7.779 408	7.898 294	8.142 008	8.393 838	8.654 021
8	8.892 336	9.051 687	9.214 226	9.549 109	9.897 468	10.259 803
9	10.159 106	10.368 496	10.582 795	11.026 564	11.491 316	11.977 989
10	11.463 879	11.731 393	12.006 107	12.577 893	13.180 795	13.816 448
11	12.807 796	13.141 992	13.486 351	14.206 787	14.971 643	15.783 599
12	14.192 030	14.601 962	15.025 805	15.917 127	16.869 941	17.888 451
13	15.617 790	16.113 030	16.626 838	17.712 983	18.882 138	20.140 643
14	17.086 324	17.676 986	18.291 911	19.598 632	21.015 066	22.550 488
15	18.598 914	19.295 681	20.023 588	21.578 564	23.275 970	25.129 022
16	20.156 881	20.971 030	21.824 531	23.657 492	25.670 528	27.888 054
17	21.761 588	22.705 016	23.697 512	25.840 366	28.212 880	30.840 217
18	23.414 435	24.499 691	25.645 413	28.132 385	30.905 653	33.999 033
19	25.116 868	26.357 180	27.671 229	30.539 004	33.759 992	37.378 965
20	26.870 374	28.279 682	29.778 079	33.065 954	36.785 591	40.995 492
21	28.676 486	30.260 471	31.969 202	35.719 252	39.992 727	44.865 177
22	30.536 780	32.328 902	34.247 970	38.505 214	43.392 290	49.005 739
23	32.452 884	34.460 414	36.617 889	41.430 475	46.995 828	53.436 141
24	34.426 470	36.666 528	39.082 604	44.501 999	50.815 577	58.176 671
25	36.459 264	38.949 857	41.645 908	47.727 099	54.864 512	63.249 030
26	38.553 042	41.313 102	44.311 745	51.113 454	59.156 383	68.676 470
27	40.709 634	42.759 060	47.084 214	54.669 126	63.705 766	74.483 823
28	42.930 923	46.290 627	49.967 583	58.402 583	68.528 112	80.697 691
29	45.218 850	48.910 799	52.966 286	62.322 712	73.639 798	87.346 529
30	47.575 416	51.622 677	56.084 938	66.438 848	79.058 186	94.460 786
31	50.002 678	54.429 471	59.328 335	70.760 790	84.801 677	102.073 041
32	52.502 759	57.334 502	62.701 469	75.298 829	90.899 778	110.218 154
33	55.077 841	60.341 210	66.209 527	80.063 771	97.343 165	118.933 425
34	57.730 177	63.453 152	69.857 909	85.066 959	104.183 755	128.258 765
35	60.462 082	66.674 013	73.652 225	90.320 307	111.434 780	138.236 878
36	63.271 944	70.007 603	77.598 314	95.836 323	119.120 867	148.913 460
37	66.174 223	73.457 869	81.702 246	101.628 139	127.268 119	160.337 400
38	69.159 449	77.028 895	85.970 336	107.709 546	135.904 206	172.561 020
39	72.234 233	80.724 906	90.409 150	114.095 023	145.058 458	185.640 292
40	75.401 260	84.550 278	95.025 516	120.799 774	154.761 966	199.635 112

TABLE,

Showing the present worth of an Annuity of one dollar per annum, from 1 *year to* 40, *inclusive.*

Yrs.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	Yrs.
1	0.970 874	0.966 184	0.961 538	0.952 381	0.943 396	0.934 579	1
2	1.913 470	1.899 694	1.886 095	1.859 410	1.833 393	1.808 017	2
3	2.828 611	2.801 637	2.775 091	2.723 248	2.673 012	2.624 314	3
4	3.717 098	3.673 079	3.629 895	3.545 951	3.465 106	3.387 209	4
5	4.579 707	4.515 052	4.451 822	4.329 477	4.212 364	4.100 195	5
6	5.417 191	5.328 553	5.242 137	5.075 692	4.917 324	4.766 537	6
7	6.230 283	6.114 544	6.002 055	5.786 373	5.582 381	5.389 286	7
8	7.019 692	6.873 956	6.732 745	6.463 213	6.209 744	5.971 295	8
9	7.786 109	7.607 687	7.435 332	7.107 822	6.801 692	6.515 228	9
10	8.530 203	8.316 605	8.110 896	7.721 735	7.360 087	7.023 577	10
11	9.252 624	9.001 551	8.760 477	8.306 414	7.886 875	7.498 669	11
12	9.954 004	9.663 334	9.385 074	8.863 252	8.383 844	7.942 671	12
13	10.634 955	10.302 738	9.985 648	9.393 573	8.852 683	8.357 635	13
14	11.296 073	10.920 520	10.563 123	9.898 641	9.294 984	8.745 452	14
15	11.937 935	11.517 411	11.118 387	10.379 658	9.712 249	9.107 898	15
16	12.561 102	12.094 117	11.652 296	10.837 770	10.105 895	9.446 632	16
17	13.166 118	12.651 321	12.165 669	11.274 066	10.477 260	9.763 206	17
18	13.753 513	13.189 682	12.659 297	11.689 587	10.827 603	10.059 070	18
19	14.323 799	13.709 837	13.133 939	12.085 321	11.158 116	10.335 578	19
20	14.877 475	14.212 403	13.590 326	12.462 210	11.469 421	10.593 997	20
21	15.415 024	14.697 974	14.029 160	12.821 153	11.764 077	10.835 527	21
22	15.936 917	15.167 125	14.451 115	13.163 003	12.041 582	11.061 241	22
23	16.443 608	15.620 410	14.856 842	13.488 574	12.303 379	11.272 187	23
24	16.935 542	16.058 368	15.246 963	13.798 642	12 550 358	11.469 334	24
25	17.413 148	16.481 515	15.622 080	14.093 945	12.783 356	11.653 583	25
26	17.876 842	16.890 352	15.982 769	14.275 185	13.003 166	11.825 779	26
27	18 327 031	17.285 365	16.329 586	14.643 034	13.210 534	11.986 709	27
28	18.764 108	17.667 019	16.663 063	14.898 127	13.406 164	12.137 111	28
29	19.188 455	18.035 767	16.983 715	15.141 074	13.590 721	12.277 674	29
30	19.600 441	18.392 045	17.292 033	15.372 451	13.764 831	12.409 041	30
31	20.000 428	18.736 276	17.588 494	15.592 811	13.929 086	12.531 814	31
32	20.338 766	19.068 865	17.873 552	15.802 677	14.084 043	12.646 555	32
33	20.765 792	19.390 208	18.147 646	16.002 549	14.230 230	12.753 790	33
34	21.131 837	19.700 684	18.411 198	16.192 204	14.368 141	12.854 009	34
35	21.487 220	20.000 661	18.664 613	16.374 194	14.498 246	12.947 672	35
36	21.832 252	20.290 494	18.908 282	16.546 852	14.620 987	13.035 208	36
37	22.167 235	20.570 525	19.142 579	16.711 287	14.736 780	13.117 017	37
38	22.492 462	20.841 087	19.367 864	16.867 893	14.846 019	13.193 473	38
39	22.808 215	21.102 500	19.584 485	17.017 041	14.949 075	13.264 928	39
40	23.114 772	21.355 072	19.792 774	17.159 086	15.046 297	13.331 709	40

PROB. CLXXI.—TO FIND THE COST OF A LOAN.

RULE.—*Add the present worth of a share (computed by Prob. CLXVI, Annuities) to the present worth of the annuity required for the loan.*

PROB. CLXXII.—TO FIND THE ACTUAL RATE OF INTEREST PAID FOR A LOAN.

RULE.—*Add to the cost of the share the amount of an annuity required for the loan, and from this sum subtract the amount loaned. Then divide this difference by the product of the time in years and the amount loaned.*

EXAMPLES.

2050. Suppose a building association required a monthly payment of $2.00, and guaranteed $500.00 to each shareholder at its close, what will a share cost, if it close in 7 years, allowing 6 per cent., simple interest?

2051. What will a share cost, as above, at compound interest?

2052. What will it cost to borrow $200.00 on a single share, from the above association, they requiring $1.00, in addition to the monthly payments, as interest, for 7 years?

NOTE.—The present worth of the share in this case is nothing.

2053. Which would be preferable, to take the above loan and its monthly payments, to continue for 8 years, or to borrow the same sum for the same time, at 6 per cent., compound interest, and what is the difference?

2054. If a building association require a monthly payment of $1.00, and guarantee $200.00 on each share at the close, what will a share cost if it close in 8 years, allowing 6 per cent. simple interest?

2055. What is the sum of the answers on this page?

Ans. $838.05.

What is the derivation of the term *annuity?* What is the present worth of an annuity? Wherein do Arithmetical and Geometrical Progression differ from each other? Give Prob. CLXXI. Rule. Prob. CLXXII. Rule. Which should you prefer, to borrow money on a note for 7 years, or obtain it from a building association for the same time? Why?

REVIEW OF PARTS I, II, III, IV, V.

MISCELLANEOUS EXAMPLES.

2056. The town of Danvers has a debt of $75000, which is to be paid in 5 years. They wish to establish a sinking fund to pay this amount when due. The taxable property of the town is estimated at $2987000. What must be the annual tax on a dollar to secure the amount, provided the fund be invested at 6 per cent., compound interest? Ans. .00445 per cent.

2057. What is the square of $9\frac{17}{181}$?

2058. What is the square root of $\frac{3}{5}$?

2059. What is the cube of $9\frac{1}{8}$?

2060. What is the cube root of $912\frac{1}{4}$?

2061. What will a share in the Potomac Building Association cost, if a monthly payment of $2 is required for 8 years?

2062. James Bright gave his note of $786.25 due in 9 months, with interest. He is offered a discount of 7 per cent. if he will pay it now. What sum will pay the debt?

2063. Harvey Wright lent Harvey Kingsbury $500 for 2 months, Dec. 28th, 1860, at 6 per cent. When was this note due?

2064. What rate per cent. did Kingsbury actually pay for the money?

2065. What is the area of a circle whose diameter is 216.826 feet?

2066. How large a square could this circle contain?

2067. The quotient is $72096\frac{9}{89}$. What is the divisor and dividend?

2068. Sum of the above answers? Ans. 6455549.6166.

2069. The product is $117\frac{9}{11}$, and two factors are $3\frac{3}{7}$ and $9\frac{11}{117}$. What is the third factor? Ans. $3\frac{651}{836}$.

2070. Change $\frac{33}{187}$ and $\frac{513}{745}$ to their lowest form.

2071. Give the prime factors of 2491.

2072. Give the least common multiple of 289672 and 937862.

2073. What is the sum of a series whose ratio is $\frac{7}{8}$, first term 912, and number of terms 18?

2074. A. holds three notes against B., one of $200, due in 6 months; one of $300, due in 8 months; and one of $600, due in 10 months. B. desires to make one payment of the whole. When should it be made? Ans. In 8 m. 22 da.

2075. What should be the face of a note drawn on 4 months at 6 per cent., that, discounted, will pay the above debt, provided the several sums are on 7 per cent. interest? Ans. $1180.23.

2076. John Parsons holds the following notes against Samuel Smith, viz:

One dated Jan. 11th, 1852, for $600, at 3 years.
" July 10th, 1852, for $400, at 2 years.
" Jan. 18th, 1854, for $300, at 6 months.

Mr. S. desires to pay the whole at one payment. When should it be made?

2077. A person borrows five thousand dollars, at 5 per cent., paying the interest at the end of the year, and loans it out at 6 per cent., receiving the interest thereof quarterly. What will he make per annum?

2078. What is the difference between the simple interest of $600 for 4 years, and the compound interest of the same sum for that time?

2079. If, by working 12 hours per day, a man can plow ¼ of a field in 16 days, how long will it take him to do the remainder, if he work 15 hours per day?

2080. In the middle of a pond, 18 yards square, stands a flag-staff 120 feet high. What will be the length of a rope reaching from the top of the staff to the nearest edge of the pond?

2081. A certain room is 32 by 40 feet, and 10 feet high. What will be the length of a cord extending from one of the lower corners to the most distant upper one?

2082. What are the dimensions of a field containing 24 acres, having its breadth equal to two-thirds of its length?

2083. What is the sum of the last six answers on this page?
Ans. 404.55.

2084. How far will a man travel, if he walks $10\frac{3}{4}$ hours at $3\frac{1}{2}$ miles per hour, $11\frac{1}{4}$ hours at $3\frac{7}{8}$ miles per hour, and $9\frac{3}{5}$ hours at $4\frac{1}{6}$ miles per hour?

2085. How long would it take a man to drive over this distance, at the rate of $7\frac{3}{5}$ miles per hour?

2086. What are the contents of a field that is $237\frac{3}{5}$ rods on one side, and $341\frac{2}{7}$ rods on the other?

2087. How many gallons in a box $3\frac{3}{7}$ feet long, $2\frac{3}{8}$ feet wide, and $2\frac{1}{6}$ feet deep?

2088. How deep must a cylinder be, 14 inches in diameter, to contain 1 bushel?

2089. There is a cubic box whose inside dimensions are 16 inches each way. Into this is placed a ball 15 inches in diameter. How many gallons of water will fill the box?

2090. What is the sum of the above answers?

Ans. 81382.6828.

2091. There is a room to be plastered, painted, papered, and carpeted. The mason agrees to plaster and find the material for \$.25 per square yard. The painter finds stock and puts 3 coats of paint on the washboards for \$.37½ per square yard; paints the windows at \$.75 each, doors at \$2.50 each, fireplaces at \$3.75 each; and finds paper and hangs it on the walls for \$.27 per square yard. An upholsterer is to furnish and put down the carpet for \$1.37½ per square yard. The room is 36 feet long, 18 feet 6 inches wide, and 10 feet 4 inches high; wash boards 14 inches high. There are 6 windows, each 4 feet 3 inches wide, and 7 feet 4 inches high; and 5 doors, 4 feet 8 inches wide, 8 feet 2 inches high; also 2 fire-places, 3 feet 6 inches high, and 3 feet wide. What will the plasterer's bill amount to?

2092. How much the painter's?

2093. What is the entire expense? Ans. \$185.30.

2094. What is the difference between $16\frac{3}{4} \times 17\frac{3}{4}$ and $16\frac{1}{4} \times 17\frac{1}{4}$?

2095. If I pay \$369.21 for 1191 gallons of molasses, for what must I sell it per gallon to gain 13 per cent.? Ans. 35 cts.

2096. How much carpeting $\frac{7}{8}$ yards wide will it take to cover a floor 35 feet wide, and 86 feet 6 inches long?

2097. How much carpet paper $2\frac{1}{4}$ yards in width will cover the same floor? Sum of answers, 533.9505.

2098. A father willed $\frac{1}{2}$ of his property to his eldest son, $\frac{1}{3}$ to his second son, and $\frac{1}{6}$ to his daughter. The father and eldest son both were killed at the same time. How must the property be divided?

2099. A farmer has several tracts of land. The first tract, of 15 acres, is worth $25 per acre. The second, of 37 acres, is worth $67 per acre. The third, of 400 acres, is worth $18 per acre. How many acres must he take from his fourth field, worth $9 per acre, to be able to sell his land at $20 per acre?

2100. If a family of 15 persons consume $23\frac{1}{2}$ tons of soft coal in 8 months, how much coal would last 27 persons 3 years $6\frac{1}{2}$ months?

2101. How many cubic feet in a cellar that would contain it?

2102. What will it cost to insure my house, valued at $6500, at $\frac{7}{8}$ per cent. per annum?

2103. What is the sum of the last four answers?

Ans. 9363.475568.

2104. What is the average due of the following account?

James Allen, bot. of R. B. Entwistle.

Jan. 12,	1859.	Merchandize,	6 mos.	$347.21.
Feb. 26,	"	"	4 mos.	637.28.
Mar. 17,	"	"	7 mos.	858.98.
May 27,	"	"	5 mos.	329.18.
July 17,	"	"	3 mos.	817.37.
" 23,	"	"	9 mos.	128.19.
Aug. 3,	"	"	4 mos.	397.63.
" 17,	"	"	6 mos.	918.17.
Sept. 12,	"	"	7 mos.	412.39.
" 28,	"	"	3 mos.	1783.41.
" 30,	"	"	5 mos.	385.31.

2105. A fish was put in a cylinder 1 foot 6 inches in diameter, containing 14 inches of water, and the water was observed to rise 6 inches. What was the cubic contents of the *fish?*

2106. Sold a quantity of flour at ten dollars per barrel, and in so doing made a profit of 20 per cent. What did the flour cost me?

2107. Jos. C. Fearson sold a quantity of butter at 25 cents per pound, which gave him a profit of 25 per cent. If he had sold it at 18 cents per pound, what would have been the gain or loss per cent.?

2108. Marshall Jones bought a quarter section of government land at $1.25 per acre. He paid $2 per acre for breaking it up, $600 for fencing it, and $1200 for his buildings. After keeping his farm two years, he sold out at $20.01 per acre. How much did he make by the transaction, not allowing interest?

2109. What rate of interest did Mr. Jones make on the money he invested?

2110. How many rails will it take to enclose a square piece of land, containing 720,000 acres, with a fence five rails high, allowing fifteen rails to every two rods?

2111. If the tract of land were an equilateral triangle, containing the same number of acres, how many rails would it take?

2112. How many rails would it take, if the same number of acres were in the form of a regular hexagon?

2113. William Brown has a field which is fenced in with a rail fence, containing 16 rails to the rod. The field is in the form of a square, and contains as many acres as there are rails in the fence that surrounds it. How many acres in the field?

Sum of answers above, 1646422.68.

Analysis of Part VI.
Exchange, Philosophical and Astronomical Questions, and Practice.

CHAPTERS.	SECTIONS.	PAGE
I. EXCHANGE	1. Domestic Exchange	269
	2. Foreign Exchange	274
II. PHILOSOPHICAL AND ASTRONOMICAL QUESTIONS	1. Philosophical Questions	278
	2. Astronomical Questions	281
III. INFINITE DECIMALS		283
IV. ANALYSIS		288
V. PRACTICE		290
GENERAL REVIEW		292

PART VI.

EXCHANGE, PHILOSOPHICAL AND ASTRONOMICAL QUESTIONS, AND PRACTICE.

CHAPTER I.

EXCHANGE.

SECTION I.

DOMESTIC EXCHANGE.

§ 1. *Orders.*

393. An Order is a written document authorizing and requesting a person that owes you to pay a specified amount to a third party.

394. The General Form of an Order.

$\$500\frac{00}{100}$. *Washington, July 29, 1857.*

Wm. Orme, Esq. *Sir:*

Please pay to the order of Chas. T. Young five hundred dollars, and charge the same to me. *Aaron Brown.*

395. This order becomes payable by Charles T. Young endorsing or writing his name on the back of it.

396. If Mr. Orme refuses to pay the money to Mr. Young's order, Mr. Young could not commence an action at law with him, but must hold Mr. Brown responsible.

397. If Mr. Orme is unable to pay on presentation, but agrees to pay in a specified time, say 15 days, he writes across the face of the order,

"Payable fifteen days from date.

Wm. Orme."

398. An order may be given on time: as, Sixty days after sight please pay, &c. &c.; or, Please pay twenty days *after sight*, &c.

399. "Days of Grace" may or may not be allowed.

§ 2. *Acceptances.*

400. If you have an account which you wish paid, you must carefully make out a "Bill," naming each item, with its date, price, quantity, and amount.

On presenting this to the debtor, if he is unable to pay at the time, he may write his name across the *face* of the bill, and then, *the bill* is called an acceptance, and becomes in all respects a note, and is negotiable. In the same way, *an order* becomes an acceptance. (See Art. 397.)

§ 3. *Notes.*

401. A Note is a written or printed document, promising to pay a specified sum, in money or other property, at some future time.

402. Notes are of several kinds, as Promissory, Negotiable, Bankable Joint and Several, &c.

403. Form of Promissory Note.

\$750$\frac{00}{100}$. Baltimore, Aug. 1, 1859.

For value received, I promise to pay to Jesse D. Bright seven hundred fifty dollars, on demand, with interest Thomas Benton.

404. Thomas Benton is the "maker" of the note.

405. Jesse D. Bright has the full title to this note, and cannot transfer the right to another, except for collection in his name. He is the "holder."

406. No person can be compelled to pay money to another, unless he has received an equivalent; hence, the words "value received" are essential to every note.

407. A promissory or other note may be on demand, or on time, and is then called a

Note on Time.

$\$476\frac{00}{100}$. Coventry, Ct., Aug. 3, 1859.

Two years after date, for value received, I promise to pay Charles Underwood four hundred seventy-six dollars, with interest.

Charles Talcott.

408. The words on interest may be omitted; legal interest will be understood.

409. In most places, days of grace are allowed, unless the note distinctly says "without grace."

410. A note is "*payable*" on the day it was promised to be paid, but not "*due*" till the last day of grace, unless that day falls on Sunday, or on a legal holiday. And if a note is payable on Sunday, and a legal holiday comes on Saturday, the note must be paid on Friday.

411. To make a legal demand for the payment of a note, the note must be *presented to the maker at the specified time and place.*

412. If he refuses or neglects to pay it, it is said to be dishonored.

Negotiable Notes.

413. A negotiable note is one that may be transferred from one person to another. They are of two forms: 1st, those payable to a person or order; 2d, those payable to a person or bearer.

414. Those payable to order become payable by the endorsement of the name of the payee on the back of the note.

415. The second class are payable to the holder, whoever he may be.

Forms of Negotiable Notes.

$\$786\frac{00}{100}$ Philadelphia, Sept. 7, 1858.

Three months after date, for value received, I promise to pay to the order of Rufus Griswold, seven hundred eighty-six dollars, with interest at 6 per cent.

Solomon Weeks.

$927. Norwich, Ct., Oct. 8, 1858.

Thirty days after date I promise to pay Reuben Young, or bearer, nine hundred twenty-seven dollars, value received, without interest and without grace.

Samuel Hepburn.

416. The first note is made payable by Mr. Griswold's placing his name on the back of the note, called "endorsing it."

417. By endorsing, the "endorser" becomes responsible for, 1st, the genuineness of the note; 2d, for legal possession and right of transfer; and, 3d, for its payment.

418. Every person who places his name on the back of a note, becomes in a like manner responsible, whether he has an interest in it or not.

419. All endorsers are freed from their obligations if the note is not presented to them for payment, or protested, on the very day it is payable.

420. A Protest is a document made out by a Notary Public, certifying that a note is dishonored, and that the endorsers are responsible.

421. A bankable note does not differ materially from a negotiable note, but usually has the place of payment named.

Form of a Bankable Note.

$1276. Boston, Oct. 31, 1859.

Sixty days after date, for value received, I promise to pay Calvin Brewer, or order, twelve hundred and seventy-six dollars. Payable at Suffolk Bank, Boston.

James L. Burgess.

Form of a Joint Note.

$370. Springfield, Mass., Nov. 5, 1858.

Thirty days after date, for value received, we jointly promise to pay Edwin Case, or order, three hundred and seventy dollars. Payable at Yale Bank, New Haven.

Oliver Brown.
Cyrus Brown.

Form of Joint and Several Note.

$50. New York, Oct. 11, 1859.

One month after date we jointly and severally promise to pay Chas. Talcott, or order, fifty dollars. Payable at Hartford Bank, Ct.

Lemuel Wilcox.

422. If a note remains a specified number of years without any payment, it becomes outlawed, and cannot be collected. It may be renewed, however, by the payment of any amount endorsed on the back before it is outlawed.

423. A note under seal or attested, is not outlawed for 20 years.

SECTION II.

FOREIGN EXCHANGE.

424. Exchange takes into consideration the payment of a sum of money by an order at a distance.

425. A Bill of Exchange is a written or printed order, authorizing the payment of a specific sum of money on the maker's account.

426. There are always three parties, and generally four: 1st, the *Drawer* or *maker* of the bill; 2d, the *Drawee*, the person receiving payment; 3d, the *Payee*, the person making payment; and 4th, the *Remitter*, or purchaser of the bill.

427. Three bills are drawn for the same amount, and together, are called a set of exchange. This is to guard against loss.

428. When the parties reside in the same country it is called Inland Exchange.

429. When the parties reside in different countries it is called Foreign Exchange.

430. Bills of Exchange may be endorsed, and the parties are liable the same as in the case of notes.

431. The Intrinsic Par of Exchange is a term used to compare the intrinsic value of the coins of different countries.

432. The Commercial Par of Exchange is a comparison of coins, according to their commercial value.

433. The Course of Exchange is the price paid at one place for bills drawn on another.

434. Bills of Exchange are at a premium when they cost more than they are drawn for.

435. Bills of Exchange are at a discount when they cost less than they are drawn for.

436. Previous to 1842, 1 pound sterling was worth $\$4.44\frac{4}{9}$, and this is called the "old par."

437. Since 1842, 1 pound sterling is worth $4.84, and this is called the "new par."

438. Exchange between England and the United States is still estimated at so much per cent. on the "old par."

439. At old par, 9 pounds sterling equal $40.

PROB. CLXXIII.—TO FIND THE VALUE OF A SET OF EXCHANGE ON LONDON.

RULE.—1. *Reduce the amount of the bill to pounds.*

2. *Take* 40 *ninths of that amount.*

3. *To this, add the amount of premium.*

ILLUSTRATION.—What is the value of a set of exchange on London, amounting to £2300, premium 8 per cent.?

	Amount in Pounds, 2300
	40
10222$\frac{2}{9}$	9)92000
.08	10222$\frac{2}{9}$
817.77$\frac{7}{9}$	817$\frac{7}{9}$
	$11040 Ans.

EXAMPLES.

2114. A merchant in Baltimore wishes to remit to Liverpool £4786 11s., exchange being 9½ per cent. What will it cost him?

2115. A man wishes to remit £8000 to his friend in Glasgow, exchange being 10¾ per cent. What will it cost him?

2116. James Brown bought on speculation a set of exchange for 7¾ per cent., amounting to £7468 9s. What did it cost?

2117. Charles Brown can pay a bill in London by remitting $21000 in specie, or by purchasing a bill of exchange for £4325 at 8¼ per cent. premium. What is the difference between the two methods of payment?

2118. Sum of the above answers? Ans. 98629.84.

PROB. CLXXIV.—TO FIND THE AMOUNT OF A BILL OF EXCHANGE, THAT A GIVEN SUM WILL PURCHASE.

RULE.—1. *Find the cost of the money unit.*

2. *Divide the given sum by that cost.*

ILLUSTRATION.—What amount on London will $8000 purchase at $9\frac{1}{4}$ per cent. premium?

1.0925	$4.8555)8000.000(1650.011
40	4855 555
9)43700	3166 666
$4.855\frac{5}{9}$ = Cost of a pound.	2913 333
	243 333
	242 775
Ans. £1650.011.	558
	485
	73

EXAMPLES.

2119. What amount of exchange will $9786 purchase on Edinburg, exchange at $9\frac{3}{8}$ per cent.?

2120. What amount of exchange on Dublin can I purchase for $15787.50, exchange at $10\frac{1}{2}$ per cent.?

2121. What amount of exchange on London can I purchase for $12757.21, exchange 9 per cent.?

2122. What amount of exchange can I purchase on Liverpool for 1576 bales of cotton, 450 lbs. each, at $7\frac{1}{2}$ cents per lb., after deducting my agent's commission of $3\frac{1}{2}$ per cent., exchange being $8\frac{3}{8}$ per cent.?

2123. Bought 12 hhds. of sugar, each 987 lbs., at $9\frac{1}{11}$ cents, at Havana, and shipped to Liverpool to a commission merchant for sale and returns. He writes he has paid freight and other charges, $367.81. Also, that he has sold it for $11\frac{1}{2}$ cents, and charged me $3\frac{1}{2}$ per cent. commission. What bill of exchange will settle the account between us, exchange being 8 per cent.?

2124. What is the sum of the answers on this page?

Ans. $18714.74.

Tables of Foreign Coins.

English Money.

4	Farthings	make	1 Penny (*d.*).
12	Pence	"	1 Shilling (*s.*).
20	Shillings	"	1 Pound (£).
21	Shillings	"	1 Guinea (G.).

Pound Sterling of Great Britain	$4.84
Pound of Canada, Nova Scotia, N. Brunswick, and Newfoundland	4.00
Franc of France and Belgium	.186
Livre Tournois of France	.185
Florin of Netherlands and the Southern States of Germany	.40
Marc Banco of Hamburg	.35
Rix Dollar of Bremen	.78¾
Thaler of Prussia and Northern States of Germany	.69
Specie Dollar of Denmark	1.05
Specie Dollar of Sweden and Norway	1.06
Rial of Vellon of Spain	.05
Rial of Plate of Spain	.10
Milree of Portugal	1.12
" of Azores	.83⅓

Silver Rouble of Russia	$ .75
Florin of Austria	.485
Lira of Lombardo-Venetian kingdom and Tuscany	.16
Lira of Sardinia	.186
Ducat of Naples	.80
Ounce of Sicily	2.40
Leghorn Livres	.16
Tael of China	1.48
Rupee, Company and of India	.445
Pagoda of India	1.84
Doubloon of Havana	17.00
Dollar of Laguayra	.75
Pezza of Leghorn	.87
Tuscan Crown	1.05
Scudo of Malta	.40
Nutkeel of Mogadore	.75
Mamoody of Morocco	.05
Scudo of Naples	.95
Dollar of Port au Prince	.33
Rix Dollar of Riga	.69

CHAPTER II.

PHILOSOPHICAL AND ASTRONOMICAL QUESTIONS.

SECTION I.

PHILOSOPHICAL QUESTIONS.

Prob. CLXXV.—To find how far a body will fall in a given time.

Rule.—*Multiply the square of the time in seconds by* 16.083, *and the product will be the answer in feet.*

Prob. CLXXVI.—To find the ascent of a body projected upwards.

Rule.—*The square of* ½ *the time in seconds, multiplied by* 16.083, *will give the distance in feet.*

Prob. CLXXVII.—To find the height of a vertical stream.

Rule.—*Determine the velocity from the quantity of water forced through the pipe in a given time. Divide the velocity by* 8.02, *and the square of the quotient will be the distance.*

Prob. CLXXVIII.—To determine distance by sound.

Rule.—*Multiply the time in seconds by* 1100.

Note.—Sound moves about 1 mile in 5 beats of the pulse.

Prob. CLXXIX.—To find the strength of a rope.

Rule.—*The square of the circumference, multiplied by* 200, *will give the weight in pounds it will safely bear.*

Note.—White ropes are ⅓ more durable.

Prob. CLXXX.—To determine the power of the lever.

Rule.—*As the short arm of the lever is to the long arm, so is the power to the weight.*

Prob. CLXXXI.—To determine the power of the wheel.

Rule.—*As the diameter of the axle is to the diameter of the wheel, so is the power to the weight.*

Prob. CLXXXII.—To determine the power of the pulley.

Rule.—*Multiply the power by the number of ropes that support the weight.*

Give Prob. 175 and Rule. 176, 177, 178, 179, 180, 181, 182.

PROB. CLXXXIII.—TO DETERMINE THE POWER OF AN INCLINED PLANE.

RULE.—*As the height of the plane is to its length, so is the power to the weight.*

PROB. CLXXXIV.—TO DETERMINE THE POWER OF THE WEDGE.

RULE.—*As ½ the back of the wedge is to its length, so is the power to its resistance.*

PROB. CLXXXV.—TO DETERMINE THE POWER OF THE SCREW.

RULE.—*As the distance between the threads is to the distance the power moves through in one revolution, so is the power to the weight.*

PROB. CLXXXVI.—TO DETERMINE THE PRESSURE OF A FLUID ON THE BOTTOM OF A VESSEL.

RULE.—*Multiply the area of the base in feet by the height in feet, and the product by the weight of a cubic foot of the fluid.*

NOTE.—Water weighs 62.5 lbs. per cubic foot.

PROB. CLXXXVII.—TO DETERMINE THE PRESSURE OF A FLUID ON THE SIDE OF A VESSEL.

RULE.—*Multiply the area of the side by ½ the depth, and the product by the weight of a cubic foot of the fluid.*

PROB. CLXXXVIII.—TO FIND THE AMOUNT OF WATER THAT WILL FLOW OUT OF A PIPE.

RULE.—*Multiply the square root of the depth by 5.4, and this product by the area of the orifice in feet, and it will give the number of cubic feet passing out.*

EXAMPLES.

2125. How far will a ten-pound weight fall in 15 seconds?

2126. How high from a plain was a body thrown that was 37 seconds in returning to the surface of the ground?

2127. How high from a plain was a bullet thrown that was 74 seconds in passing from the musket to the ground?

2128. How high from a plain will an engine throw a vertical stream, through an inch pipe, which passes out with a velocity of 96.24 feet per second?

2129. What is the sum of the answers? Ans. 31284.7087.

Give Prob. 183 and Rule. 184, 185, 186, 187, 188.

2130. Where must the fulcrum be placed to raise a weight of 7365 lbs. with a power 825, and lever 7 ft. long? Ans. $8\frac{6}{13}$ in.

2131. What weight will a lever raise, long arm 4 feet, short arm 3 inches, and power 300 lbs.?

2132. What power will it require to raise 6420 lbs. with a lever, long arm 6 feet, short arm 6 inches?

2133. What weight can be raised with pulleys sustaining the weight with 7 ropes, by a power of 318 lbs.?

2134. What weight can be raised with a wheel and axle, diameter of the wheel 4 feet, and of the axle 8 inches, power 87 lbs.? Sum of the last 4 answers, 8083.

2135. What weight can be rolled up an inclined plane 37 ft. long and 27 in. high, with a power of 187 lbs.? Ans. $3075\frac{1}{9}$.

2136. What is the force of a wedge 19 inches long and 8 inches across the back, if a power of 2000 lbs. be applied to the head?

2137. What will be the force of a screw whose threads are an $\frac{1}{8}$ of an inch apart, moved by a lever 7 feet long, with a power of 71 lbs.?

2138. What is the pressure per square foot on the bottom of a ship, that draws 20 feet of water?

2139. What is the pressure on one side of a square cistern filled with water, 8 feet on a side, and 10 feet deep?

2140. What is the pressure on one side of the same cistern, $\frac{1}{2}$ full?

2141. What amount of water will be thrown upon a water-wheel from a vertical outlet 9 inches wide and 8 inches deep, and 11 feet below the surface of the water?

2142. Observed the flash of a cannon, and 24 seconds after heard the report. What distance was the cannon from me, making no allowance for the movement of light?

2143. What is the sum of the last 7 answers?

Ans. 341798.8329.

2144. What is the strength of a rope $2\frac{1}{3}$ inches in circumference? Ans. $1088\frac{8}{9}$.

SECTION II.

ASTRONOMICAL QUESTIONS.

PROB. CLXXXIX.—TO FIND THE DOMINICAL LETTER, OLD STYLE.

RULE.—*To the given year add one fourth of itself, plus* 4, *and divide the sum by* 7. *If there is no remainder, the dominical letter is G; if* 1 *remainder, F, and so on in inverse order. The dominical letter for the first two months of leap year will be the following letter.*

NOTE.—Old Style closed in 1752 in England and America, and New Style commenced. In Russia the change was made in 1830.

PROB. XCC.—TO FIND THE DOMINICAL LETTER, NEW STYLE.

RULE.—1. *Divide the centuries by* 4, *and take the remainder from* 3; *then add twice this remainder to* $\frac{5}{4}$ *of the odd years, rejecting fractions, and divide the sum by* 7.

2. *If no remainder, the dominical letter is G; if* 1 *remainder, F, and so on in inverse order.*

3. *The dominical letter for leap year as in Problem CLXXXIX.*

PROB. XCCI.—TO FIND THE DAY OF THE WEEK CORRESPONDING TO ANY GIVEN DAY OF THE MONTH.

RULE.—1. *The dominical letter indicates the day on which the first Sunday in January will fall, the* 1*st day being A,* 2*d B,* 3*d C, &c.*

2. *From this, by counting backwards from the dominical letter, ascertain the day of the week for the* 1*st of January.*

3. *The following couplet will give the day for the commencement of each month:—*

At Dover Dwells George Brown, Esquire,
Good Colonel French and David Friar.

4. *From the* 1*st of the month any given day may be easily computed.*

EXAMPLES.

2145. What was the Dominical Letter for 1492?

2146. On what day of the week did Columbus discover America, Oct. 11th, 1492? Ans. Thursday.

2147. On what day of the week was Washington born, 22d February, 1732? Ans. Friday.

2148. On what day of the week did he die, 14th December, 1799? Ans. Saturday.

2149. The Great Fire in New York happened the 16th December, 1835. What day of the week? Ans. Wednesday.

2150. Marshal Ney was shot the 7th of December, 1815. On what day of the week? Ans. Thursday.

2151. Tea was destroyed the 16th of November, 1773, in Boston. What day of the week? Ans. Tuesday.

2152. Daniel Webster died the 24th of October, 1852, at Green Harbor, Marshfield, Mass. On what day of the week? Ans. Sunday.

2153. The Battle of Lake Erie took place the 10th of September, 1813. What day of the week? Ans. Friday.

2154. The Dudley Observatory was founded at Albany, N. Y., on the 28th of August, 1856. What day of the week? Ans. Thursday.

2155. President Zachary Taylor died at Washington, D. C., the 9th day of July, 1850. What day of the week? Ans. Tuesday.

2156. Henry Clay died the 29th of June, 1852, at Ashland, Kentucky. On what day of the week? Ans. Tuesday.

2157. Is it a fact "that the Battle of Waterloo was fought on Friday, the 18th of June, 1815"? Ans. Jan. 18th was Sunday.

2158. Did "the pilgrims land at Plymouth on the 11th of December, 1620, on Monday, O. S."? Ans. Yes.

2159. Did "the Battle of New Orleans occur on Saturday, Jan. 8th, 1815"? Ans. No. Jan. 8th was on Sunday.

2160. On what day of the week did the inauguration of Washington take place, April 30th, 1789? Ans. Thursday.

CHAPTER III.

INFINITE DECIMALS.

440. If, when a fraction is reduced to its lowest terms, the denominator contains any prime factor besides 2 and 5, the fraction cannot be changed to a decimal.

441. If new ciphers be annexed to the numerator, and it be divided by the denominator, the division will never cease, and the quotient is called an Infinite Decimal.

442. This quotient will consist of one or more figures continually repeated in the same order, and hence these Infinite Decimals are called Circulating Decimals.

443. *A Single Repetend* is when a single figure repeats, and is marked thus: $.\dot{3} = 3333$, &c.

444. *A Compound Repetend* is when several figures repeat, and is marked thus: $.\dot{3}4\dot{2} = 342342342$, &c.

445. *A Pure Repetend* is one that contains only repeating figures: as, $.\dot{6}65\dot{4}$.

446. *A Mixed Repetend* is one that contains figures that do not repeat: as, $.425\dot{6}2\dot{4}$.

447. *Similar Repetends* are those that commence with the same denomination: as, $.53\dot{3}\dot{6}$, $.83\dot{2}5\dot{6}$.

448. *Dissimilar Repetends* are those that do not commence with the same denomination: as, $.53\dot{3}\dot{6}$, $.1\dot{3}8\dot{8}$.

449. *Conterminous Repetends* are those that end with the same denomination: as, $.98\dot{4}33\dot{3}$, $.333\dot{3}3\dot{3}$.

450. *Similar* and *Conterminous Repetends* are those that commence at the same denomination, and close at the same denomination: as, $.4\dot{3}6\dot{2}$, $.4\dot{6}6\dot{6}$.

451. *Similar* and *Conterminous Repetends* are of the same denomination, for they have the same denominator.

Prob. XCII.—To change a pure repetend to a common fraction, without changing its value.

Rule.—*Take as many 9's for a denominator as the repetend has figures.*

Illustration.—Change $.\dot{3}$ to a common fraction.

$\dot{3} = \frac{3}{9} = \frac{1}{3}$ Ans.

EXAMPLES.

2161. Change $.6$ to a common fraction.
2162. Change $7.\dot{1}$ to a common fraction.
2163. Change $.\dot{1}2\dot{3}$ to a common fraction.
2164. Change $.\dot{7}\dot{1}$ to a common fraction.
2165. Change $92.\dot{7}\dot{8}$ to a common fraction.
2166. Change $32.\dot{7}69\dot{1}$ to a common fraction.

Prob. XCIII.—To change a mixed repetend to a common fraction, without changing its value.

Rule.—1. *Change the repetend to a fraction, as in the last problem. This will give a mixed fraction.*

2. *Under this write* 1, *with as many ciphers as there were figures in the given finite decimal.*

Illustration.—Change $.7\dot{3}$ to a common fraction.

$.7\dot{3} = .7\frac{3}{9} = \frac{7\frac{3}{9}}{10} = \frac{66}{90} = \frac{22}{30} = \frac{11}{15}$ Ans.

EXAMPLES.

2167. Change $5.0\dot{2}$ to a common fraction. Ans. $5\frac{2}{90}$.
2168. Change $13.15\dot{7}\dot{8}$ to a common fraction.
2169. Change $.3\dot{2}\dot{7}$ to a common fraction.
2170. Change $.17\dot{1}$ to a common fraction. Ans. $\frac{77}{450}$.
2171. Change $3.5\dot{7}2\dot{3}$ to a common fraction. Ans. $\frac{1906}{3330}$.
2172. Change $.5\dot{3}46\dot{6}$ to a common fraction. Ans. $\frac{53461}{99990}$.
2173. Change $\dot{7}.0\dot{7}$ to a common fraction. Ans. $7\frac{77}{999}$.

PROB. XCCIV.—TO CHANGE REPETENDS TO A COMMON DENOMINATOR.

RULE.—*Make them similar and conterminous.*

ILLUSTRATION.—Change $4.\dot{3}$, $8.\dot{5}\dot{2}$, and $.5$ to a common denominator.

It will be seen at once that it will make no difference in the value of $4.\dot{3}$, whether the 3 repeats, or 33, or 333; also, that the value will remain the same wherever it begins to repeat.

In the number $8.\dot{5}\dot{2}$, the value will be the same if $\dot{5}25\dot{2}$ or $\dot{5}2525\dot{2}$ repeat, and there will be no difference if $\dot{2}5$ or $\dot{2}52\dot{5}$ repeat.

Hence, we may commence the repetends at the same distance from the decimal point, and continue them as many places as is equal to the least common multiple of the number of repeating figures in each repetend. Hence,

$$\begin{aligned} 4.\dot{3} &= 4.3\dot{3}\dot{3} \\ 8.\dot{5}\dot{2} &= 8.5\dot{2}\dot{5} \\ .5 &= .5\dot{0}\dot{0} \end{aligned}$$

EXAMPLES.

2174. Change $.13\dot{8}$, $.5\dot{3}$, and $.62\dot{3}$ to a common denominator.
2175. Change $.57\dot{6}$, $9.\dot{2}8\dot{7}$, and $.36$ to a common denominator.
2176. Change $8.7\dot{6}$ and $5.\dot{9}2\dot{7}$ to a common denominator.
2177. Change $56.\dot{7}$ and $42.3\dot{5}$ to a common denominator.
2178. Change $93.7\dot{6}$ and $4.\dot{1}73\dot{2}$ to a common denominator.
2179. Change $81.\dot{3}\dot{3}$ and $\dot{1}9.87\dot{6}$ to a common denominator.
2180. Change $5.0\dot{2}$ and $13.15\dot{7}\dot{8}$ to a common denominator.
2181. Change $72.3\dot{8}$ and $24.\dot{0}8\dot{1}$ to a common denominator.
2182. Change 3.6 and $3.5\dot{7}2\dot{3}$ to a common denominator.
2183. Change $32.\dot{7}69\dot{1}$ and $35.$ to a common denominator.
2184. Change $58\dot{3}.03\dot{0}$ and $.70\dot{7}$ to a common denominator.

PROB. XCCV.—TO ADD CIRCULATING DECIMALS.

RULE.—1. *Change to a common denominator.*

2. *Add the numerators, observing to add to the sum of the right hand column of the repetends as many as you would carry to the right hand column of the finite decimal.*

ILLUSTRATION.—Add $7.\dot{3}$, $6.28\dot{6}$, and $.2\dot{5}6\dot{7}$.

$$\begin{array}{rcr} 7.\dot{3} & = & 7.33\dot{3}3\dot{3} \\ 6.28\dot{6} & = & 6.28\dot{6}6\dot{6} \\ .2\dot{5}6\dot{7} & = & .25\dot{6}7\dot{5} \\ \hline & \text{Ans.} & 13.95\dot{6}7\dot{5} \end{array}$$

If these repetends had been repeated, the left hand column of the repetend would come on the right of the first column to be added, and then we should have 1 to carry to 14, making 15. But the value is the same, whether written or not.

EXAMPLES.

2185. What is the sum of $31.6\dot{2} + 60.\dot{8}\dot{2} + 71.\dot{1}6\dot{4} + 45.\dot{2}\dot{4}$?
2186. What is the sum of $24.\dot{1}3\dot{2} + 2.2\dot{3} + 85.\dot{2}\dot{4}$?
2187. What is the sum of $3.6 + 78.3\dot{4}7\dot{6}$?
2188. What is the sum of $99.083 + 1.5 + .\dot{8}1\dot{4}$?
2189. What is the sum of $37.2\dot{3} + .2\dot{6} + 9.928$?
2190. What is the sum of $48.\dot{2}\dot{3} + 16.1\dot{3}$?
2191. Sum of the answers? Ans. 615.604602511′.

PROB. XCCVI.—TO MULTIPLY CIRCULATING DECIMALS.

RULE.—1. *Change them to common fractions; then proceed to multiply as in Prob.*

2. *Change the product to a circulating decimal.*

EXAMPLES.

2192. What is the product of $72.3\dot{8}$ by $45.\dot{2}\dot{4}$?
2193. What is the product of $24.\dot{1}3\dot{2}$ by $2.2\dot{3}$?
2194. What is the product of 60.25 by $45.\dot{2}\dot{4}$?
2195. What is the product of $24.\dot{0}8\dot{1}$ by $.2\dot{5}$?
2196. What is the product of $48.\dot{2}\dot{3}$ by $16.1\dot{3}$?
2197. What is the product of $.2\dot{4}\dot{5}$ by $7.\dot{3}$?
2198. What is the product of $17.\dot{3}$ by $19.\dot{6}$?

PROB. XCCVII.—TO SUBTRACT CIRCULATING DECIMALS.

RULE.—1. *Change them to a common denominator.*

2. *Subtract, observing to add* 1 *to the right hand figure of the subtrahend when its repetend is greater than that of the minuend.*

ILLUSTRATION.—Subtract $7.5\dot{6}3\dot{2}$ from $12.6\dot{3}7\dot{1}$.

$$\begin{array}{r} 12.6\dot{3}71 \\ 7.5\dot{6}3\dot{2} \\ \hline 5.0\dot{7}3\dot{8} \end{array}$$

In this example, if the repetends were repeated, there would be 1 to carry to 2, the right hand figure of the subtrahend. But then it would be the same, whether it was actually written or not.

EXAMPLES.

2199. What is the difference between $85.\dot{6}2$ and $13.7\dot{6}43\dot{2}$?
2200. What is the difference between $46.\dot{1}2\dot{3}$ and $41.\dot{3}$?
2201. What is the difference between $801.\dot{6}$ and 400.75?
2202. What is the difference between 1419.6 and $1312.\dot{9}$?
2203. What is the difference between $4234.\dot{3}\dot{6}$ and $16.2\dot{5}$?
2204. What is the difference between $963.\dot{5}\dot{6}$ and $842.1\dot{5}$?
2205. What is the difference between $796.03\dot{5}$ and $82.\dot{4}2\dot{1}$?

PROB. XCCVIII.—TO DIVIDE CIRCULATING DECIMALS.

RULE.—*Change to common fractions, divide, and change the quotient to a circulating decimal.*

EXAMPLES.

2206. What is the quotient of 24.081 by $.\dot{3}8\dot{6}$?
2207. What is the quotient of $124.\dot{2}9\dot{7}$ by $7.\dot{7}\dot{2}$?
2208. What is the quotient of $8574.\dot{3}$ by $.\dot{1}2\dot{3}$?
2209. What is the quotient of $963.\dot{2}9\dot{7}$ by $3.\dot{9}7\dot{3}$?
2210. What is the quotient of 1263.6 by $41.\dot{3}$?
2211. What is the quotient of $16342.03\dot{5}$ by $16.2\dot{5}$?
2212. What is the quotient of $24.\dot{1}3\dot{2}$ by $2.2\dot{3}$?
2213. What is the quotient of $37.2\dot{3}$ by $85.\dot{6}\dot{2}$?
2214. What is the quotient of $97.2\dot{6}$ by $31.6\dot{2}$?

CHAPTER IV.

ANALYSIS.

452. Analysis is the name given to the exposition of the principles to be employed in solving a problem?

453. Every example has its own peculiarity; hence, no specific directions can be given for analysis?

454. The conditions of the question will suggest the method of solution, and the success in discovering the best process, and of making the shortest application, will depend upon the ingenuity and skill of the pupil.

455. This may emphatically be called the COMMON SENSE RULE.

EXAMPLES.

2215. If 25 yards of cloth cost $150, what will 81 yards cost?

1. What will 1 yard cost?
2. What will 81 yards cost? Ans. $486.

2216. My house, which is worth $6000, is $\frac{7}{8}$ the value of Lemuel Packard's. How much is his worth?

1. What is $\frac{7}{8}$ worth?
2. What is $\frac{1}{8}$ worth? Ans. $6857\frac{1}{7}$.

2217. Richard can mow a piece of grass in 11 days. George can mow the same piece in 15 days. In what time will they both mow the grass?

1. How much will Richard mow in 1 day?
2. How much will George mow in 1 day?
3. How much will they both mow in 1 day?
4. How many days to do the whole? Ans. $6\frac{9}{26}$.

2218. A. can plow a field in 10 days, B. in 7 days, and C. in 12 days. How long will it take if all work together?

2219. If $25\frac{7}{9}$ yards of cloth cost $157\frac{1}{3}$, what will $81\frac{3}{7}$ yards cost?

2220. If, when flour is \$9 per barrel, the four-cent loaf weighs 8 oz., what ought it to weigh when flour is \$6 per barrel, allowing nothing for its manufacture?

1. What would it weigh if flour was \$1 per barrel?

2. What if flour was \$6 per barrel?

2221. Brown owned $\frac{7}{20}$ of a ship, which sold for \$33570. What is $\frac{2}{7}$ worth, at the same rate?

1. What is $\frac{1}{20}$ worth?

2. What is the whole ship worth?

3. What is $\frac{1}{7}$ worth?

4. What are $\frac{2}{7}$ worth? Ans. \$$27404\frac{4}{49}$.

2222. If 35 horses eat 64 bushels of oats in 9 days, how many bushels would 17 horses eat in the same time? Ans. $31\frac{3}{35}$.

2223. If a shed 13 feet high cast a shadow 4 feet, how high is a monument whose shadow, at the same time, is $106\frac{3}{5}$ feet in length? Ans. $346\frac{9}{20}$ ft.

2224. What part of 5 mi. 2 fur. 12 rds. 3 yds. 2 ft. is 37 rds. 2 ft. 9 in.? Ans. $\frac{2453}{111716}$.

2225. If $57\frac{1}{2}$ bushels of wheat are ground in 2 hours 37 minutes, how long will it take to grind $117\frac{9}{10}$ bushels of wheat?

2226. If $57\frac{3}{19}$ yards of cloth cost \$$212.37\frac{1}{2}$, what will $117\frac{13}{15}$ yards of the same kind of cloth be worth? Ans. $437943\frac{953}{1929}$.

2227. If the steamer perform a trip to Liverpool in $9\frac{1}{9}$ days, travelling $213\frac{3}{5}$ miles per day, how long would it take the ship Superior to pass the same distance, sailing $161\frac{3}{11}$ miles per day? Ans. $12\frac{196}{13305}$.

2228. If 25 horses eat $19\frac{1}{2}$ tons of hay in 27 weeks, how many horses will eat $37\frac{82}{1575}$ tons in $19\frac{1}{7}$ weeks? Ans. 67.

2229. If $137\frac{1}{2}$ gallons of water in one hour run into a tank with a capacity for 1200 gallons, and by a pipe $39\frac{1}{3}$ gallons are discharged, in how many hours will the tank be filled?

2230. If $9\frac{1}{2}$ acres of wheat can be harvested in $3\frac{1}{7}$ days of $9\frac{3}{4}$ hours each, in what time will $13\frac{1}{7}$ acres be harvested, working $8\frac{1}{3}$ hours per day?

CHAPTER V.

PRACTICE.

456. Practice is a short and concise method of solving many questions that occur in trade or business.

457. Although there is a general similarity in the methods of practice, yet each question has its own peculiarity.

458. No specific rules can be given. The student must rely upon his own capability and ingenuity for acquiring facility in this rule.

ILLUSTRATION 1.—If corn is 50 cents per bushel, what will 75 bushels cost?

Evidently, ½ as many dollars as there are bushels.

$$\begin{array}{r} 2\overline{)75} \\ \hline 37.50 \text{ Ans.} \end{array}$$

ILLUSTRATION 2.—If rye is 85 cents per bushel, what will 117 bushels cost?

If 1 bushel costs 85 cents,

$$\begin{array}{rll} 100 & \text{will cost} \ldots\ldots\ldots & \$85.00 \\ 10 & \text{will cost} \ldots\ldots\ldots & 8.50 \\ 7 & \text{will cost } 85\times 7 = & 5.85 \\ \hline 117 & \text{will cost} \ldots\ldots\ldots & \$99.35 \text{ Ans.} \end{array}$$

ILL. 3.—What is the interest of $150 for 2 yr. 3 mo. 15 da.?

$$\begin{array}{lr} \text{Int. for 2 years} \quad = & \$18.00 \\ \text{Int. for 3 mo.} = (\tfrac{1}{8} \text{ of 2 yr.}) & 2.25 \\ \text{Int. for 15 da.} = (\tfrac{1}{6} \text{ of 3 mo.}) & .375 \\ \hline \text{Int. of \$150 for 2 yr. 3 mo. 15 da.} = & \$20.625 \text{ Ans.} \end{array}$$

---

456. What is Practice? 457. Is there any similarity in the methods of solving examples by Practice? 458. Can any rules be given that will be applicable generally? Why not? Upon what must the pupil depend?

Illustration 4.—What is the value of 12 A. 3 R. 20 rods of land at \$60 per acre?

$$
\begin{array}{lr}
 & \$60 \\
 & \underline{12} \\
\text{Value of 12 acres} \quad = & 720 \\
\text{“} \quad 2\text{ R.} = \frac{1}{2}\text{ acre} \quad = & 30 \\
\text{“} \quad 1\text{ R.} = \frac{1}{2}\text{ of 2 R.} \quad = & 15 \\
\text{“} \quad 20\text{ rds.} = \frac{1}{2}\text{ of 1 R.} = & \underline{7.50} \\
 & \$772.50\text{ Ans.}
\end{array}
$$

Ill. 5.—What will 2 gals. 2 qts. 1 pt. of molasses cost at 50 cents per gallon?

$$
\begin{array}{lr}
\text{Cost of 2 gals.} \quad = & \$1.00 \\
\text{“} \quad 2\text{ qts.} = \frac{1}{4}\text{ of 2 gals.} = & 25 \\
\text{“} \quad 1\text{ pt.} = \frac{1}{4}\text{ of 2 qts.} \quad = & \underline{6\frac{1}{4}} \\
\text{Whole cost} = & \$1.31\frac{1}{4}\text{ Ans.}
\end{array}
$$

Ill. 6.—What will 600 bushels of potatoes cost at $37\frac{1}{2}$ cents per bushel?

If the price were 25 cts., the cost would be $\frac{1}{4}$ of \$600 = \$150

If the price were $12\frac{1}{2}$ c., the cost would be $\frac{1}{2}$ of \$150 = 75

And since the price is 25 + $12\frac{1}{2}$ cts., the cost must be \$225 Ans.

Ill. 7.—What is the cost of 848 yards of cloth at 75 cents per yard?

If the price were 50 cts. per yard, the cost would be $\frac{1}{2}$ of \$848 = ... \$424

If the price were 25 cts., the cost would be $\frac{1}{2}$ of \$424 = 212

And since the price is 50 + 25 cents, the cost must be ... \$636

Ill. 8.—What is the cost of $3\frac{7}{8}$ yards of silk at \$1.25 per yard?

$$
\begin{array}{lr}
\text{The cost of 3 yds. at \$1.25} = & \$3.75 \\
\text{“} \quad \text{“} \quad \frac{1}{2}\text{ yd.} \quad \text{“} \quad = & 62\frac{1}{2} \\
\text{“} \quad \text{“} \quad \frac{1}{4}\text{ yd.} \quad \text{“} \quad = & 31\frac{1}{4} \\
\text{“} \quad \text{“} \quad \frac{1}{8}\text{ yd.} \quad \text{“} \quad = & \underline{15\frac{5}{8}} \\
\text{Whole cost} = & \$4.84\frac{3}{8}
\end{array}
$$

MISCELLANEOUS EXAMPLES.

2231. What is the sum of $\frac{3}{5}$, $\frac{11}{13}$, .27, and .785?

2232. What will 84 ft. 7 in. of hemp rope cost at $12\frac{1}{3}$ cents per foot?

2233. What will 31 tubs of butter cost at $17\frac{3}{7}$ cents per lb., each weighing $53\frac{2}{9}$ lbs.?

2234. Suppose 4 of the above tubs of butter are spoiled in transportation, and are sold at 93 per cent. loss, what must be the price per lb. of the remainder, that 11 per cent. on the cost of the whole may be gained?

2235. A speculator went from Boston to Racine in July, 1858. He was absent 4 weeks. Travelling and other expenses, $198.56. He purchased 20000 bushels of wheat at 15 cents per bushel. Paid 25 cents per bushel for transportation to Rochester. Draft 5 per cent. Pays $6\frac{1}{4}$ cents per bushel for getting ground. Pays $58\frac{1}{3}$ cents per bbl. for getting it barreled up, allowing 5 bushels to the barrel, and 25 cents per bbl. for transportation to Boston. What does his flour cost him?

2236. He sells it for 4.87\frac{1}{2}$ per bbl., on a note of 4 months, without interest. Discounts the note at the Suffolk Bank, and allowing himself 6 per cent. on $15000 taken with him for 2 months, what is his profit?

2237. On a certain subscription paper for the purchase of a church bell is entered $1500, 13 per cent. of which is considered worthless. 2 per cent. is paid for collecting the remainder. The bell fixtures are worth $137.50, and the cost of transportation and other expenses for hanging will be $57.81. How large a bell can be purchased with the remaining money, at 25 cents per pound?

2238. What is the difference between the legal and accurate interest of $7562.31 from August 5th, 1859, to May 30th 1862?

2239. The largest bell on this continent is the one on the Notre Dame Cathedral, Montreal. It was imported from England in 1843, and weighs 29400 lbs. The largest bell in England is the "Great Peter of York," which weighs 26000 lbs. If the diameter of the Montreal bell be 7 feet 6 inches, what will be the diameter of the Great Peter?

2240. How much sand will pass in 15 minutes in an hour-glass, the cone being 4 inches in diameter at the base, 6 inches high, and $\frac{2}{3}$ full?

2241. What is the government tonnage of a double-decked vessel, length 187 feet, width 36 feet, depth 25 feet?

2242. A father left $\frac{5}{13}$ of his estate to his son, $\frac{5}{13}$ of the remainder to his daughter, and the balance to the widow. The difference between the son's and daughter's portion was $7000. What was the widow's portion?

2243. If A. and B. can do a piece of work in 6 days, B. and C. in 8 days, and A. and C. in 10 days, in what time can each do it alone?

2244. How many reams of paper will it take to print an edition of one thousand copies of this Arithmetic, allowing one quire waste to each ream?

2245. If a person put his note of $1000 for 60 days in bank, how much money would he receive thereon, at the usual rate of discount?

2246. For what amount must a note be drawn to obtain $600 from a bank for 60 days?

2247. A person sold a horse worth $150 for $180, taking one half cash, and the balance in a note for 6 months, without interest. How much does he make?

2248. What price must I put on flour that cost me $7.31 per bbl., so that I can afford to fall 6 per cent., yet make 12 per cent., allowing interest on the investment for 60 days?

2249. What will a telegraph wire $\frac{1}{8}$ of an inch in diameter weigh, extending from Washington to Baltimore, a distance of 42 miles?

2250. What will be the surface of the above wire?

2251. If a cask holding six gallons contains 15000 bullets $\frac{1}{2}$ an inch in diameter, how much water will fill the cask?

2252. How many tons of iron would be necessary to lay a railroad track between Philadelphia and Baltimore, distance 98 miles, allowing each rail of twenty feet in length to weigh 340 pounds?

2253. The range of the thermometer each day at 12, M., for the week ending July 10th, 1858, was as follows: 98°, 85°, 90°, 94°, 95°, 96°, 97°. What was its average height?

2254. How many bricks will it take to build a house 24 by 36 feet, 30 feet high, with a back building adjoining 16 by 28 feet, 20 feet high, the walls being 12 inches thick, and the bricks 8 by 4 by 2; there are in the house 24 windows 6 by 3, and two outer doors 8 by 4?

2255. Two men purchased a stick of timber 30 feet long, three feet square at the large end, and one foot square at the other end. At what distance from the small end must the stick be divided, so that each may have one half?

2256. What is the weight of a cast-iron water pipe, bore 12 inches, thickness $\frac{1}{2}$ inch, length 10 feet?

2257. Three men own a stack of hay in the shape of a cone, 20 feet high, and 15 feet diameter at the base. They wish to divide it in such a manner that the second man shall have 10 per cent. more than the first, and the third 10 per cent. more than the second. How many feet must the first take from the top to get his share?

2258. How many feet will be left for the third man's share?

2259. How much ground will a horse feed over that is tied by a halter 30 feet long?

2260. There are two numbers, such that $\frac{3}{5}$ of one exceeds $\frac{5}{7}$ of the other by $107\frac{1}{9}$. What is the ratio of the numbers?

2261. What will pay the balance on a note of $2000, on which are the following endorsements, viz:

Jan. 25th, 1860	$257.82	May 5th, 1862	$487.31
Feb. 3d, 1860	128.93	July 3d, 1862	523.91
June 17th, 1861	97.71	Aug. 5th, 1862	281.27

Date of note, Dec. 12th, 1859. Settlement, Sept. 23d, 1862.

2262. If I purchase tea at 48 cents per lb., pay 7 per cent. for expenses, and allow 8 per cent. for bad debts, what must be my price to gain 17 per cent. on the cost?

2263. How many feet in length is the Atlantic cable, allowing it to be 3000 statute miles long?

2264. The population of the United States, for the year 1840, was 17069454; for 1850, 23191074. What will it probably be in 1860?

2265. Two men carry a weight of 300 lbs. suspended from a bar 3 feet long, 2 inches from the middle. What proportion will each sustain?

2266. Goodyear & Co. send a set of exchange, amounting to $7000, to a commission merchant in London for investment, on a commission of 6 pence per £. What was the amount left for investment, exchange being 9½ per cent. premium?

2267. What is the value of a perpetuity of $1200 per annum?

2268. The County Road Commissioners of Rockland laid out a road across Ethan Allen's farm 3½ rods wide, and ¾ of a mile long, and allowed him $2.50 per rod for fencing, and $37.00 per acre for the land taken. What did he receive for damages?

2269. What will be the expense of filling up 1½ acres of land with gravel 2 feet deep, at 23 cents per cubic yard?

2270. What will it cost to pave Pennsylvania Avenue from the Treasury Building to the Capitol, length 1 mile, width 120 feet, at $5.00 per square yard?

2271. A. and B. owned a stick of mahogany timber 2 feet square at one end, 3 feet at the other, and 12 feet long. How many square feet of veneering, $\frac{1}{16}$ of an inch in thickness, shall A. take off to obtain his half, allowing 45 per ct. waste in cutting?

2272. How long a tether must be attached to a horse that he may graze over $\frac{3}{4}$ of an acre?

2273. What number multiplied by $\frac{7}{8}$ will produce $31\frac{1}{4}$?

2274. What part of $\frac{3}{5}$ is 45?

2275. What is the side of a cubical mound containing 5822 cubic feet?

2276. An army being marshalled with the number of rank and file equal, wanted 35 men to make the last rank full. There were 75 men on file. How many men in the army?

2277. What time between eleven and twelve o'clock will the minute and hour hands be together?

2278. The time past 1 P. M. is equal to $\frac{3}{7}$ of the time until midnight. What is the time?

2279. How long must a board be, $9\frac{1}{4}$ inches wide, to contain two square feet?

2280. Fifteen years after a man's marriage he was twice the age of his wife. At marriage he was three times her age. How old were they when married?

2281. What is the entire surface of a cubical stone containing 389017 cubic feet?

2282. How much sheet iron will it take to make forty feet of stove pipe six inches in diameter, allowing one inch for seaming and two feet for jointing?

2283. How many feet of iron wire, $\frac{1}{16}$ inch in diameter, can be made from a block of iron 8 by 4 by 2 inches?

2284. A. can do a piece of work in 5, B. in 7, and C. in 11 days. How long will it take them all to do it?

2285. Bought 20 dozen eggs at the rate of 3 for 2 cents, and sold them for 20 cents a dozen. What is the per cent. gain?

2286. Reduce $\frac{9523}{10272}$, $\frac{10961}{12204}$, and $\frac{1829}{2240}$ to lowest terms.

2287. Standing on the tower of a church I dropped a bullet, and observed $3\frac{1}{2}$ seconds elapse before it struck the ground. What was my height?

2288. A note of $514.47 was on interest from Aug. 1, 1859, to July 7, 1861. It then amounted to $6000. What was the rate?

2289. A person, after spending $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of his money, finds he has $80 left. How much had he at first?

2290. What is the interest on $11111.11 for 11 yrs. 11 mo. 11 days?

2291. What will it cost to pave a sidewalk 67 feet 5 inches long and 16 feet wide, the curbstone costing $\$.87\frac{1}{2}$ per foot, bricks $9.00 per thousand, and the labor $\$.62\frac{1}{2}$ per square yard, allowing 5 per cent. waste on the bricks?

2292. A person borrows a certain sum of money at 6 per cent., and lends $\frac{1}{2}$ of what he borrowed at 7 per cent., and the other half at 9 per cent. At the end of the year he finds that he has gained $96 by the operation. What was the sum borrowed?

2293. A person uses $\frac{1}{2}$ of a tree for rails, $\frac{1}{4}$ of the remainder for posts, then finds he has 60 feet left. What was the height of the tree?

2294. A owns $\frac{1}{2}$ of a ship, and sells $\frac{3}{4}$ of his share to B. B. sells $\frac{1}{2}$ of his part to C. What is C.'s part worth, if the whole ship is worth $60000?

2295. If A. can do a piece of work in $6\frac{3}{5}$ days, B. in $9\frac{2}{7}$ days, and C. in $15\frac{1}{3}$ days, in what time can they do it if all work together?

2296. A.'s age is double that of B.'s, and B.'s is three times that of C.'s. The sum of their ages is 140 years. What is the age of each?

2297. How many more acres will 36 miles perimeter enclose in the form of a circle than in the form of a square?

2298. How many acres if in the form of a hexagon?

2299. What would it cost to dig a canal from Baltimore to Washington, 36 miles, 20 feet wide and 6 feet deep, at 12 cents per cubic foot?

2300. On what day of the week will the 4th of July, 1866, fall?

2301. From a cone 10 inches high, and base 6 inches, was cut a sphere 4 inches in diameter. What was the ratio between the parts?

2302. What was the length of the string with which Queen Dido surrounded the site of Carthage, the area being a circular "Hide"?

2304. Of what width was it necessary to cut the string, the ox hide being 5 feet square?

2305. An hour-glass, with a cone 3 inches in diameter at the base, 4 inches high, and $\frac{3}{4}$ filled with sand, was set at 12 o'clock. What was the height of the sand in the upper cone at 12 o'clock and 20 minutes?

2306. If a four-cent loaf weigh 8 ounces when flour is \$8.37½ per bbl., what ought it to weigh when flour is \$6.00 per bbl., allowing 1 cent for making?

2307. On a circular railway, 60 rods in diameter within the inner rail, is a car, with its wheels firmly attached to an axletree 8 feet 3 inches long. What must be the diameter of the outer wheel, the inner one being 3 feet, that the car may run without difficulty?

2308. What must be the dimensions of a cylindrical cistern 10 feet deep to contain 6000 gallons of water?

2309. Required the proof of the last example.

2310. A bank failing, assigns its property to a trustee, and after two months he agrees to pay the creditors in instalments of 10 per cent. each, without interest, as often as he can collect sufficient funds, till all the assets are paid out. In three months more he makes the first payment of 10 per cent. Twelve months after he made a second payment. Two months after this, A., whose original claim was \$7000, sells his demand for 37½ cents on a dollar, and takes in payment a note at 4 months. In two months he finds this note not bankable, and sells it for 70 per cent., taking pay in *coin*. What per cent. of the original debt did he realize, allowing interest?

2311. A man bought a horse and gave his note, payable in 6 months, with interest. Afterwards he sold the horse, taking a note for 6 months, with interest, and finds the note he got for the horse will amount to \$208, which is a gain of 30 per cent. *on the amount* of the first note. What was its face?

2312. A Loan and Fund Association guaranteed $500 to a shareholder at the expiration of 8 years, for the monthly payment of $3.00 per share. Charles Johnson took 4 shares, and held them 5 years, paying his monthly dues. He then drew a loan from the association of $1200.00 on his whole stock, and made an additional payment of $6 per month as interest. What did the loan cost on the day of purchase?

2313. An hour-glass, with a cone 8 inches high, 6 inches in diameter at the base, and $\frac{3}{5}$ full of sand, was set at 1 o'clock. What time was it when the sand had fallen 1 inch?

2314. How many gallons in a cistern 16 feet long, 11 feet wide, and 13 feet deep?

2315. How many cubic feet in the hold of a vessel that will contain 3237 bushels of corn?

2316. Bought of Baring & Brothers a set of exchange dated Sept. 29th, 1859, amounting to £4000 6*s.* 6*d.*, and drawing 4 per cent. interest. What is it worth in New York Aug. 12th, 1861, exchange being 11¼ per cent.?

NOTE.—The English compute accurate interest.

2317. What is the length of the side of an equilateral triangle, whose area is just one acre?

2318. Required the number of revolutions a log 12 feet in length would perform in describing a circle, diameters of its ends 2 and 3 feet, respectively.

2319. How many bushels of wheat must a person take to mill to obtain 6000 pounds of flour, the miller taking $\frac{1}{16}$ for toll and $\frac{1}{10}$ being bran?

2320. The solid granite columns of the Treasury Building, Washington, D. C., are 31 feet 7½ inches high, 4 feet 6 inches diameter at the base, and 3 feet 6 inches diameter at the top. What would be the weight of 80 such columns?

2321. The G. G. string of a piano consists of a steel wire $\frac{1}{16}$ of an inch in diameter, wound with a wire $\frac{1}{40}$ of an inch in diameter. How many feet of covering wire will wind a G. G. string 5 feet in length?

2322. A, B, and C made a contract to sprinkle 4½ miles of a street in a city, during 7 months, for $500. They were to distribute, daily, 1000 gallons of water equally over the street. They then agreed to divide the contract, A taking the first third, B the second, and C the third; and, in consideration of there being a hill in the middle of B's section, 6 per cent was to be allowed on all carried beyond the first third of that section. They obtained their water ½ a mile from A's section, so that B hauled his water over A's, and C hauled his over A's and B's sections. They further agreed to divide the amount received in proportion to the number of hundred weight actually carried 1 rod. What was each man's share?

2323. Zalmon Brewster agreed to sink a shaft for the Willamette Mining Company, 8 feet in diameter and 100 feet deep, for $4000. The material brought out was to be deposited in a ring around the mouth of the shaft 3 feet deep, commencing 12 feet from the centre. It was estimated to cost 10 cents per cubic foot for digging gravel, and 50 cents for stone, and 1 cent per cubic foot for every foot the material was carried from the mouth of the shaft. After digging 70 feet the Company deemed it advisable to stop the labor, and agreed to pay the contractor 5 per cent. advance on what was then actually due to relinquish his contract. What did the shaft cost the Company, 30 feet being gravel and the remaining distance rock?

THE END.

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